B3.2-R3 BASIC MATHEMATICS

NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) If $\frac{(1+i)(1+\sqrt{3}\,i)}{1-i} = 2(\cos\theta + i\sin\theta)$, then find the value of θ .

b) If **A** and **B** are symmetric matrices, then show that **AB** – **BA** is skew-symmetric matrix.

c) If *w* (#1) is a cube root of unity, then show that

$A = w + w^{(1/2+3/8+9/32+\ldots\ldots\infty)}$

is real. Hence, find the value of A.

d) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^3}{2^n}$.

e) Evaluate the integral $I = \int_0^2 \frac{3x^2}{(x^2+1)(x^2+4)}\,dx$

f) Find the coefficient of x^2 in the binomial expansion of $\left(x - \frac{1}{2x}\right)^8$

g) Let $|\mathbf{a}| = 7,\ |\mathbf{b}| = 2,\ \mathbf{a}\times\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$. Find the acute angle between the vectors **a** and **b.**

(7x4)

2.

a) Find the inverse of the matrix

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

using Gauss-Jordan elimination method.

b) Find all the eigen values of the matrix $\mathbf{B} = 2\mathbf{I} + 3\mathbf{A} - \mathbf{A}^2$, where **I** is an identity matrix of order 3 and

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$$

c) Using the concept of rank, find the values of *a* and *b* for which the system of equations

$$3x - y + 2z = 3$$
$$2x + y + 3z = 5$$
$$x - 2y + az = b$$

has no solution.

(6+6+6)

3.

a) Show that

$$\tan\left[i \log_e \left(\frac{a - ib}{a + ib} \right) \right] = \frac{2ab}{a^2 - b^2} .$$

b) Find the value of x, when

$$\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - .....\infty \right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - .....\infty \right) = \frac{\pi}{2} .$$

c) Find all the asymptotes to the curve

$$y\sqrt{x^2 - 4} = x^2 .$$

(6+6+6)

4.

a) Find the limit

$$\lim_{x \to 0} \frac{\cos(2x^3) - 1}{\sin^6(2x)} .$$

b) Find the domain of the function

$$f(x) = \sqrt{\sin^{-1}(2x) + \pi / 6} .$$

c) The function $y = a \cos x + b \tan x + x$ has extreme values at $x = 0$ and $x = \pi / 6$. Find the values of a and b.

d) Find $\frac{dy}{dx}$ when $y = (\sin x)^{\cos^{-1} x}$.

(4+4+4+6)

5.

a) Show that $\int_0^{\pi} f(\sin x)dx = 2\int_0^{\pi/2} f(\sin x)dx$.

b) Evaluate the integral

$$I = \lim_{x \to 0} \left[\frac{\int_0^x \sin^3 t dt}{x^4} \right]$$

c) The area of the region bounded by the curves $y = x - x^2$ and $y = mx$ equals 9/2. Find the value of m.

(6+6+6)

6.

a) Find the value of p, for which the equation

$$px^2 + xy + y^2 - 5x - y + p = 0$$

represents a pair of straight lines.

b) Write the equation

$$4x^2 + 9y^2 - 32x + 54y + 109 = 0$$

in standard form of the equation of the ellipse. Hence, determine the eccentricity and the coordinates of foci.

c) Find the product of the perpendicular distances from the foci to a tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(6+6+6)

7.

a) Find the sum of the series

$$\frac{3}{1^2} + \frac{5}{1^2 + 2^2} + \frac{7}{1^2 + 2^2 + 3^2} + \ldots\ldots\ldots\ldots \text{upto } \infty$$

b) Find a unit vector **a** which is horizontal and perpendicular to the vector **b** = 4**i** – 3**j** + 7**k**.

c) Two vectors **a** = 2**i** – 2**j** + **k** and **b** = 2**i** + **j** – **k** are given. Write the vector **b** as sum of vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ such that $\mathbf{b}_1$ is parallel to **a** and $\mathbf{b}_2$ is perpendicular to **a**. Find the vectors $\mathbf{b}_1$ and $\mathbf{b}_2$.

(5+5+8)