Code: DE-23 / DC-23 **Subject: MATHEMATICS - II**

JUNE 2007

Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. Modules of $\left(\sqrt{i}\right)^{\sqrt{i}}$ is

(A) $e^{\pi/4}$ **(B)** $e^{-\pi/4}$
(C) $e^{-\pi/4\sqrt{2}}$ **(D)** $e^{\pi/4\sqrt{2}}$

b. If $\tan\frac{x}{2} = \tanh\frac{y}{2}$ then the value of cos x cos hy is

(A) –1 **(B)** 0
(C) 1/2 **(D)** 1

c. The two non-zero vectors $\overline{A}$ and $\overline{B}$ are parallel if

(A) $\overline{A}\times\overline{B}=0$ **(B)** $\left|\overline{A}\times\overline{B}\right|=1$
(C) $\overline{A}\cdot\overline{B}=0$ **(D)** $\left|\overline{A}\right|=\left|\overline{B}\right|$

d. The volume of the parallelopipid with sides $\overline{A}=6\hat{i}-2\hat{j}$, $\overline{B}=\hat{j}+2\hat{k}$, $\vec{C}=\hat{i}+\hat{j}+\hat{k}$ A is

(A) 5 cubic units **(B)** 10 cubic units
(C) 15 cubic units **(D)** 20 cubic units

e. If $\begin{bmatrix}1 & 2 & 3\\ 0 & 2 & 5\\ 0 & 0 & 3\end{bmatrix}$ then eigen value of A^{-1} are

(A) $1, \frac{1}{2}, \frac{1}{3}$ **(B)** 1, 2, 3

(C) 0, 1, 2 **(D)** $0, 1, \frac{1}{2}$

f. The sum and product of the eigen values of $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are

(A) Sum = 5, Product = 7 **(B)** Sum = 7, Product = 5
(C) Sum = 5, Product = 5 **(D)** Sum = 7, Product = 7

g. If $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ then the value of f(0) is

(A) 0 **(B)** $\frac{\pi}{2}$
(C) $-\frac{\pi}{2}$ **(D)** π

h. The inverse Laplace transform of $(s+2)^{-2}$

(A) e^{-2t} **(B)** e^{2t}
(C) te^{2t} **(D)** te^{-2t}

i. The solution of the differential equation $y'' + y = 0$ satisfying the condition y(0)=1, $y\left(\frac{\pi}{2}\right) = 2$ is

(A) $y = 2\cos x + \sin x$ **(B)** $y = \cos x + 2\sin x$
(C) $y = \cos x + \sin x$ **(D)** $y = 2(\cos x + \sin x)$

j. Fourier Sine transform of 1/x is

(A) S **(B)** S/2
(C) $S^2/2$ **(D)** $-S^2/2$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. A rigid body is spinning with angular velocity 27 radians per second about an axis parallel to $2\hat{i} + \hat{j} - 2\hat{k}$ passing through the point $\hat{i} + 3\hat{j} - \hat{k}$. Find the velocity of the point of the body whose position vector is $4\hat{i} + 8\hat{j} + \hat{k}$. **(8)**

b. Find the sides and angles of the triangle whose vertices are $\hat{i}-2\hat{j}+2\hat{k}$, $2\hat{i}+\hat{j}-\hat{k}$ and $3\hat{i}-\hat{j}+2\hat{k}$. **(8)**

Q.3 a. Find the volume of the tetrahedron formed by the point (1,1,1) (2,1,3) (3,2,2,), (3,3,4). **(8)**

b. The centre of a regular hexagon is at the origin and one vertex is given by $\sqrt{3}+i$ on the Argand diagram. Determine the other vertices. **(8)**

Q.4 a. Prove that the general value of θ which satisfies the equation

$$(\cos\theta+i\sin\theta)(\cos 2\theta+i\sin 2\theta)\cdots(\cos n\theta+i\sin n\theta)=1 \text{ is } \frac{4m\pi}{n(n+1)},$$

where m is any integer **(8)**

b. Use De Moivre's theorem to solve the equation $x^4-x^3+x^2-x+1=0$ **(8)**

Q.5 a. Show that

$$\begin{vmatrix} a^2+\lambda & ab & ac & ad \\ ab & b^2+\lambda & bc & bd \\ ac & bc & c^2+\lambda & cd \\ ad & bd & cd & d^2+\lambda \end{vmatrix} = \lambda^3\left(a^2+b^2+c^2+d^2+\lambda\right)$$

(8)

b. Express the following matrix as a sum of a symmetric matrix and a skew symmetric matrix.

$$\begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}.$$

(8)

Q.6 a. Find the values of λ, for which following system of equations has non-trivial solutions. Solve equations for all such values of λ.

$$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$$
$$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$$
$$2x+(3\lambda+1)y+3(\lambda-1)z=0$$

(8)

b. Find the characteristic equation of the matrix $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence evaluate the matrix equation $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. **(8)**

Q.7 Expand $f(x) = \sqrt{1-\cos x},\ 0 < x < 2\pi$ in a Fourier Series.

Hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots$ **(16)**

Q.8 a. Find the Laplace transform of $\frac{1-\cos t}{t}$. **(8)**

b. Find the Inverse Laplace transform of $\cot^{-1}\left(\frac{s+3}{2}\right)$. **(8)**

Q.9 a. Solve the initial value problem

$2y'' + 5y' + 2y = e^{-2t},\quad y(0) = 1 = y'(0)$.

Using Laplace transforms. **(8)**

b. Solve

$(D-1)^2 (D^2 + 1) y = \operatorname{Sin} \frac{x}{2}$ **(8)**