JUNE 2008

Code: DE23/DC23 **Subject: MATHEMATICS - II**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. If $-3+ix^2y$ and x^2+y+4i represent conjugate complex numbers then the value of x and y is

(A) $x=\pm 1, y=-4$. **(B)** $x=-4, y=\pm 1$.
(C) $x=-4, y=-1$. **(D)** $x=1, y=4$.

b. Imaginary part of $\sin \bar{z}$ is

(A) – cos x cosh y **(B)** – cos x sinh y
(C) – sin x cosh y **(D)** – sin x sinh y

c. Three vectors $\bar{A}, \bar{B}, \bar{C}$ are coplanar, the value of their scalar triple product is

(A) 0 **(B)** 1
(C) –1 **(D)** *i*

d. If θ is the angle between the vectors $\bar{a}$ and $\bar{b}$ such that $|\bar{a}\times\bar{b}|=|\bar{a}\cdot\bar{b}|$ then θ is

(A) 0° **(B)** 45°
(C) 120° **(D)** 180°

e. The value of the determinant $\begin{vmatrix} 1989 & 1990 & 1991 \\ 1992 & 1993 & 1994 \\ 1995 & 1996 & 1997 \end{vmatrix}$ is

(A) 1 **(B)** 2
(C) –1 **(D)** 0

f. If the product of two eigen values of the matrix $\begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$ is 16, then the third eigen value is

(A) 0 **(B)** 5
(C) 2 **(D)** –2

g. If f(x) is defined in (0, L), then the period of f(x) to expand it as a half range sine series is

(A) L. **(B)** 0.
(C) 2L. **(D)** $L/2$.

h. The inverse Laplace transform $L^{-1}\left(1/s^n\right)$ is possible only when n is

(A) 0 **(B)** –ve integer
(C) –ve rational number **(D)** +ve integer

i. The differential equation of a family of circles having the radius r and centre on the x axis is

(A) $y^2\left[1+\left(\frac{dy}{dx}\right)^2\right]=r^2$ **(B)** $x^2\left[1+\left(\frac{dy}{dx}\right)^2\right]=r^2$

(C) $\left(x^2+y^2\right)\left[1+\left(\frac{dy}{dx}\right)^2\right]=r^2$ **(D)** $r^2\left[1+\left(\frac{dy}{dx}\right)^2\right]=x^2$

j. If y satisfies $y''-3y'+2y=e^{-t}$ with $y(0)=y'(0)=0$ then Laplace transform $L\{y(t)\}$ is

(A) $\frac{1}{(s+1)(s+2)^2}$ **(B)** $\frac{1}{(s+1)(s-2)^2}$

(C) $\frac{1}{(s+1)^2(s-2)}$ **(D)** $\frac{1}{(s+1)^2(s+2)}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find the moment of the force $\bar{F}$ about a line through the origin having direction of $2\hat{i}+2\hat{j}+\hat{k}$, due to a 30 Kg force acting at a point (– 4, 2, 5) in the direction of

$12\hat{i} - 4\hat{j} - 3\hat{k}$. **(8)**

b. Prove that the right bisectors of the sides of a triangle intersect at its circum centre. **(8)**

Q.3 a. Show that the components of a vector $\overline{B}$ along and perpendicular to $\overline{A}$ in the plane of $\overline{A}$ and $\overline{B}$ are $\left(\frac{\overline{A}\cdot\overline{B}}{A^2}\right)\overline{A}$ and $\frac{(\overline{A}\times\overline{B})\times\overline{A}}{A^2}$. **(8)**

b. If $\tan(\theta + i\varphi) = e^{i\alpha}$ show that $\theta = \left(n + \frac{1}{2}\right)\frac{\pi}{2}$ and $\phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$. **(8)**

Q.4 a. If $(a_1 + ib_1)(a_2 + ib_2)\ldots\ldots\ldots(a_n + ib_n) = A + iB$ then

$\tan^{-1}\frac{b_1}{a_1} + \tan^{-1}\frac{b_2}{a_2} + \ldots\ldots\ldots + \tan^{-1}\frac{b_n}{a_n} = \tan^{-1}\frac{B}{A}$. **(8)**

b. Show that the origin and the complex numbers represented by the roots of the equation $z^2 + az + b = 0$, where *a, b* are real, form an equilateral triangle if $a^2 = 3b$. **(8)**

Q.5 a. Prove that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right).$$ **(8)**

b. Determine the values of α, β, γ when $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal. **(8)**

Q.6 a. Find the values of k such that the system of equations $x + ky + 3z = 0$, $4x + 3y + kz = 0$, $2x + y + 2z = 0$ has non-trivial solution. **(8)**

b. Find the characteristic equation of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$. Hence find A^{-1}. **(8)**

Q.7 Find the Fourier series for $f(t)=\begin{cases} 0, & -2<t<-1 \\ 1+t, & -1<t<0 \\ 1-t, & 0<t<1 \\ 0, & 1<t<2 \end{cases}$. **(16)**

Q.8 a. Find $L\left(e^{-4t}\dfrac{\sin 3t}{t}\right)$. **(8)**

b. Find the inverse Laplace transform of $\dfrac{s+4}{s(s-1)(s^2+4)}$. **(8)**

Q.9 a. Using Laplace transformation, solve the following differential equation:

$\dfrac{d^2x}{dt^2}+9x=\cos 2t$ if $x(0) = 1$, $x(\pi/2)=-1$. **(8)**

b. Solve $(D^2+D+1)y=\cos 2x$. **(8)**