KINGS
COLLEGE OF ENGINEERING

DEPARTMENT OF MATHEMATICS
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QUESTION BANK

SUBJECT NAME: MA1253 - PROBABILITY AND RANDOM PROCESSES
YEAR/SEM: II/IV

UNIT – I
RANDOM VARIABLE

PART A(2 Marks)

1.Define random Variable.

2.If the random variable X has the following probability distribution

X :	-2	-1	0	1
P(x):	0.4	K	0.2	0.3

Find k and the mean of x.

3. Let X be a continuous random variable with pdf f(x)= $f(x)=\begin{cases}3x, & if\ \ 0<x<1\\ 0 & , \ iotherwise\end{cases}$

find p(x≤0.6)

4. A random variable x has p.d.f f(x) given by $f(x)=\begin{cases}cxe^{-x}, & if\ \ x>0\\ 0 & , \ if\ \ x\le o\end{cases}$

Find the value of c and C.D.F of x.

5. Is the function defined as follows a density by function?

$$f(x)=\begin{cases}0 & for\ \ x<2\\ \frac{1}{18}(3+2x) & for\ \ 2\le x\le 4\\ 0 & for\ \ x>4.\end{cases}$$

6. The first four moments of a distribution about x=4 are 1,4,10 and 45 respectively. Show that the mean is 5, variance is 3, $\mu_3=0\ and\ \mu_4=26$.

7. Define moment generating function and write the formula to find mean and variance

8. Find the M.G.F of the random variable X having the p.d.f f(x)=$\frac{1}{4}$, -2≤x≤2

9. Find the moment generating function of binomial distribution

10. The mean and variance of the binomial distribution are 4 and 3 respectively. Find P(X=0)

11. State any two instances where poisson distribution may be successfully employed.

12. In which probability distribution, variance and mean are equal.

13. If X and Y are independent poisson variates such that p[X = 1]=p[X=2] and p[Y=2] = P[Y=3] find v[X-2Y].

14. Write the moment generating function of Geometric distribution
15. Show that the uniform distribution $f(x) = \frac{1}{2a}, -a < x < a$, the moment generating function about origin is $\frac{\sinh at}{at}$
16. If X is uniformly distributed over (0,10) calculate the probability that 3<x<8.
17. Obtain the mean and variance of the Exponential distribution with p.d.f f(x)=λ $e^{-\frac{x}{5}}$,x>0
18. Define generalized form of the gamma distribution.
19. Write two characteristics of the Normal distribution.
20. If the probability density function of X is $f_x(x)$=2x, 0<x<1, find the probability density function of Y=3X+1.

PART – B(16 Marks)

1.a) A random variable x has the following distribution

x:	-2	-1	0	1	2	3
P(x):	0.1	k	0.2	2k	0.3	3k

(i) Evaluate k.
(ii) Evaluate P(-2<x<3).
(iii) Find the cumulative distribution function of x. **(8)**

b) A continuous random variable has the p.d.f

f(x)= kx^4, -1<x<0

Find the value of k and $P\left[x > \frac{-1}{2} \mid x < \frac{-1}{4}\right]$ **(8)**

2.a) Find the moment generating function of a random variable x having the probability density function $f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}}, & x > 0 \\ 0 & , otherwise \end{cases}$

and hence find the mean and variance. **(8)**

b) A continuous random variable X has the p.d.f f(x)= Kx^2e^{-x} , x≥0. **(8)**
Find the r^{th} moment of X about the origin.Hence find mean and variance of X.

3.a) For Binomial distribution mean is 6 and standard deviating is $\sqrt{2}$. Find the first two terms of the distribution. **(8)**

b) Derive mean and variance of binomial distribution. **(8)**

4.a) Obtain the MGF of Poisson distribution and hence compute the first four moments. **(8)**

b) Derive MGF of Poisson distribution and hence find mean and variance. **(8)**

5.a) Prove the memory less property of the Geometric distribution. **(8)**

b) A die is cast until 6 appears. What is the probability that it must be cast more than 5 times? **(8)**

6.a) If X is uniformly distributed is [-2,2], find (i) P(x<0) and (ii) $P\left(|X-1| \geq \frac{1}{2}\right)$ **(8)**

b) If X is uniformly distributed with E(x)=1 and Var(x)=4/3, find P(X<0) **(8)**

7.a) Obtain the MGF of exponential distribution and hence compute the first four moments. **(8)**

b) The daily consumption of milk in excess of 20,000 gallons is approximately exponential with θ = 3000.The city has a daily stock of 35,000 gallons. What is the probability that of two days selected at random, the stock is insufficient for both days. **(8)**

8.a) Define Gamma distribution and find the mean and variance of the same. **(8)**

b) The daily consumption of milk in a city, in excess of 20,000 L is approximately distributed as a Gamma variate with the parameters α=2 and $\lambda = \frac{1}{10,000}$.The city has a daily stock of 30,000 litres. What is the probability that the stock is insufficient on a particular day? **(8)**

9.a) State and explain the properties of Normal $N(\mu,\sigma^2)$ distribution (**8)**

b) For a normal distribution with mean 2 and variance 9, find the value of x_1, of the variable such that the probability of the variable lying in the interval $(2,x_1)$ is 0.4115 **(8)**

10.a) If X is a random variable following normal distribution with μ=1,σ=2.Find

P[$|X-2| < \frac{1}{2}$, X>0]. **(8)**

b) Let X and Y be independent random variables with common p.d.f $f_X(x) = e^{-2x}$, x>0. Find the joint pdf of U=X+Y and $V=e^x$ **(8)**

UNIT - II

TWO DIMENSIONAL RANDOM VARIABLES

PART-A(2 Marks)

1. The joint probability density function of the random variable (X,Y) is given by f(x,y)=Kxy $e^{-(x^2+y^2)}$ x>0, y>0. Find the value of K.
2. The joint probability density function of two random variables given by $f_{xy}(x.y)$ = x(x-y)/8, 0<x<2; -x<y<x and find $f_{y/x}(y/x)$
3. If X and Y are random variables having the joint density function f(x,y) = (6-x-y)/8, 0<x<2; 2<y<4,find P(X+Y<3).
4. Define joint distributions of two random variables X and Y and state its properties.
5. If two random variables X and Y have p.d.f f(x,y) = k(2x+y) for 0≤x≤2, 0≤y≤3, evaluate k.
6. Find the marginal density functions of X and Y if

$$f(x,y) = \begin{cases} \frac{2}{5}(2x+5y) & for\ \ 0 \le x \le 1 \\ 0 & for\ \ 0 \le y \le 1 \end{cases}$$

7. Find the marginal p.d.f of X and Y for the joint pdf of a bivariate random variable ,given f(x,y) =

$$\begin{cases} \frac{1}{8}(x+y) & for\ \ 0 \le x \le 2, 0 < y < 2 \\ 0 & otherwise \end{cases}$$

8. If Y=-2x+3, find the cov(x,y).
9. Show that $Cov^2(X,Y) \le Var(X).Var(Y)$
10. Prove that if one of the regression coefficient is greater than unity then the other must be less than unity.
11. Prove that the correlation coefficient P_{xy} takes value in the range -1 to 1.

12. Distinguish between correlation and regression.
13. Find the acute angle between the two lines of regression.
14. The regression equations of two variables x and y are 3x+y=10 and 3x+4y=12 find the coefficient of correlation between them.
15. The two regression lines are x+6y=4, 2x+3y =1. Find the mean values of x and y.
16. If x and y are independent random variable with variance 2 and 3. Find the variance of 3x+4y..
17. State central limit theorem.
18. State central limit theorem in Liapunoff's form.
19. State central limit theorem in Lindberg-Levy's form.
20. Write the applications of central limit theorem.

PART-B(16 Marks)

1. a) Given is the joint distribution X and Y

		X		
		0	1	2
Y	0	0.02	0.08	0.10
	1	0.05	0.20	0.25
	2	0.03	0.12	0.15

Obtain (i) Marginal Distributions and
(ii) The Conditional Distribution of X given Y=0. **(8)**

b) The joint probability mass function of X and Y is given by

P(x,y)		0	1	2
	0	0.1	0.04	0.02
X	1	0.08	0.20	0.06
	2	0.06	0.14	0.30

Compute the marginal PMF of X and Y, $P[x \le 1, y \le 1]$ and check if X and Y are independent. **(8)**

2.a). If the joint probability density function of a two dimensional random variable (X,Y) is given

by $f(x,y) = x^2 + \frac{xy}{3}$, $0<x<1, 0<y<2$

$= 0$, elsewhere

Find

(i) $P(X>1/2)$

(ii) $P(Y>X)$ and

(iii) $P(Y<1/2 / X<1/2)$.

(iv) $P(Y<1)$

(v) Find the conditional density functions **(8)**

b) The joint probability mass function of (X,Y) is given by $p(x,y) = K(2x+3y)$, x = 0,1,2; y =1,2,3. Find all the marginal and conditional probability distributions. **(8)**

3.a) The joint probability density function of the two dimensional random variable is

$f(x,y) = \frac{8}{9}xy$, $1<x<y<2$

= 0 , otherwise **(8)**

(i)Find the marginal density functions of X and Y

(ii) Find the conditional density function of Y given X

b)The joint pdf of a bivariate random variable (X,Y) is given by

$f(x,y)= kxy, 0<x<1, 0<y<1$

= 0 , otherwise

Find (i) k (ii) $P(X+Y<1)$ (iii) Are X and Y independent **(8)**

4. a) Two random variable X and Y have the joint density

$f(x,y) = 2-x-y$; $0<x<1$, $0<y<1$

= 0, otherwise.

Show that $Cov(X,Y) = -1/11$. **(8)**

b) Suppose the joint probability density function is given by

$f(x,y) = \frac{6}{5}(x+y^2)$ $0\le x\le 1$, $0\le y\le 1$

= 0 , otherwise.

Obtain the marginal PDF of X and that of Y.

Hence or otherwise find $P[1/4\le y\le 3/4]$ **(8)**

5. a) Two dimensional random variable (X,Y) has the joint PDF

$f(x,y) =2, 0<y<x<1$; 0 otherwise. Find

(i) marginal and conditional distributions

(ii) joint distribution function F(x,y)

(iii) Check whether X and Y are independent.

(iv) $P(X<1/2/Y<1/4)$ **(8)**

b) Two dimensional random variable (X,Y) has the joint PDF

$f(x,y) = 8xy, 0<x<y<1$; 0 otherwise. Find

(i) marginal and conditional distributions

(ii) Check whether X and Y are independent **(8)**

6. a)Given $f_{xy}(x,y) = cx(x-y)$, $0<x<2$, $-x<y<x$

= 0, otherwise

(1) Evaluate C

(2) Find $f_x(x)$

(3) $f_{y/x}(y/x)$ and

(4) $f_y(y)$ **(8)**

b) If the equations of the two lines of regression of y on x and x on y are respectively $7x-16y+9=0$; $5y-4x-3=0$, calculate the coefficient of correlation. **(8)**

7 a) Two random variables X and Y are defined as $Y = 4X+9$. Find the coefficient of correlation between X and Y. **(8)**

b) Calculate the correlation coefficient for the following heights(in inches) of fathers X and their sons Y. **(8)**

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

8 a) Two random variables X and Y have the joint pdf

$$f(x,y) = \begin{cases} k(4-x-y), & 0\le x\le 2,\ 0\le y\le 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the correlation coefficient between X and Y. **(8)**

b) From the following data, find **(8)**

I. The two regression equations

II. The coefficient of correlation between the marks in mathematics and statistics

III. The most likely marks in statistics when marks in mathematics are 30.

Marks in Mathematics:	25	28	35	32	31	36	29	38	34	32
Marks in Statistics:	43	46	49	41	36	32	31	30	33	39

9 a) Let (X,Y) be a two-dimensional non-negative continuous random variable having the joint density f(x,y) = $\begin{cases} 4xye^{-(x^2+y^2)}, x \ge 0, y \ge 0 \\ 0, elsewhere \end{cases}$

Find the density function of U=$\sqrt{(X^2+Y^2)}$ **(8)**

b) A distribution with unknown mean μ has variance equal to 1.5. Use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be atleast 0.95 that the sample mean will be within 0.5 of the population mean. **(8)**

10. a) A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using Central limit theorem , with what probability can we assert that the mean of the sample will not differ from μ = 60 by more than 4? **(8)**

b) The joint p.d.f of X and Y is given by f(x,y) = $e^{-(x+y)}$ x>0,y>0,find the probability density function of U = (X+Y)/2. **(8)**

UNIT – III
CLASSIFICATION OF RANDOM PROCESSES

PART A(2 Marks)

1.State the four types of stochastic processes .

2. {X(s,t)} is a random process , what is the nature of X(s,t) when (a) s is fixed (b) t is fixed?

3. Define strict sense stationary process .

4. Consider the random process X(t) = cos (t+ϕ),where ϕ is uniformly distributed in($-\pi/2,\pi/2$). Check whether the process is stationary?

. .

5.Define wide sense stationary process.

6. State Chapman-kolmogorov theorem.

7. When is a stochastic process said to be ergodic?

8. Give an example of an ergodic processes..

9. Define Markov chain and one-step transition probability.

10. Define Markov process.

11. Prove that the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$ is the tpm of an irreducible Markov chain.

12. Find the invariant probabilities for the Markov chain $\{x_n; n \geq 1\}$ with state space

S={0,1}and one step TPM P= $\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

13. If the transition Probability matrix of a Markov chain is $\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$, we find the limiting distribution of the chain.

14.Define Binomial process.

15.State the properties of Bernoulli process.

16. Prove that the sum of two independent Poisson process is a Poisson process.

17.State any two properties of Poisson process

18. If patients arrive at a clinic according to poisson process with mean rate of 2 perminute.Find the probability that during a 1- minute interval no patient arrives.

19.The probability that a person is suffering from cancer is 0.001.Find the probability that out of 4000 persons.Exactly 4 suffer because of cancer.

20. Define sine wave process**.**

PART B(16 Marks)

1. a). Classify the random process and explain with an example. **(8)**

b). Give a random variable y with characteristic function $\phi(w)$ and a random process $x(t) = \cos(\lambda t + y)$. show that $\{x(t)\}$ is stationary is the wide sense if $\phi(1) = 0$ and $\phi(2) = 0$ **(8)**

2. a). Show that the random process $x(t) = A\cos(wt + \theta)$ is a wide sense stationary process if A and w are constants and is a uniformly distributed random variable is (0,2π). **(6)**

b). The probability distribution of the process $\{x(t)\}$ is given by

$$p(x(t) = n) = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1,2,3....... \\ \frac{at}{1+at}, n = 0 \end{cases}$$

Show that it is not stationary. **(10)**

3.a) If x(t) = y coswt + z sinwt, where y and z are two independent normal RVS with E(y) = E(z) = 0, E(y^2) = E(z^2) = σ^2 and w is a constant, prove that $\{x(t)\}$is a SSS process of order 2. **(8)**

b) Show that the random process X(t)=Asin(wt+θ) is WSS, A and w are constants and θ is uniformly distributed in (0,2π). **(8)**

4. a).Show that the random process $x(t) = \cos(t + \phi)$ where ϕ is uniformly distributed is (0,2π) with probability density function $f_\phi(x) = \frac{1}{2\pi}, 0 < x < 2\pi$ is

(i). First order stationary.

(ii). Stationary is wide sense.
(iii). Ergodic. **(8)**

b). Distinguish between 'stationary' and weakly stationary stochastic processes. Given an example to each type. Show that poisson process is an evolutionary process. **(8)**

5. a) The random binary transmission process {X(t)} is a WSS process with zero mean and autocorrelation function $R(\tau) = 1 - \frac{|\tau|}{T}$ where T is a constant. Find the mean and variance of the time average of {X(t)} over (0, T). Is {X(t)} mean Ergodic?. **(8)**

b) Prove that the random processes x(t) and Y(t) defined by $X(t) = A \cos \omega_0 t + B \sin \omega_0 t$, $Y(t) = B \cos \omega_0 t - A \sin \omega_0 t$ are jointly wide – sense stationary, if A and B are uncorrelated zero mean random variables with the same variance. **(8)**

6. a)The transition probability matrix of a Markov chain $\{x_n\}$ three states 1,2 and 3 is P = $\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)}=(0.7,0.2,0.1)$. Find
(i). $P\{X_2=3\}$ and
(ii). $P\{X_3=2, X_2=3, X_1=3, X_0=2\}$ **(8)**

b)Three boys A,B, and C are throwing a ball to each other. A always throws ball to B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian to B as to A. Find the transition matrix and classify the states **(8)**

7. a) Let (Xn: n=1,2,..) be a Markov chain with state space S = {0,1,2} and one step transition probability matrix: P = $\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 \end{bmatrix}$ Is the chain ergodic? Explain. **(8)**

b). If x(t) and y(t) are two independent poisson processes, show that the conditional distribution $\{x(t)\}$ gives {x(t)+y(t)} is binomial **(8)**

8 a) Suppose that customers arrive at a bank according to a poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes (i) Exactly 4 customer arrive and (ii) more than 4 customers arrive**.** **(8)**

b).Write a detailed note on Normal process. **(8)**

9.a).Derive the distributions of poisson process and find its mean and variance. **(8)**

b) If {X(t)} is a Gaussian process with $\mu(t) = 10$ and $C(t_1,t_2) = 16e^{-|t_1-t_2|}$ find the probability that (i) $X(10) \leq 8$ and (ii) $|X(10) - X(6)| \leq 4$ **(8)**

10 a) If a Gaussian random process X(t) is WSS, then show that it is strictly stationary. **(8)**

b).Write a critical note on 'sine wave' process and its applications **(8)**

UNIT IV
CORRELATION AND SPECTRAL DENSITY

PART – A (2 Marks)

1. Define auto correlation function and prove that for a WSS process {x(t)}, $R_{xx}(-\tau) = R_{XX}(\tau)$.
2. State any two properties of an auto correlation function.
3. Find the variance of the stationary process {X(t)} whose ACF is given by $R(\tau) = 16 + \frac{9}{1+6\tau^2}$
4. Find the mean of a stationary random process whose autocorrelation function is $R_{XX}(\tau) = 36 + \frac{8}{1+4\tau^2}$
5. Define cross correlation and its properties.
6. Prove that $R_{XY}(\tau) = R_{YX}(-\tau)$
7. State any two properties of cross correlation.
8. Define Spectral density
9. The power spectral density of a random process {X(t)}, is given by
   $$S_{XX}(\omega) = \begin{cases} \pi, |\omega| \langle 1 \\ 0, elsewhere \end{cases}$$
   Find its autocorrelation function.
10. The power spectral density function of a wide sense stationary process is given by
    $$S(W) = \begin{matrix} 1; |w| < w_0 \\ 0, otherwise \end{matrix}$$
    Find the auto correlation function of the process.
11. If $R(\tau) = e^{-2\lambda|\tau|}$ is the autocorrelation function of a random process X(t), obtain the spectral density of X(t).
12. What is meant by spectral analysis?
13. State any two uses of spectral density. .
14. Define cross –spectral density and its examples.
15. Find the power spectral density function of a WSS process with autocorrelation function $R(\tau) = e^{-\alpha\tau^2}$
16. State Wiener –Khintchine relation.
17. Check whether the following functions are valid auto correlation functions. (i) $\frac{1}{1+4\tau^2}$ (ii) $2\sin(\pi\tau)$
18. State any two properties of cross-power density spectrum.
19. Find the power spectral density function of a stationary process if $R(\tau) = e^{-|\tau|}$
20. Find the variance of the stationary process {X(t)} where $R(\tau) = 2+4e^{-2|\tau|}$

PART - B(16 Marks)

1. a) Given that the autocorrelation function for a stationary ergodic process with no periodic components is $R(T)=25 + \frac{4}{1+6T^2}$ find the mean and variance of the process {X(t)} . **(8)**

   b) Derive the mean, autocorrelation and autocovariance of poisson process. **(8)**

2. a) The auto correlation function for a stationary process X(t) is given by

$R_{XX}(\tau) = 9 + 2e^{-|\tau|}$. Find the mean of the random variable Y = $\int_0^2 X(t)dt$

and variance of X (t). **(8)**

b) Consider two random processes X(t) = 3 cos (ωt +θ) and Y(t) = 2cos(ωt +θ - $\frac{\pi}{2}$)

where θ is a random variable uniformly distributed in (0,2π).
Prove that $\sqrt{R_{XX}(0)R_{YY}(0)} \geq |R_{XY}(\tau)|$ **(8)**

3. a) If X(t) is a WSS process and if Y(t) = X(t+a)-X(t-a), Prove that
$R_{YY}(T) = 2R_{xx}(T) = 2R_{xx}(T) - R_{xx}(T+2a) - R_{xx}(T-2a)$. **(8)**

b) Given that a process X(t) has the autocorrelation function
$R_{XX}(\tau)=Ae^{-\alpha|\tau|}\cos(\omega_0\tau)$ where A > 0, α>0 and ω_0 are real constants .
Find the power spectrum of X(t). **(8)**

4. a) The power spectrum of a WSS process X = {x(t)} is given by
$s(w) = \frac{1}{(1 + w^2)^2}$.
Find the auto – correlation function and average power of the process. **(8)**

b) The Power spectral density of a zero mean wide stationary process
X(t) is given by.

S(w) = { K ; |w| <wo
0 ; otherwise where k is a constant

Show that X(t) and X(t + $\frac{\pi}{100}$) are uncorrelated **.** **(8)**

5. a) Calculate the power spectral density of a stationary random process
whose auto correlation is $R_{XX}(\tau)= e^{-\alpha|\tau|}$ **(8)**

b) If the cross – correlation of two processes {X(t)} and {Y(t)} is
$R_{xy}(t, t+\tau) = \frac{AB}{2}[\sin(w_o\tau) + \cos(w_o(2t+\tau))]$ where A,B are w_o and
constants. Find the cross power spectrum. **(8)**

6. a) The cross power specrum of real random process X(t) and Y(t)is given
by $S_{xy}(\omega)$ = {a+jb ω, | ω |<|
0 , otherwise. Find the cross correlation function. **(8)**

b) Show that if Y(t) = X(t + a) – x(t-a), $S_{YY}(\omega) = 4\sin^2 a\omega\, S_{xx}(\omega)$,
where X(t) is WSS. **(8)**

7. a) If the auto correlation function of a WSS process in $R(\tau) = pe^{-p}|\tau|$.

Show that its spectral elements is given by $S(\omega) = \frac{2}{1+\left(\frac{\omega}{p}\right)^2}$. **(8)**

b) The auto correlation function of the random binary transmission X(t) is given by

$$R(\tau) = \begin{cases} 0 \; for |\tau| \rangle T \\ 1 - \frac{|\tau|}{T} \quad for |\tau| \langle T \end{cases}$$ Find the power spectrum of the process X(t) **(8)**

8.a) A random process is given by $\omega(t) = AX(t) + BY(t)$ where A and B are real constants and X(t) and Y(t) are jointly WSS processes.
(i) Find the power spectrum $S_{\omega\omega}(\omega)$ of $\omega(t)$
(ii) Find $S_{\omega\omega}(\omega)$ if X(t) and Y(t) are correlated
(iii) $S_{xz}(\omega)$ and $S_{yz}(\omega)$. **(8)**

b) The auto correlation function of the Poisson increment process is given by $R(\tau) = \begin{cases} \lambda^2 + \frac{\lambda}{\varepsilon}\left(1 - \frac{|\tau|}{\varepsilon}\right); for|\tau| \rangle \varepsilon \\ \lambda^2; for|\tau| \le \varepsilon \end{cases}$

Prove that its spectral density is given by $S(w) = 2\pi \lambda^2 \delta \omega + \frac{4\lambda \sin^2(\omega\varepsilon/2)}{\varepsilon^2\omega^2}$. **(8)**

9. a) The cross power spectrum of a real random process {X(t)} and {Y(t)} is given by
$S_{xy}(W) = a+jbw$, $|w| <1$
0 , elsewhere
Find the cross correlation function. **(8)**

b) Find out the output power density spectrum and output autocorrelation function for a system with $h(t) = e^{-t}$, $t \ge 0$ for an input with power density spectrum $\eta_0 / 2$, $-\infty < f < \infty$. **(8)**

10. a) If $R(\tau) = e^{-2\lambda|\tau|}$ is the autocorrelation function of a random process {X(t)}, obtain the spectral density of {X(t)}. **(8)**

b). A WSS process {X(t)} with $A(\tau) = Ae^{-\alpha|\tau|}$ where A and α are real positive constants is applied to the input of a linear time invariant system with $h(t) = e^{-bt}U(t)$ where b is a real positive constant. Find the power spectral density of the output of the system. **(8)**

11. a) If {X(t)} is a Gaussian process with $\mu(t) = 10$ and $C(t_1,t_2) = 16e^{-|t_1-t_2|}$ find the probability that (i) $X(10) \le 8$ and (ii) $|X(10) - X(6)| \le 4$ **(8)**

b) If X(t) and Y(t) are two WSS processes then show that $\sqrt{R_{XX}(0)R_{YY}(0)} \ge |R_{XY}(\tau)|$ **(8)**

12.a) The power spectral density of a WSS process is given by

$$S(w) = \begin{cases} \frac{b}{a}(a-|\omega|; & |\omega| \le a \\ 0 & ; |\omega| > a \end{cases}$$

Find auto-correlation function of the process **(8)**

b) The Autocorrelation of a random telegraph signal process is given by $R(\tau) = A^2 e^{-2\alpha|\tau|}$ Determine the power spectral density random telegraph signal. **(8)**

UNIT – V
LINEAR SYSYTEM WITH RANDOM INPUTS

PART - A(2 Marks)

1. Define a system. When is it called linear system?
2. Describe a linear system.
3. State the properties of a linear filter.
4. Describe a linear system with a random input.
5. Give an example for a linear system.
6. Define time-invariant system.

7. State causal system
8. State memory less system
9. State Stable system. .
10. Define unit impulse response to the system.
11. Define White Noise. .
12. Define thermal noise. .
13. Define Band Limited White Noise.
14. Find the autocorrelation function of a Gaussian white noise**.**

PART - B(16 Marks)

1. a) Consider the white Gaussian Noise of zero mean and power spectral density No/2 applied to a low pass RC filter where transfer function is $H(f) = \frac{1}{1 + 2\pi i f RC}$. Find the output spectral density and auto correlation function of the output process. **(8)**

   b) Show that for an input output system (X(t), Y(t), (y(t)); $S_{yy}(w) = S_{xx}(w).\ |H(w)|^2$ where H(w) is the system transfer function, and input X is wide sense stationary. **(8)**

2. a) Show that $S_{yy}(w) = |H(w)|^2 S_{xx}(w)$ where $S_{xx}(w)$ and $S_{yy}(w)$ are the power spectral density functions of the input X(t) and the output Y(t) and H(w) is the system transfer function**.** **(8)**

   b) If the input X(t) and its output Y(t) are related by Y (t) = $\int_{-\infty}^{\infty} h(u) X(t-u)\, du,$ then prove that the system is a linear time – invariant system. **(8)**

3 a) If the input to a time-invariant, stable linear system is a WSS process, then prove that the output will also be WSS process . **(8)**

   b) Show that if {X(t)} is a WSS process then the output {Y(t)} is WSS process. **(8)**

4. a) If X(t) is the input voltage to a circuit and Y(t) is the output voltage, {X(t)} is a stationary random process with μ_x = o and $R_{xx}(\tau) = e^{-2|\tau|}$.
   Find μ_y , S_{xx} (w) and S_{yy}(w) if the system function is given by H (w) = $\frac{1}{\omega^2 + 2^2}$ **(8)**

   b) If the input X(t) and the output Y(t) are connected by the differential equation $T\frac{dy(t)}{dt} + y(t) = x(t)$ prove that they can be related by means of a convolution type integral. Assume that x(t) and y(t) are zero for t ≤ 0. **(8)**

5. a) Find the input auto correlation function, output auto correlation function and output spectral density of the RC – low pass filter when the filter is subjected to a white noise of spectral density $\frac{N_0}{2}$. **(8)**

   b) A Circuit has an impulse response given by h(t) = $\begin{cases} 0, elsewhere \\ \frac{1}{T}, 0 \le t \le T \end{cases}$

   Evaluate S_{yy} (w) in terms of S_{xx} (w). **(8)**

6. a) Obtain the transfer function of the pre-whitener whose input is

$S_{ni}(w_o) = N_0 \frac{w^2+2}{w^2+4}$ and the output is $S_{no}(w) = N_o$. **(8)**

b) The occurrence of a pulse is supposed to follow the Poisson Law(Poisson Process). Calculate the auto correlation of such a random telegraph signal. **(8)**

7 a) X(t) is the input voltage and Y(t) is the output voltage also {X(t)} is a stationary process with $\psi_x = 0$ and $R_{xx}(\tau) = e^{-\alpha|\tau|}$. Find μ_y, $S_{yy}(w)$, $R_{yy}(\tau)$. **(8)**

b) Suppose that the auto correlation function of the input random process x(t) is $R_{xx}(\tau) = z\delta(t)$ and H(w), the impulse response is $\frac{1}{b+iw}$ then find the following.

(i) Auto correlation of output random process y(t)

(ii) Power spectral density of y(t). **(8)**

8. a) A random process X(t) having the auto correlation function $R_{XX}(\tau) = pe^{-\alpha|\tau|}$ where p and α are real positive constants , is applied to the input of the system with impulse response h(t) = $\begin{cases} \lambda e^{-\lambda t}; t > 0 \\ 0; t < 0 \end{cases}$ where λ is a real positive constant. Find the autocorrelation function of the network's response Y(t). **(8)**

b) If y(t) = A cos ($w_o t + \theta$) + N(t) where A is a constant, θ is a random variable with a uniform distribution in $(-\pi,\pi)$ and {N(t)} is a band limited Gaussian white noise with a power spectral density.

$S_{NN}(w) = \begin{cases} 0, elsewhere \\ \frac{N_0}{2} \quad for |-\omega_0| \langle \omega_B \end{cases}$

Find the power spectrum density of {Y(t)} Assume that N(t) and θ are independent . **(8)**

9. a) If {X(t)} is a band limited process such that $S_{XX}(\omega) = 0$, $|\omega| > \sigma$ prove that $2[R_{XX}(0) - R_{XX}(\tau)] \leq \sigma^2 \tau^2 R_{XX}(0)$ **(8)**

b) Find out the output power density spectrum and output autocorrelation function for a system with $h(t) = e^{-t}$, $t \geq 0$ for an input with power density spectrum $\eta_0 / 2$, $-\infty < f < \infty$. **(8)**

10.a) If $R(\tau) = e^{-2\lambda|\tau|}$ Is the autocorrelation function of a random process {X(t)}, obtain the spectral density of {X(t)}. **(8)**

b). A WSS process {X(t)} with $A(\tau) = Ae^{-\alpha|\tau|}$ where A and α are real positive constants is applied to the input of a linear time invariant system with $h(t) = e^{-bt}U(t)$ where b is a real positive constant. Find the power spectral density of the output of the system. **(8)**

11. a)Define white noise. Find the A.C.F of the white noise **(8)**

b)If {X(t)} is the input to a linear time invariant system and {y(t)} is the output, find the A.C.F of {y(t)}. **(8)**