DECEMBER 2006

Code: D-23 / DC-23 **Subject: MATHEMATICS - II**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$ is

(A) 8 **(B)** 12
(C) 16 **(D)** None of these

b. A square root of 3 + 4i is

(A) $\sqrt{3}+i$ **(B)** $2-i$
(C) $2+i$ **(D)** None of these

c. Any vector a is equal to

(A) $(a\cdot\hat{i})\hat{i}+(a\cdot\hat{j})\hat{j}+(a\cdot\hat{k})\hat{k}$ **(B)** $(a\cdot\hat{j})\hat{i}+(a\cdot\hat{k})\hat{j}+(a\cdot\hat{i})\hat{k}$
(C) $(a\cdot\hat{k})\hat{i}+(a\cdot\hat{i})\hat{j}+(a\cdot\hat{j})\hat{k}$ **(D)** $(a\cdot a)(\hat{i}+\hat{j}+\hat{k})$

d. If a and b are two unit vectors inclined at an angle θ and are such that a + b is a unit vector, then θ is equal to

(A) $\pi/4$ **(B)** $\pi/3$
(C) $\pi/2$ **(D)** $2\pi/3$

e. The value of the determinant $\begin{vmatrix} 1 & \omega^3 & \omega^5 \\ \omega^3 & 1 & \omega^4 \\ \omega^5 & \omega^4 & 1 \end{vmatrix}$, where ω is an imaginary cube root of unity is

(A) $(1-\omega)^2$ **(B)** 3
(C) -3 **(D)** 4

f. The value of the determine $\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$ is equal to

(A) -4 **(B)** 0
(C) 1 **(D)** 4

g. The inverse of a diagonal matrix is

(A) not defined **(B)** a skew-symmetric matrix
(C) a diagonal matrix **(D)** a unit matrix

h. The period of function sin 2x + cot 3x + sec 5x is

(A) π **(B)** 2π
(C) $\pi/2$ **(D)** $\pi/3$

i. The Laplace transform of $\sin^2 t$ is

(A) $\frac{2}{s(s^2+4)}$ **(B)** $\frac{1}{s(s^2+4)}$
(C) $\frac{2}{(s+4)(s-2)}$ **(D)** $\frac{1}{(s+4)(s-2)}$

j. The solution of the differential equation $(D^2+4)y = e^x$ is

(A) $c_1 \cos 2x - c_2 \sin 2x + \frac{e^x}{4}$ **(B)** $c_1 \cos 2x + c_2 \sin 2x + \frac{e^x}{4}$
(C) $c_1 \cos 2x + c_2 \sin 2x + \frac{e^x}{5}$ **(D)** $c_1 \cos 4x - c_2 \sin 4x + \frac{e^x}{5}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If n is a positive integer, prove that $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos\frac{n\pi}{6}$. **(8)**

b. Find all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{3/4}$ and show that the product of all these values is 1. **(8)**

Q.3 a. If the roots of $z^3 + iz^2 + 2i = 0$ represent vertices of a triangle in the Argand plane, then find area of the triangle. **(8)**

b. Find the value of $(\vec{a} \times \vec{b}) \times \vec{c}$ if
$\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$. **(8)**

Q.4 a. Prove that the sum of all the vectors drawn from the centre of a regular octagon to its vertices is the zero vector. **(8)**

b. Find the moment about the point $M(-2,4,-6)$ of the force represented in magnitude and position by $\overrightarrow{AB}$, where the point A and B have the co-ordinates $(1,2,-3)$ and $(3,-4,2)$ respectively. **(8)**

Q.5 a. Show that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$. **(8)**

b. Write the following system of equations in the matrix form AX = B and solve this for X by finding A^{-1}.
$2x_1 - x_2 + x_3 = 4$
$x_1 + x_2 + x_3 = 1$
$x_1 - 3x_2 - 2x_3 = 2$ **(8)**

Q.6 a. Using matrix methods, find the values of λ and μ so that the system of equations
$2x + 3y + 5z = 9$
$7x + 3y - 2z = 8$
$2x + 3y - \lambda z = \mu$.

has (i) unique solution and (ii) has no solution **(8)**

b. Verify Cayley Hamilton theorem for the matrix
$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$.

Use Caley Hamilton theorem to evaluate A^{-1} and hence solve the equations
$x + 2y = 3$
$3x + y = 4$ **(8)**

Q.7 Find the Fourier series for the functions
$f(x) = \frac{1}{4}(\pi - x)^2, \ 0 < x < 2\pi$ **(16)**

Q.8 a. Find the Laplace transform $L\{te^{at}\sin at\}$ **(8)**

b. Find the inverse Laplace transform $L^{-1}\left\{\frac{2s+1}{(s+1)(s^2+1)}\right\}$ **(8)**

Q.9 a. Solve the differential equation
$(D^2+9)y = \cos 3x$ **(8)**

b. By using Laplace transform, solve the differential equation
$\frac{d^2y}{dt^2}+9y = \cos 2t$, with initital conditions $y(0)=1, y\left(\frac{\pi}{2}\right)=-1$ **(8)**