DipIETE – ET / CS (OLD SCHEME)

Code: DE23 / DC23 **Subject: MATHEMATICS - II**
Time: 3 Hours **UNE 2009** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2x10)

a. Argument of $\sqrt{\frac{1+i}{1-i}}$ is

(A) 0 **(B)** $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$ **(D)** π

b. $\frac{(\cos 3\theta + i\sin 3\theta)^4}{(\cos 4\theta + i\sin 4\theta)^3} \times \frac{(\cos 4\theta - i\sin 4\theta)^5}{(\cos 5\theta + i\sin 5\theta)^{-4}}$ is equal to

(A) 1 **(B)** –1
(C) 0 **(D)** None of these

c. $(i \times j)\cdot k + (j \times k)\cdot i + (k \times i)\cdot j$ is equal to

(A) 0 **(B)** 1
(C) 2 **(D)** 3

d. If $2i + j - mk$ is perpendicular to the sum of the vectors $i - j + 2k$ and $3i + 2j + k$, then m is equal to
(A) 1 **(B)** 2
(C) 3 **(D)** 4

e.
$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$$ is equal to

(A) a **(B)** b
(C) c **(D)** 0

f. The sum and product of eigen values of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ are

(A) 7, 8 **(B)** 7, 3
(C) 3, 0 **(D)** 3, 1

g. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$ and I is the unit matrix of order 3, then A^3 is equal to

(A) $pA^2 + qA + rI$ **(B)** $rA^2 + qA + pI$
(C) $qA^2 + rA + pI$ **(D)** $rA^2 + pA + qI$

h. The inverse Laplace transform of $\frac{1}{s(s^2+1)}$ is

(A) 1 + sin t **(B)** 1 – sin t
(C) 1 + cos t **(D)** 1 – cos t

i. The period of the function $\text{Sin}\frac{2\pi x}{n}$ is

(A) 2 **(B)** π

(C) n **(D)** $\frac{1}{n}$

j. The solution of the differential equation $\frac{d^2y}{dx^2} - \frac{4dy}{dx} + 4y$ = Sin 2x is

(A) $y = (c_1 + c_2x)e^{2x} + \frac{1}{8}\text{Cos}2x$ **(B)** $y = (c_1 + c_2x)e^{2x} + \frac{1}{8}\text{Sin } 2x$
(C) $y = c_1e^{2x} + c_2e^{2x} + \text{Cos}2x$ **(D)** $y = (c_1 + c_2x)e^{2x} + \text{Sin } 2x$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find the real and imaginary parts of tan (x + iy). **(8)**

b. Use De-Moivre's Theorem to solve the equation $x^5 + 1 = 0$. **(8)**

Q.3 a. If Z_1 and Z_2 are two complex numbers, show that
$|Z_1 + Z_2|^2 + |Z_1 - Z_2|^2 = 2|Z_1|^2 + 2|Z_2|^2$. **(8)**

b. ABCDEF is a regular hexagon whose centroid is 0. Show that
$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 6\overrightarrow{AO}$ **(8)**

Q.4 a. Show that $(\vec{A} \times \vec{B})^2 = (\vec{A} \cdot \vec{A})(\vec{B} \cdot \vec{B}) - (\vec{A} \cdot \vec{B})^2$. **(8)**

b. A particle acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. Find the total work done. **(8)**

Q.5 a. Evaluate $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$. **(8)**

b. Use Cramer's rule to solve the equations

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$ **(8)**

Q.6 a. For what values of k the equations

$$x + y + z = 1$$
$$2x + y + 4z = k$$
$$4x + y + 10z = k^2$$

has a solution and solve them completely in each case. **(8)**

b. Use Cayley-Hamilton theorem to find inverse of

$$\begin{bmatrix} 3 & 2 & 4 \\ 4 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix}$$ **(8)**

Q.7 a. Find the Laplace transform of $te^{2t}\sin 3t$. **(8)**

b. Find the inverse Laplace transform of $\log \frac{s+a}{s+b}$. **(8)**

Q.8 a. Solve the differential equation

$$\frac{d^2y}{dx^2}+4y=e^x+\text{Sin } 2x$$. **(8)**

b. Use Laplace transform to solve the initial value problem

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=e^{2x}$$

Given that $y=0, \frac{dy}{dx}=0$ at x = 0. **(8)**

Q.9 a. Find all values of $\left[\frac{1}{2}+i\frac{\sqrt{3}}{2}\right]^{3/4}$. **(6)**

b. Find a Fourier series expansion of the function

$$f(x)=x^2, \quad -\pi<x<\pi$$ **(10)**