B3.2-R3: BASIC MATHEMATICS

NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) If $\vec{a} = 2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$, then find a vector of magnitude 4 perpendicular to both $\vec{a}$ and $\vec{b}$.

b) Find $\lim_{x \to 0} \frac{3^x - 2^x}{x}$

c) Find the value of the determinant

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$$

d) Evaluate $\int \frac{dx}{x(x^4 + 1)}$

e) Find the area of region bounded by the parabola $y^2=4x$ and its latus rectum.

f) Find the equation of the tangent to the curve $x=\cos\theta$ and $y=\sin\theta$ at $\theta = \frac{\Pi}{4}$.

g) Test the convergence/divergence of the series: $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(7x4)

2.

a) Find the value of the constant p so that the vectors $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}, \vec{b} = \vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{c} = 3\vec{i} + p\vec{j} + 5\vec{k}$ are coplanar.

b) Using DeMoivre's theorem, find the value of $\left(1 + \cos\frac{\Pi}{3} + i\sin\frac{\Pi}{3}\right)^6$.

c) Reduce the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -6 & 2 & -4 \\ -3 & 1 & -2 \end{bmatrix}$ to echelon form and hence find its rank.

(6+6+6)

3.

a) Examine the continuity of the following function at $x = 0$

$$f(x) = \begin{cases} \frac{\cos 4x - \cos 2x}{x^2} & \text{, if } x \neq 0 \\ 5 & \text{, if } x = 0, \end{cases}$$

b) If $y=\log(1+\cos x)$, and $2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) + \frac{d^3y}{dx^3} = c$, determine the value of c.

d) Solve the following system of equations by Cramer's rule:

$$2x + y + z = 7$$
$$3x - y - z = -2$$
$$x + 2y - 3z = -4$$

(6+6+6)

4.

a) If $\int_2^5 f(x)dx = 7$ and $\int_4^5 f(x)dx = 3$, find $\int_2^4 f(x)dx$.

b) Examine the convergence and absolute convergence of the series: $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$.

c) Find the points of local maxima or local minima, if any, of the function
$f(x) = \sin 2x - x$, $\frac{-\Pi}{2} \le x \le \frac{\Pi}{2}$.

d) Evaluate $\int_1^2 |2x - 3|dx$.

(3+6+6+3)

5.

a) Obtain the asymptotes of the curve: $(x+y)^2=xy^2$.

b) Find the vertex and focus of the parabola
$y^2-4y-2x-8=0$

c) Obtain the Taylor's series for the function $f(x) = \cos 2x$ at x=0.

d) Draw the graph of the function

$$f(x)=\begin{cases} x, & \text{when } o\le x<½ \\ 1, & \text{when } x=½ \\ 1-x, & \text{when } ½<x\le 1. \end{cases}$$

Is $f(x)$ continuous at $x = \frac{1}{2}$?

(6+4+4+4)

6.

a) i) Examine the validity of the Rolle's theorem for the function
$f(x) = \cos x$ in $\left[0, \frac{\Pi}{2}\right]$.

ii) Verify Lagrange's mean value theorem for the function
$f(x) = \sqrt{x^2 - 4}$ in [2, 4].

b) Evaluate $\int_0^2 (x\sqrt{x+2})\,dx$.

c) Find the equation of the circle which touches y-axis and whose center is (1, 2).

d) If $\vec{a} = 3\vec{i} - \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} + 3\vec{j} + \vec{k}$, then find $\left|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})\right|$

(6+4+4+4)

7.

a) Classify the following Conics in terms of parabola, ellipse or hyperbola:
i) $x^2-3xy+y^2+10x-10y+21=0$
ii) $22x^2-12xy+17y^2-112x+92y+178=0$

b) Find the values of x and y if $A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} x & y \\ 1 & 2 \end{pmatrix}$ and $(A+B)^2=A^2+B^2$.

c) Find $\frac{dy}{dx}$, when $y = \frac{x(x^2 + 4)^{1/3}}{(x^2 + 3)^{1/2}}$.

d) Show that the following system of equations has infinite number of solutions:
$x-2y+3z=0$, $2x+4y+z=0$, $3x+2y+4z=0$

(6+4+4+4)