B3.2-R3: BASIC MATHEMATICS

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) If $z = \frac{\sqrt{3}+i}{2}$, find the value of z^{69}.

b) Given that $\int_o^\infty e^{-x^2}\,dx = \sqrt{\pi}/2$, find $\int_o^\infty x^{-\frac{1}{2}} e^{-2}\,dx$.

c) If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, find α, β so that $(\alpha I + \beta A)^2 = A$.

d) Find $\lim_{x\to\infty} \left(\frac{x+2}{x+1}\right)^{x+3}$.

e) If w is an imaginary cube root of unity, then find the value of

$$\begin{vmatrix} 1+w & w^2 & -w \\ 1+w^2 & w & -w^2 \\ w^2+w & w & -w^2 \end{vmatrix}.$$

f) Test the convergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots$ to ∞. It is given that $x^2 < 1$.

g) Find the projection of vector $5\hat{i} + 2\hat{j} + 3\hat{k}$ along $2\hat{i} + \hat{j} + \hat{k}$.

(7x4)

2.

a) Show that the system of equation x - 3y - 8z = -10; 3x + y - 4z = 0 and 2x + 5y + 6z = 13 is consistent and find the solution.

b) Find the characteristic roots and characteristic vectors of the matrix A given by $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$.

c) Find the vertex, focus and directrix of the conic section $4y^2 + 12x - 12y + 39 = 0$.

(6+6+6)

3.

a) Find the equation and length of common chord of the circles

$2x^2 + 2y^2 + 7x - 5y + 2 = 0$

and $x^2 + y^2 - 4x + 8y - 18 = 0$.

b) Find the locus of the intersection of the lines

$$\frac{x}{3}-\frac{y}{2}=m$$

and $\frac{x}{3}+\frac{y}{2}=m^{-1}$,

if m is variable.

c) Find $\frac{dy}{dx}$, if

$$y=\frac{(x+2)^3(3x+5)^{-4}\sin x}{(2x+2)^2}.$$

(6+6+6)

4.

a) Find the value of $\int_{\pi/4}^{3\pi/4}\frac{1}{1+\cos x}\,dx$.

b) $\int_{-\pi/2}^{\pi/2}\sin^2 x\ \cos^2 x\,(\sin x+\cos x)\,dx$.

c) Find the area of the region

$\{(x,y): x^2 \le y \le |x|\}$.

(6+6+6)

5.

a) Normal is drawn at a variable point P of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Find the maximum distance of the normal from the centre of the ellipse.

b) Find the asymptote of the curve $x^3 + 4x^2y + 4xy^2 + 5x^2 + 15xy + 10y^2 - 2y + 1 = 0$.

c) Integrate $\int_{-1}^{1}|1-x|\,dx$.

(6+6+6)

6.

a) If sin y = x sin (a+y) and $\frac{dy}{dx}=K\frac{\sin^2 y}{x^2}$, find K.

b) Using mean value theorem, prove that tan x > x for all $x \in \left(0,\frac{\pi}{2}\right)$.

c) Show that the line $5x + 3y + \lambda x = 2\lambda y - 6$ always passes from a fixed point. Determine the coordinate of the point.

(6+6+6)

7.

a) Discuss the position of point (1,2) and (6,0) with respect to the circle $x^2 + y^2 - 4x + 2y - 11 = 0$.

b) If $x(1+y)^{1/2} + y(1+x)^{1/2} = 0$, determine $\frac{dy}{dx}$ at $x = 1$.

c) Find the sum of the following series as $n \to \infty$

$$\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \ldots + \frac{n}{n^2+n^2}.$$

(6+6+6)