Code: DE23/DC23 **Subject: MATHEMATICS - II**
Time: 3 Hours **Max. Marks: 100**

DECEMBER 2007

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. The complex numbers Z = x + iy, which satisfy the equation $\left|\frac{Z-5i}{Z+5i}\right| = 1$ lie on

(A) the x-axis.
(B) the line y = 5.
(C) A circle passing through the origin.
(D) None of these.

b. If $Z^2 = |Z|^2$, then

(A) $Re(Z) = 0$ **(B)** $Im(Z) = 0$
(C) Z=0 **(D)** $Z = x(1 \pm i)$, with x real

c. If $\bar{a}$ and $\bar{b}$ are two unit vectors and ϕ is the angle between them, then $\left(\frac{1}{2}\right)|\bar{a} - \bar{b}|$ is equal to

(A) $\pi/2$ **(B)** 0
(C) $|\text{Sin}\ \phi/2|$ **(D)** $|\text{Cos}\ \phi/2|$

d. A vector which makes equal angles with the vectors $(1/3)(\hat{i} - 2\hat{j} + 2\hat{k})$, $(1/5)(-4\hat{i} - 3\hat{k})$ and $\hat{j}$ is

(A) $5\hat{i} + \hat{j} + 5\hat{k}$ **(B)** $-5\hat{i} + \hat{j} + 5\hat{k}$
(C) $-5\hat{i} - \hat{j} + 5\hat{k}$ **(D)** $5\hat{i} + \hat{j} - 5\hat{k}$

e. If $\omega\ (\neq 1)$ is a cube root of unity and $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$, then

(A) $x = 1$ **(B)** $x = \omega$
(C) $x = \omega^2$ **(D)** none of these

f. If $\Delta = \begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & a \\ c-a & -a & 0 \end{vmatrix}$, then Δ is equal to

(A) (a+b) (b+c) (c+a) **(B)** bc + ca + ab
(C) 2abc **(D)** none of these

g. If A is a skew-symmetric matrix and n is a positive integer, then A^n is

(A) a symmetric matrix.
(B) skew-symmetric matrix for even n only.
(C) diagonal matrix.
(D) symmetric matrix for even n only.

h. The period of the function sin x + sin 2x + sin 3x is

(A) π **(B)** $\pi/2$
(C) $\pi/3$ **(D)** 2π

i. The Laplace transform of $L\left(\frac{e^t}{\sqrt{t}}\right)$ is

(A) $\sqrt{\frac{\pi}{s-1}}$ **(B)** $\sqrt{\frac{\pi}{(s+1)}}$
(C) $\sqrt{\frac{\pi}{s^2-1}}$ **(D)** $\sqrt{\frac{\pi}{s^2+1}}$

j. The solution of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$ is

(A) $C_1e^{2x} - C_2e^{3x} + \frac{e^{3x}}{3}$ **(B)** $C_1e^{2x} + C_2e^{4x} + \frac{e^{4x}}{4}$
(C) $C_1e^{2x} + C_2e^{3x} + \frac{e^{4x}}{2}$ **(D)** $C_1e^{2x} - C_2e^{3x} - \frac{e^{4x}}{2}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Simplify $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n$. **(8)**

b. Find all the values of $(1+i)^{1/5}$. **(8)**

Q.3 a. If z_1 and z_2 are two complex numbers, prove that $|z_1+z_2|^2=|z_1|^2+|z_2|^2$ If and only if $\frac{z_1}{z_2}$ is purely imaginary. **(8)**

b. A vector $\vec{x}$ satisfies the equation $\vec{x}\times\vec{b}=\vec{c}\times\vec{b};\ \vec{x}\times\vec{a}=0$. Prove that $\vec{x}=\vec{c}-\frac{(\vec{a}.\vec{c})}{(\vec{a}.\vec{b})}\vec{b}$ provided $\vec{a}$ and $\vec{b}$ are not perpendicular. **(8)**

Q.4 a. Using vector methods prove that the diagonals of a parallelogram bisect each other. **(8)**

b. The constant forces 2i – 5j + 6k, -i+2j-k and 2i + 7j act on a particle which is displaced from position 4i – 3j – 2k to position 6i + j – 3k. Find the total work done. **(8)**

Q.5 a. Show that

$$\begin{vmatrix} 1^2+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}=\left(1+a^2+b^2\right)^3$$

(8)

b. Write the following equations in the matrix form AX = B and solve for X by finding A^{-1}.

$$x+y-2z=3$$
$$2x-y+z=0$$
$$3x+y-z=8$$

(8)

Q.6 a. Test the consistency of the following equations and if possible, find the solution

$$4x-2y+6z=8$$
$$x+y-3z=-1$$
$$15x-3y+9z=21$$

(8)

b. Obtain the characteristic equation of the matrix $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$ and use Cayley-Hamilton theorem to find its inverse. **(8)**

Q.7 Find the Fourier series expansion for the function

$$f(x)=\frac{1}{2}(\pi-x),0<x<2\pi.$$

(16)

Q.8 a. Find the Laplace transform of $L\{t^2e^t\sin 4t\}$. **(8)**

b. Find the Inverse Laplace transform of $L^{-1}\left(\frac{s+1}{s^2+6s+25}\right)$ **(8)**

Q.9 a. Solve the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \text{Sin } 2x$$. **(8)**

b. By using Laplace transform solve the differential equation
$\frac{d^2y}{dt^2} + y = t\cos 2t,$ with initial conditions $y = 0, \frac{dy}{dt} = 0$, when t = 0. **(8)**