Code: DE23/DC23 | Subject: MATHEMATICS - II

Time: 3 Hours | Max. Marks: 100

DECEMBER 2010

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $x + iy = \sqrt{2} + 3i$, then $x^2 + y$ is

(A) 7 **(B)** 5

(C) 13 **(D)** $\sqrt{2} + 3$

b. If $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$, then cos x is equal to,

(A) $\dfrac{e^{ix} + e^{-ix}}{2}$ **(B)** $\dfrac{e^{-ix} + e^{x}}{2}$

(C) $\dfrac{e^{x} - e^{-x}}{2}$ **(D)** $\dfrac{e^{x} - e^{-ix}}{2}$

c. If three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar then,

(A) $(\vec{a} \times \vec{b}) \times \vec{c} = 0$ **(B)** $(\vec{a} \times \vec{b}) \cdot \vec{c} = 0$

(C) $(\vec{a} \times \vec{b}) \cdot \vec{c} = 1$ **(D)** $(\vec{a} \times \vec{b}) \cdot \vec{c} = -1$

d. If $|\vec{A} + \vec{B}| = 30$, $|\vec{A} - \vec{B}| = 20$ and $|\vec{B}| = 23$, then $|\vec{A}|$ is equal to

(A) 12 **(B)** 13

(C) 11 **(D)** 14

e. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, then A^{-1} is

(A) $\dfrac{1}{2}\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ **(B)** $\dfrac{1}{3}\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$

(C) $\dfrac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ **(D)** $\dfrac{1}{2}\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$

f. For what value of x is the matrix

$$\begin{bmatrix} 3-x & 2 & 2 \\ 1 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$$ singular?

(A) 1,2,3 **(B)** 1,-2,-3
(C) 1,2,5 **(D)** 1,-2,-5

g. The characteristic roots of the matrix $\begin{bmatrix} -6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is

(A) 2,3,6 **(B)** 1,2,3
(C) 2,2,8 **(D)** 1,2,6

h. The period of $|\cos x|$ is

(A) $\pi/2$ **(B)** π
(C) $3\pi/2$ **(D)** 2π

i. The inverse Laplace transform of $\frac{1}{S(S+2)}$ is

(A) $-\frac{1}{2}\left[e^{2t}-1\right]$ **(B)** $-\frac{1}{2}\left[e^{-2t}-1\right]$
(C) $-\frac{1}{2}\left[e^{-2t}-2\right]$ **(D)** $-\frac{1}{2}\left[e^{-2t}+2\right]$

j. The solution of differential equation $\frac{d^2y}{dx^2}+9y=e^x-\cos 2x$ is

(A) $y=c_1\cos 3x+c_2\sin 3x+\frac{1}{10}e^x-\frac{1}{5}\cos 2x$

(B) $y=c_1\cos 3x-c_2\sin 3x-\frac{1}{10}e^{-x}+\frac{1}{5}\cos 2x$

(C) $y=c_1\cos 3x+c_2\sin 3x+10e^x-5\cos 2x$

(D) $y=c_1\cos 3x-c_2\sin 3x+5e^x-5\cos 2x$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that the roots of the equation $x^{10}+11x^5-1=0$ are $\left(\frac{\pm\sqrt{5}-1}{2}\right)\left(\cos\frac{2n\pi}{5}+i\sin\frac{2n\pi}{5}\right)$, where n =0,1,2,3,4. **(8)**

b. If $\sin(\alpha + i\beta) = x + iy$, then show that, $\frac{x^2}{\cosh^2\beta} + \frac{y^2}{\sinh^2\beta} = 1$. **(8)**

Q.3 a. The centre of a regular hexagon is at the origin and one vertex is given by $\sqrt{3} + i$ on the Argand plane. Find the complex number represented by the other vertices. **(8)**

b. Show that the line joining one vertex of parallelogram to the mid-point of an opposite side trisects the diagonal and is trisected there at. **(8)**

Q.4 a. Forces of magnitude 5,3,1 Kg acting on the directions $6i + 2j + 3k$, $3i - 2j + 6k$, $2i - 3j - 6k$ respectively act on a particle which is displaced from the point $(2,-1,-3)$ to $(5,-1,1)$. Find the work done by the forces, the unit of length being metre. **(8)**

b. Find an unit vector parallel to the sum of the vectors $\vec{a} = 2i + 4j - 5k$ and $\vec{b} = i + 2j + 3k$. **(8)**

Q.5 a. Evaluate $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-7 & 3x-64 \end{vmatrix} = 0$. **(8)**

b. Solve the equation using Cramer's Rule.

$$3x - 4y - z = 2$$
$$6x + 6y + 3z = 7$$
$$9x - 8y - 5z = 0$$

(8)

Q.6 a. Find the values of λ for which the following system of equation is consistent and has nontrivial solution. Solve the equation for all such values of λ

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$
$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$
$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$$

(8)

b. Find the characteristic equation of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ and hence find the inverse of the matrix A. **(8)**

Q.7 a. Find the Laplace transform of $e^{-3t}(\cos 4t + 3\sin 4t)$. **(8)**

b. Find the Inverse Laplace Transform of $\frac{3s-2}{s^2 - 4s + 20}$. **(8)**

Q.8 a. Use Laplace transform technique to solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$, given that $x(0) = 1$ and $x\left(\pi/2\right) = -1$. **(8)**

b. Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x \cdot e^x \cdot \sin x$. **(8)**

Q.9 a. Show that any real valued function can be uniquely expressed as the sum of an even function and an odd function. **(8)**

b. Find the Fourier series for the function $f(x) = x$ in the interval $[-\pi, \pi]$. **(8)**