B3.2-R3: BASIC MATHEMATICS

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Find the real values of *x* and *y* such that $\frac{(1+i)x-i}{1-i}+\frac{(1-2i)y+i}{1+i}=\frac{1-7i}{2}$.

b) Express the matrix $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

c) Test whether the function $f(x)=\sin\{\pi(x-[x])\}$ is differentiable at *x* = 1, where [*x*] is the greatest integer function. (Note: [x]=1 for x > 1 and = 0 for x < 1.)

d) Using the formula $\int_0^{2a} g(x)dx=\int_0^{2a} g(2a-x)dx$, evaluate the integral $\int_0^{2a}\frac{f(x)dx}{f(x)+f(2a-x)}$.

e) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{n^3}{2^n}$.

f) Find the equation of the straight line passing through (2, -6) and the point of intersection of the lines $5x - 2y + 14 = 0$ and $2y = 8 - 7x$.

g) If $|\underline{a}|=2, |\underline{b}|=7$ and $\underline{a}\times\underline{b}=3\hat{i}+2\hat{j}+6\hat{k}$, find the angle between $\underline{a}$ and $\underline{b}$.

(7x 4)

2.

a) A matrix C is orthogonal if $C^{-1} = C^T$. Is the matrix $C=\begin{bmatrix} -1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & \sqrt{2/3} \\ 1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{6} \end{bmatrix}$ orthogonal?

b) Solve the system of equations:

$$\begin{aligned} 3x + y + z &= 1 \\ x - 3y + z &= -1 \\ 2x + 2y + 7z &= 10 \end{aligned}$$

by Gauss elimination method.

c) If $\sin y = x\sin(a+y)$, then $\frac{dy}{dx}=A\sin^2(a+y)$. Find A.

(6+6+6)

3.

a) The volume of a cube is increasing at the rate of 7cm^3/sec. How fast is the surface area increasing when the length of the edge is 12 cm?

b) Find the sides of the rectangle of greatest area that can be inscribed in the ellipse $x^2+9y^2=81$.

c) Evaluate the definite integral $\int_{0}^{\pi/2} \frac{\cos^2 x \, dx}{\sin x + \cos x}$.

(6+6+6)

4.

a) Find all solution of the trigonometric equation $5 \sin x + 12 \cos x = 13$.

b) Discuss the convergence of the series

$$1 + \frac{x}{2} + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \frac{4!}{5^4}x^4 + \ldots, \ (x \neq e).$$

c) Find the equations of the tangent and normal to the ellipse $4x^2 + 36y^2 - 4x + 24y + 1 = 0$ at the point $(1/2, -2/3)$.

(6+6+6)

5.

a) Find the area of the region bounded by $y = x(x-1)(x-3)$, $x = 0, x = 3$ and the *x*–axis.

b) Find the equation of a circle of radius 10 and concentric with the circle $2x^2 + 2y^2 + 20x + 10y - 20 = 0$.

c) Let $\vec{a} = i + 2j - k, \vec{b} = 2i + j + k, \vec{c} = 6i + j - k$. Determine a vector $\vec{p}$ such that $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \bullet \vec{a} = 0$.

(6+6+6)

6.

a) Find the locus of *z, g*iven that $\left| \frac{1 - iz}{z - i} \right| = 1$.

b) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$.

c) Use Lagrange mean value theorem to determine a point *P* on the curve $y = \sqrt{x - 1}$ defined on the interval [1, 2], where the tangent is parallel to the chord joining the end points of the curve.

(6+6+6)

7.

a) Test the convergence of the series

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \ldots$$

b) Find whether the points (2, 2) and (5, 3) lie inside or outside the hyperbola $4x^2 - 9y^2 - 8x + 36y - 68 = 0$.

c) Find the area of the region bounded by the curves $y \geq x^2$ and $y = |x|$.

(6+6+6)