NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) If $\mathbf{a} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{c} = -\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, then determine the vector $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$.

b) Express $z = \dfrac{\sqrt{2} + \sqrt{6}\,i}{(\sqrt{3} + i)(\sqrt{2} - \sqrt{6}\,i)}$ in the form $x + iy$ where x and y are real numbers

c) Find the characteristic roots of the matrix

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

d) Find the area of the region bounded by $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and above the x-axis.

e) Find $\lim\limits_{x \to \infty} \left(\dfrac{x+5}{x+1} \right)^{x+3}$

f) Test the convergence of the series

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots$$

g) Find the equation of the hyperbola whose eccentricity is $\sqrt{2}$ and distance between foci is 16.

(7x4)

2.

a) Find a vector **b** such that $\mathbf{a} \cdot \mathbf{b} = 1$, **a** x **b = j – k** and **a = i + j + k**.

b) Given

$$\begin{vmatrix} b^2 + c^2 & ab & ac \\ ba & c^2 + a^2 & bc \\ ca & cb & b^2 + a^2 \end{vmatrix} = \begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix} = Ka^2b^2c^2,$$

find K.

c) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, then find the eigen values of $\mathbf{B = I + A + A^2}$, where $\mathbf{I}$ is the identity matrix of order 3.

(6+6+6)

3.

a) If 1, w, w^2 are cube roots of unity, then find the roots of $(z - 1)^3 + 8 = 0$.

b) Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 1 & 5 \\ -1 & 2 & 5 \\ 3 & 2 & 9 \end{bmatrix}$$

c) If one of the roots of

$$A(x) = \begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 0$$

is x = -9, then find the other roots.

(6+6+6)

4.

a) How will you classify the following curves in terms of parabola, ellipse or hyperbola?
i) $17x^2+12xy+8y^2-46x-28y+33=0$,
ii) $x^2-5xy+y^2+8x-20y+15=0$?

b) Put the equation $9x^2+25y^2=225$ in the standard form and sketch it. Clearly indicate center, vertices, foci, distance between the center and the focus.

c) Find the equation of the tangent to a parabola $y^2=4ax$ at the given point (x_1,y_1) on it. What can you say about the tangent at (0,0)?

(6+6+6)

5.

a) Evaluate $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$.

b) Find the length of the portion of the tangent to the curve x = a $\cos^3\theta$, y = a $\sin^3\theta$ intercepted between coordinate axes.

c) Evaluate $\lim_{x \to 0} (Cos\, x)^{\frac{1}{x^2}}$.

(6+6+6)

6.

a) Evaluate $\int_{-1}^{1} \frac{3+\sin^3 x}{1+x^2} dx$.

b) Find the minimum value of
$f(x) = \sin^4 x + \cos^4 x$.

c) Find all the asymptotes to the curve
$(2x + 3)y = (x - 1)^2$.

(6+6+6)

7.

a) If $x = \cos\theta + \theta \sin\theta$, $y = \sin\theta - \theta \cos\theta$, then show that

$$\frac{dy}{dx} = \tan\theta .$$

Also find $\frac{d^2y}{dx^2}$.

b) Using Lagrange mean value theorem show that $e^x > 1+x,\ x > 0.$

c) Test the convergence of the series

$$\frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots$$

for all x.

(6+6+6)