Code: D-23 / DC-23 **Subject: MATHEMATICS - II**
Time: 3 Hours **June 2006** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. Let $z_1 = 2-5i;\ z_2 = -1+4i;\ z_3 = 6+i$ and $z_4 = 3-7i$. Express $\frac{(z_1+\overline{z_2})z_3}{z_4}$ in the form a + bi, a , b $\in$ R.

(A) $\frac{208}{29}+\frac{27}{29}i$ **(B)** $\frac{208}{29}-\frac{27}{29}i$
(C) $\frac{28}{209}+\frac{27}{29}i$ **(D)** $\frac{28}{209}-\frac{27}{29}i$

b. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1-z_3}{z_2-z_3}=\frac{1-i\sqrt{3}}{2}$ are vertices of the a triangle which is

(A) acute-angled and isosceles **(B)** right-angled and isosceles
(C) obtuse-angled and isosceles **(D)** equilateral

c. A unit vector parallel to 3i+4j-5k is

(A) $-\frac{3}{5\sqrt{2}}i-\frac{4}{5\sqrt{2}}j+\frac{1}{\sqrt{2}}k$ **(B)** $\frac{3}{5\sqrt{2}}i-\frac{4}{5\sqrt{2}}j-\frac{2}{\sqrt{2}}k$
(C) $-\frac{3}{5\sqrt{2}}i+\frac{4}{5\sqrt{2}}j+\frac{2}{\sqrt{2}}k$ **(D)** $\frac{3}{5\sqrt{2}}i-\frac{4}{5\sqrt{2}}j+\frac{1}{\sqrt{2}}k$

d. Let $\vec{a}$ = (1, 2, 0), $\vec{b}$ = (-3, 2, 0), $\vec{c}$ = (2, 3, 4). Then $\vec{a}\cdot(\vec{b}\times\vec{c})$ equals

(A) 33 **(B)** 30
(C) 31 **(D)** 32

e. If ω is complex cube root of unity, and $A=\begin{pmatrix}\omega & 0\\ 0 & \omega\end{pmatrix}$, then A^{100} is equal to

(A) 0 **(B)** -A

(C) A (D) none of these

f. If A and B are symmetric matrices, then AB + BA is a

(A) diagonal matrix **(B)** null matrix
(C) symmetric matrix **(D)** Skew-symmetric matrix

g. The function $x^3 \sin x$ is

(A) odd **(B)** even
(C) neither **(D)** none of these

h. The function cos x + sin x + tan x + cot x + sec x + cosecx is

(A) both periodic and odd **(B)** both periodic and even
(C) periodic but neither even nor odd **(D)** not periodic

i. The Laplace Transform for sin at is

(A) $\frac{s}{s^2 - a^2}$ **(B)** $\frac{a}{s^2 + a^2}$
(C) $\frac{s}{s^2 + a^2}$ **(D)** $\frac{a}{s^2 - a^2}$

j. The Inverse Laplace Transform for $\frac{s+9}{s^2 + 6s + 13}$ is

(A) $e^{3t}(\cos(2t) + 3\sin(2t))$ **(B)** $e^{-3t}(\cos(2t) + 3\sin(2t))$
(C) $e^{3t}(\cos(2t) - 3\sin(2t))$ **(D)** $e^{-3t}(\cos(2t) - 3\sin(2t))$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If a, b, c are real numbers such that $a^2 + b^2 + c^2 = 1$ and b + ic = (1 + a)z, where z is a complex number, then show that $\frac{1+iz}{1-iz} = \frac{a+ib}{1+c}$. **(8)**

b. Given that $z_1 + z_2 + z_3 = A$, $z_1 + z_2\omega + z_3\omega^2 = B$ and $z_1 + z_2\omega^2 + z_3\omega = C$, where ω is a cube root of unity. Express z_1, z_2, z_3 in terms of A, B, C and ω. **(8)**

Q.3 a. Show that for all real μ, $\cos(6\mu) = 32\cos^6(\mu) - 48\cos^4(\mu) + 18\cos^2(\mu) - 1$. **(8)**

b. For any four vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ prove that $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right)=\left(\vec{a}\cdot\vec{c}\right)\left(\vec{b}\cdot\vec{d}\right)-\left(\vec{a}\cdot\vec{d}\right)\left(\vec{b}\cdot\vec{c}\right)$. Hence prove that $\left(\vec{b}\times\vec{c}\right)\cdot\left(\vec{a}\times\vec{d}\right)+\left(\vec{c}\times\vec{a}\right)\cdot\left(\vec{b}\times\vec{d}\right)+\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right)=0$ **(8)**

Q.4 a. In ΔOAB let $OA=\bar{a}$, $OB=\bar{b}$. Then find the vector representing AB and OM, where M is the midpoint of AB. **(4)**

b. Prove that the straight line joining the mid-points of two non-parallel sides of a trapezium is parallel to the parallel sides and is half their sum. **(12)**

Q.5 a. For reals A, B, C, P, Q, R find the value of determinant

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix}$$ **(8)**

b. Using matrix method find the values of λ and μ so that the system of equations:

$2x-3y+5z=12$

$3x+y+\lambda z=\mu$

$x-7y+8z=17$ has infinitely many solutions. **(8)**

Q.6 a. Solve the system of equations

$x+y+z=6$

$x-y+2z=5$

$3x+y+z=8$

by using inverse of a suitable matrix. **(8)**

b. Using Cayley-Hamilton theorem find A^3 for $A=\begin{pmatrix}1 & 2\\ 4 & 3\end{pmatrix}$. **(8)**

Q.7 State whether the function f(x) having period 2 and defined by

$f(x)=1-x^2, -1\le x\le 1$

is even or odd. Find its Fourier Series. **(16)**

Q.8 a. Find the Laplace transform of $f(t) = e^{2t}t^2$. **(8)**

b. Find the Inverse Laplace transform for $L(s) = \dfrac{e^{-3s}}{(s-1)^4}$. **(8)**

Q.9 a. Solve the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3\sin x$$

given that y = -0.9 and $\frac{dy}{dx} = -0.7$, when x=0 **(8)**

b. Using the Laplace transform solve the differential equation

$$f''(t) - 4f'(t) + 3f(t) = 1$$

with initial conditions $f(0) = f'(0) = 0$. **(8)**