CERTIFICATE IN DERIVATIVES: MATHEMATICS AND BASIC PRINCIPLES

Specimen Examination Paper

April 1999

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 9 questions leaving sufficient space between each answer.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available actuarial tables, derivatives formula sheet and an electronic calculator.

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1 (i) Explain the credit risks involved in dealing in FTSE futures contracts tarded on LIFFE and the means by which these are minimised. [6]

(ii) Discuss the operation of margin payments on LIFFE under the following headings:

(a) factors that determine the size of the initial margins

(b) the margining system used by LIFFE and how it would apply to a portfolio of derivative contracts with the exchange

(c) the form which initial margin can take [8]

[Total 14]

2 Consider an option on a non-dividend paying stock when the stock price is \$30, the exercise price is \$29, the continuously compounded risk free rate of interest is 5% p.a., the volatility is 25% p.a. and the time to maturity 4 months.

(i) Using the Black-Scholes model, calculate the price of the option if it is:

(a) a European call
(b) an American call
(c) a European put [5]

(ii) Verify that put-call parity holds. [2]

(iii) If the stock is now assumed to be dividend paying and goes ex-dividend in 1.5 months time and the expected dividend is \$0.5 recalculate the prices in (i)(a) and (i)(c). [7]

[Total 14]

3 (i) The current price of a share is £1.85. It is known that a dividend payment of 7p per share will be received in 3 months time. Using a binomial lattice, calculate the value of a four-month European put option on the share with an exercise price of 1.90. Use the following parameters:

$$u,\ d\ (=1/u),\ p,\ r,\ \Delta t\ (=\text{one month})$$

Where u is the ratio of the share price to the price one month before if the price has moved up, d is the corresponding ratio if the share price has moved down, p is the risk neutral probability of an upwards movement, r *is* the appropriate risk neutral interest rate and Δt is the time period between consecutive lattice points.

You are given $r = 0.06$ and $\sigma = 0.3$. [7]

(ii) Using the same binomial lattice, calculate approximate values of delta, gamma and theta for the option at time zero. [6]

[Total 13]

4 In November 1998 an options trader enters the following portfolio of derivatives on the June 1999 FTSE100 futures contract. The current futures price is 5717 and the options expire on 19 June 1999.

Contract	*Strike*	*Expiry*	*Current Price*	*Delta*
Short 100 Calls	5800	June 1999	278	+48%
Short 100 Puts	5600	June 1999	257	–40%

(i) Sketch a diagram of the profit and loss on the portfolio at expiry of the options against a suitable range of futures prices (assuming no futures hedge), and add to this graph a line indicating approximately the current profit. [4]

(ii) What futures position is required to delta hedge this portfolio at the current market level? [2]

(iii) Discuss the effect on the trader's profit and loss of the market falling sharply in the next few days suggesting the likely action he would take (assume the trader has hedged his position as suggested in part (ii)). Would your answer be any different if the fall took place a week before expiry? [5]

[Total 11]

5 (i) (a) State (without derivation) the partial differential equation derived from Ito's lemma that is satisfied by the prices of derivatives in a Black-Scholes world, defining the variables concerned.

(b) State the main assumptions underlying the equation. [3]

(ii) State the main constraints on the behaviour of a stock price for Ito's lemma to be applicable. [3]

[Total 6]

6 Discuss briefly the role of speculators, hedgers and arbitrageurs in the derivatives market and the use to which these groups put derivatives. [4]

7 (i) List the factors that affect the price of a European option. [2]

(ii) Explain how the value of a European option varies with each of the factors listed in (i). [6]

(iii) Briefly explain how an institution selling options protects itself against losses arising from changes in each of the factors in (i). [6]

[Total 14]

8 Draw a diagram to show bounds (ignoring transaction costs) on the possible values of an American call option as a function of the price of the underlying security. The bounds illustrated should be independent of any model for the price process followed by the underlying security. You should explain the rationale behind the bounds shown.

[5]

9 The price of a stock follows a binomial tree. The price of the stock at each node is shown on the tree and the probability of an upward move in the stock from any node is p.

		70
	60	
50		50
	40	
		30
t=0	1	2

An investor can choose between investing in the stock or cash. Cash deposits accumulate at a continuously compounded rate of 4.879%.

(i) Show that the investor can synthesise the payoff from a derivative exerciseable at $t = 1$, whose payoff depends on the stock price at that time.

(ii) (a) Find the value at $t = 0$ of a put with strike price 55 which is exerciseable at $t = 1$.

(b) Show that the value of the put is the discounted expectation with respect to a different probability measure Q.

(iii) With reference to the binomial tree explain what is meant by a "filtration".

(iv) Consider a European put with strike price 55 and expiry date $t = 2$.

(a) Find the risk neutral measure.

(b) Show that the discounted stock price process is a martingale with respect to the risk neutral measure.

(c) The stock price decreases from 50 at $t = 0$ to 40 at $t = 1$, and then increases to 50 at $t = 2$. Show that the hedging strategy is self-financing.

[Total 19]

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Specimen Solutions

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1 (i) A fund that buys (sells) a FTSE future obtains (sheds) UK equity market exposure. Thus if FTSE rises the counterparty with whom the fund has dealt will owe it some money (conversely if FTSE falls the fund will owe the counterparty money). [1]

To protect against the risk of default, each side to a futures transaction on LIFFE puts up a returnable deposit, called "initial margin", which is lodged with the exchange's central clearing house (which for LIFFE is the London Clearing House). [1]

The contract is marked-to-market daily with capital gains or losses arising from market movements paid between the two parties on a daily basis. Effectively the futures contract is closed out and rewritten each day through the marking-to-market process. [2]

An important point to note for a third party is that the central clearing house only provides a guarantee of performance for clearing members of LIFFE. [2]

Third parties will need to deal through a clearing member, called a "clearing agent". [1]

Thus the third party is actually exposed to the credit risk of the clearing agent it has used for the deal. [2]

The fund would need to lodge initial margin with its clearing agent (and would pay/receive variation margin from/to that agent). [1]

However, in practice the initial margin that the fund lodges with its clearing agent would be held in a client account under client money rules, and should therefore be more secure than, say, merely a loan to the clearing agent. [2]

(ii) (a) The rationale behind initial margins is that exchanges want initial margins to cover all likely daily changes in the value of the futures contract — this should cover all likely customer losses. [2]

Initial margin requirements can be changed at any time by the Clearing House if market conditions change suddenly. In practice the amount (per contract) rarely changes (to avoid administrative hassle). [2]

The level of initial margin is different for different contracts. In general, it's related to the price volatility of the underlying asset. The greater the price volatility the greater the initial margin requirement. [2]

Exchanges commonly set initial margin levels to equal

(average daily absolute changes in value of a futures contract)
+ 3(standard deviation of these daily price changes)

where the statistical data is measured over some recent time period. [2]

(b) The system LIFFE uses to calculate margin positions is called SPAN. [1]

If you have several option positions on LIFFE the initial margin you have posted is not necessarily the sum of the LIFFE calculated margins on the individual positions, for two reasons: [2]

Some contracts have offsetting risks, which the SPAN system takes into account when determining the overall initial margin required. [1]

You do not need to post exactly the right amount of margin with the clearing agent. Many investment managers have a looser system in which margin payments only move when the amount owed from one party to the other exceeds a certain amount, e.g. £50,000 (to avoid the administrative burden of large numbers of small payments). [1]

For options on futures contracts (eg the option on the long gilt future on LIFFE) the contract price is not paid at the time of purchase. Option positions are marked-to-market daily giving rise to variation margin flows. If such an option is exercised by the buyer, the buyer is required to pay the original contract price to the Clearing House. [1]

(c) Initial margin can also be lodged in the form of stocks pledged as collateral (an obvious approach if selling a FTSE future). These stocks may be ones similar to those underlying the contract (an obvious way of margining if the fund is say selling FTSE futures or call options on individual stocks). [1]

Other sorts of collateral, e.g. treasury bills and gilts may also be acceptable as collateral. Such collateral could be lodged with the clearing agent (or perhaps lodged in a custodial account controlled by a third party custodian, but not available to the original investor until the pledging of the stock has ceased). [1]

2 $S = 30$ $X = 29$ $r = 0.05$ $\sigma = 0.25$ $T = 0.3333$

$$d_1 = \frac{\ln\left(\frac{30}{29}\right) + \left(0.05 + \frac{0.25^2}{2}\right) \times 0.3333}{0.25\sqrt{0.3333}} = 0.4225$$

$$d_2 = \frac{\ln\left(\frac{30}{29}\right) + \left(0.05 - \frac{0.25^2}{2}\right) \times 0.3333}{0.25\sqrt{0.3333}} = 0.2782$$

$N(0.4225) = 0.6637$ $N(0.2782) = 0.6096$

$N(-0.4225) = 0.3363$ $N(-0.2782) = 0.3904$

(i) (a) The European call price is

$$30 \times 0.6637 - 29e^{-0.05\times 0.3333} \times 0.6096 = 2.52$$

\$2.52 [4]

(b) The American call price is the same as the European call price. [2]

(c) The European put price is

$$29e^{-0.05\times 0.3333} \times 0.3904 - 30 \times 0.3363$$

$= 1.05$

$= \$1.05$ [4]

(ii) $p + S = c + Xe^{-rT}$

$c = 2.52$ $p = 1.05$ $S = 30$ $X = 29$ $e^{-rT} = 0.9835$

$p + S = 31.05 = c + Xe^{-rT} = 31.05$ [4]

(iii) The present value of the dividend must be subtracted from the stock price to give a new stock price for the BS formula. [4]

$$30 - 0.5e^{-0.125\times0.05} = 29.5031$$

$$d_1 = \frac{\ln\left(\frac{29.5031}{29}\right) + \left(0.05 + \frac{0.25^2}{2}\right)(0.3333)}{0.25\sqrt{0.3333}} = 0.3068$$ [3]

$$d_2 = \frac{\ln\left(\frac{29.5031}{29}\right) + \left(0.05 - \frac{0.25^2}{2}\right)(0.3333)}{0.25\sqrt{0.3333}} = 0.1625$$ [3]

The price of the call option is

$$= 29.5031 \times 0.6205 - 29e^{-0.125\times0.05} \times 0.5645$$

$$= 2.04$$ [2]

The price of the put option is

$$= 29e^{-0.125\times0.05} \times 0.4355 - 29.5031 \times 0.3795$$

$$= 1.35$$

as $N(-d_1) = 0.3795$ and $N(-d_2) = 0.4355$ [2]

3 (i) **Binomial lattice calculation**

The binomial tree is constructed for the share price less the discounted value of future dividends (S^*) assuming that the volatility given applies to this figure rather than the actual share price. [2]
Actual share prices on the nodes prior to the dividend payment can be modelled by adding back the value of dividends but, as we are valuing a European option, this is unnecessary in this case. [1]

We have $S = 1..85$, $X = 1.90$, $r = 0.06$, $\sigma = 0.30$ and $\Delta t = 0.08333$. [1]

The required parameters for the binomial tree are:

$$u = \exp(\sigma\sqrt{\Delta t}) = 1.0905$$

$$d = \frac{1}{u} = 0.9170$$

$$a = \exp(r\Delta t) = 1.0050$$

$$p = \frac{a-d}{u-d} = 0.5073$$

$S^* = 185 - 7 \div 1.0050^3 = 178.1$ [2]

The tree is therefore:

				251.8 / **0**
			230.9 / **0**	
		211.8 / **2.9**		211.8 / **0**
	194.2 / **9.1**		194.2 / **5.8**	
178.1 / **17.6**		178.1 / **15.6**		178.1 / **11.9**
	163.3/ **26.6**		163.3 / **25.7**	
		149.8 / **38.3**		149.8 / **40.2**
			137.4 / **51.7**	
				126.0 / **64.0**

[8]

Where the first number given for each node is S^* and the second is the option value. The option price at time zero is thus 17.6p.

(ii) **Approximate values for Greeks**

Given the approximations involved it is sufficient to work with the tree for S^* rather than S.

Delta

Delta = Δoption price / $\Delta S \approx \frac{(9.1 - 26.6)}{(194.2 - 163.3)} = -0.57$ [4]

Gamma

Gamma = ΔDelta / ΔS

$$\approx \frac{\left[\frac{(2.9-15.6)}{(211.8-178.1)} - \frac{(15.6-38.3)}{(178.1-149.8)}\right]}{(211.8-149.8)/2}$$

$= \underline{0.014 \text{ (per penny)}}$ [4]

Theta

$$\text{Theta} = \frac{\Delta \text{ Option price}}{\Delta t} = \frac{(15.6-17.6)}{(2\times 0.833)}$$

$= \underline{-1.2 \text{ (pence per annum)}}$ [4]

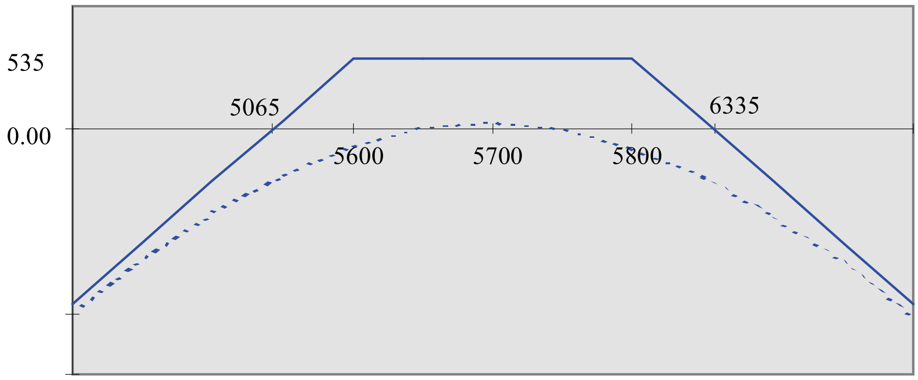
4 An approximate "strangle" diagram such as the one below is required.

Vertical axis is profit on strategy; horizontal is price of future.

The expiry lines must have 45° angles on the sides, through points shown (which should be marked).

The dotted line is the current time curve: this should pass through P&L zero at the points 5717 and 5683. [2]

The maximum profit at expiry is 535 if the underlying lies in the range 5600–5800. Unlimited loss. [2]



[4]

Delta position = –100 *(48% – 40%) = –8 i.e. short 8 contracts.

Therefore delta hedge is to buy 8 contracts of June 1998 FTSE future. [4]

If the market falls sharply, the call will be worthless and the put will become almost identical to a future (all intrinsic value). [2]

Hence the option position will be equivalent to being long 100 futures contracts. [1]

What actually happens to the trader's P&L depends on his next action, but he will probably make a loss, since the volatility outcome is likely to be higher than that assumed in the prices. [1]

The trader will also lose because the options will increase in volatility. [1]

Note that the option premiums imply a large volatility from the market prices.

The correct action for a delta hedger is to sell contracts until the position is neutral. [1]

This could mean selling up to 500 futures, plus any ones he had bought as an earlier delta hedge. [1]

A week before expiry will exacerbate the gamma effect. [1]

Hence:

- the final loss will be greater because the options will have little time value and hence little protection against sudden moves [1]
- there will be no loss due to volatility, since the time remaining is too small to allow any meaningful vega exposure [1]

5 (i) (a) The partial differential equation is:

$$\frac{\partial u}{\partial t} + (r-q)S\frac{\partial u}{\partial S} - ru + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 u}{\partial S^2} = 0 \qquad [1]$$

S is the value of the security underlying the derivative, σ is the volatility of (log) movements in S, u is the value of the derivative, r is the risk-free rate of interest and q is the dividend yield on the underlying security. [2]

[In the Black-Scholes p.d.e. q would be zero, since the security is assumed to have a zero dividend yield, but it would be non-zero in the

Garman-Kohlhagen formulation, so either would be correct as an answer, as long as the relevant assumption was clearly stated.]

(b) The main assumptions involved are:

Markets are arbitrage-free (i.e. there are at least some participants able to carry out arbitrage) [1]

Markets are frictionless (i.e. short-selling/borrowing is possible, transaction costs are nil and taxes payments/tax credits on profits/losses are symmetric) [1]

The volatility, σ, exists and is constant or a deterministic function of time. [1]

(ii) Ito's lemma applies to a variable x which follows an Ito process, i.e.

$$dx = a(x,t)dt + b(x,t)dz$$

where dz is a Weiner process (i.e. a Brownian motion). [2]

The main constraints are therefore that S follows such a process in which a and b are just functions of x and t. [2]
There are also regularity conditions on a and b to permit integration over time. [2]

6 Speculators wish to take a position in the market betting that a price will either go up or down. [1]

Futures, forwards, options and other derivative instruments allow speculators to take positions in an asset with much less capital than would be required to achieve the same position in the cash market. [1]

Speculators add liquidity to the derivatives markets. [1]

Hedgers want to eliminate or reduce an exposure to movements in the price of an asset. [1]

Forward contracts, say, allow hedgers to reduce their exposure or eliminate it without an initial payment. Hedging using forwards or futures makes the outcome more certain but does not necessarily make it better relative to the unhedged position. [1]

Option strategies allow hedgers to **insure** (upside benefits only — for a premium) their positions against adverse market movements for the payment of an up-front premium. [1]

Arbitrage involves locking in a riskless profit by simultaneously entering into transactions in two or more markets. [1]

Many apparent arbitrage opportunities are eliminated by the transaction costs involved. The main arbitrageurs are major capital markets players (e.g. market making departments of large international investment banks) which face very low transactions costs. [1]

Very large positions are required to profitably exploit the small size of the arbitrage opportunities. [1]

[Maximum marks to be awarded for any answer: 8]

7 (i) Stock price
Strike price
Time to expiry
Volatility
Risk free rate
Dividends

if all correct [4]
extra or missing factors [−1]

(ii) Effect on price of option. (Assuming 1 factor altered at a time.)

	Call	*Put*	
Stock price	+	+	[2]
Strike price	–	+	[2]
Time to expiry	?	?	[2]
Volatility	+	+	[2]
Risk free rate	+	–	[2]
Dividends	–	+	[2]

(iii) When an option is old the stirke price is agreed between the counterparties. There is no risk from changing strike price for a "vanilla" option. [1]

An institution will monitor its exposure to each of the factors that affect option prices. This is achieved by calculating the sensitivity of the portfolios to changes in each factor. These sensitivities are often called the "Greeks". [1]

An institution will calculate

Delta	Δ	the change in value of the portfolio from changes in the price of the underlying [1]
Gamma	Γ	change in Δ from changes in the price of the underlying [1]
Vega	γ	change in the value of the portfolio arising from changes in the implied volatility [1]
Theta	θ	change in the portfolio value over time [1]
Rho	ρ	change in the value of the portfolio from changes in interest rates [1]

Dividend uncertainty is difficult to measure and protect against. If a larger dividend is paid than expected, and all other things are equal, then we would expect the stock price to fall and so the effect on the option price would be captured by the Δ and Γ. [1]

In practice "all other things are not equal" and a larger than expected dividend may be the result of favourable trading conditions which would have a positive effect on the stock price. [1]

After calculating the "Greeks" an institution can gauge the level of risk to each factor. If a portfolio is Δ neutral then it has no exposure to changes in the price of the underlying. [1]

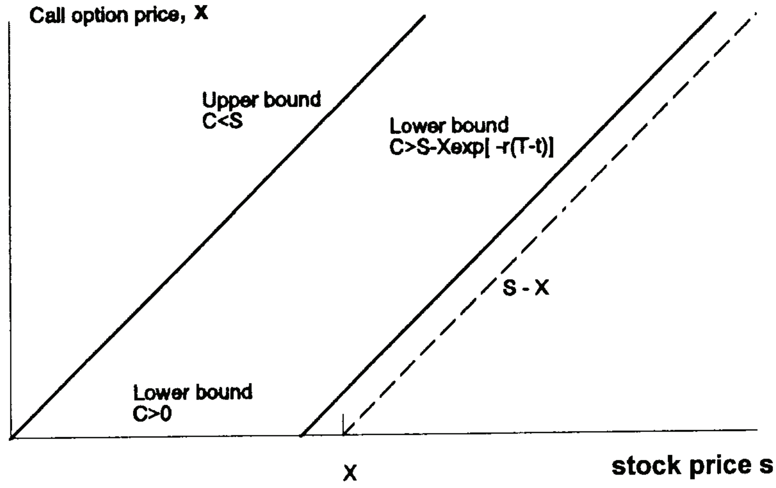
The portfolio position will be calculated for all Greeks and value at risk (VaR) calculations should be carried out to assess the exposure to the portfolio of changes in each of the factors. [2]

Also VaR calculations should be performed for different factors moving together.

8 Bounds for an American call option

The underlying share price is an upper bound because if the call were priced higher than the share it would be possible to sell the call and buy the share to make a risk free immediate profit. [2]

The value of the call must also be greater than zero and greater than the share price less the present value of the exercise price discounted at the risk free rate. If the call were priced less than this it would be possible to make a risk free profit by buying the call, selling the share and investing the discounted exercise price at the risk free rate. [2]



Bounds on the price of a call option

[4]

r is the risk free interest rate
t is the current time
T is the expiry date of option [2]

9 (i) Let ψ be the holding of the cash/bond at $t = 0$

ϕ be the holding of stock at $t = 0$

Consider a derivative that pays

$f(60)$ if the stock price = 60 at $t = 1$
$f(40)$ if the stock price = 40 at $t = 1$

Require

$$f(60) = \phi \,.\, 60 + \psi e^r$$
$$f(40) = \phi \,.\, 40 + \psi e^r$$ [1]

$\Rightarrow \quad f(60) = \phi \,.\, (60) + f(40) - \phi \,.\, 40$

$$\phi = \frac{f(60) - f(60)}{60 - 40} \qquad [1]$$

$$\psi = e^{-r}\,(f(60) - \phi \,.\, 60) \qquad [1]$$

Hence if investor holds ϕ stock and ψ cash/bond at $t = 0$ can synthesise a derivative whose payoff at $t = 1$ depends on the stock price at $t = 1$. [1]

(ii) (a) Now $\phi = \dfrac{0 - 15}{20} = -¾$

$$\psi = (0 - (-¾) \,.\, 60)\, e^{-0.04879} = 42.86$$

Check value at $t = 1$

if stock price $= 60 \Rightarrow -¾ \times 60 + 42.86 \times e^{r} = 0$

$= 40 \Rightarrow -¾ + 40 + 42.86 \times e^{r} = 15$ [2]

Value at $t = 0$ $= -¾ \times 50 + 42.86 = \underline{\underline{5.36}}$ [2]

(b) Let the value of the derivative be $f(50)$ at $t = 0$.

$$f(50) = \phi \,.\, 50 + \psi$$

$$= \frac{f(60) - f(40)}{60 - 40} + 50 + e^{-r}\left(f(60) - \frac{f(60) - f(40)}{60 - 40} \,.\, 60\right)$$

[2]

$$= e^{-r}\,(f(60)\, q + f(40)\,(1 - q))$$

where $q = \dfrac{50 \,.\, e^{r} - 40}{60 - 40} = 0.625$ [2]

Hence $f(50) = E_Q[F_{t=1}(.) \mid \text{at } t = 0 \text{ stock price} = 50]$

Under the measure Q probability of stock price moving up is 0.625 and probability of stock price moving down is 0.375. [2]

$$\therefore f(50) = 0.625 \times f(60)\, e^{-r} + 0.375 \times f(40)\, e^{-r}$$

$$= \underline{\underline{5.36}}$$ [2]

(iii) A filtration F_t is the stock price history up to time t.

If s_t is the stock price at time t, then a filtration $F_t = (s_0, s_1, \ldots, s_t)$

if the stock price at $t = 1$ is 60 then

$$F_1 = \{50, 60\}$$ [4]

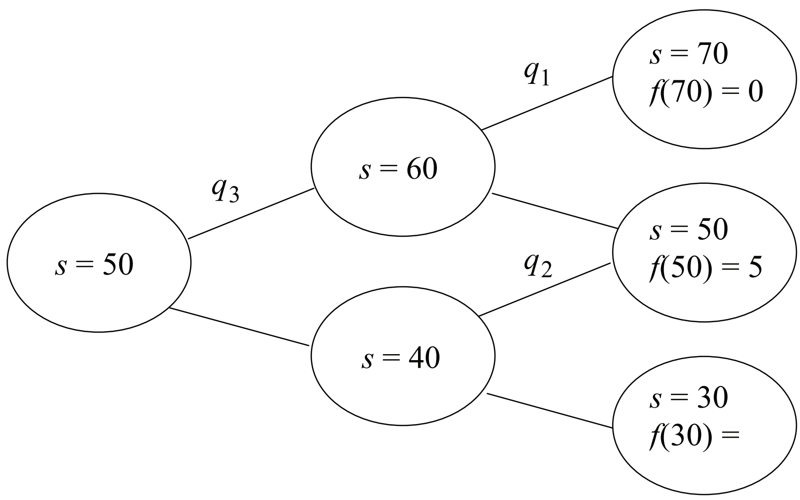
(iv) Let S_{now} be the stock value at the node being considered. S_{up} and S_{down} are the possible values of the stock at the end of the time period.

From (ii)(b) we know that

$$q = \frac{S_{\text{now}}\, e^r - S_{\text{down}}}{S_{\text{up}} - S_{\text{down}}}$$

where q is the probability of the up move under the new probability measure Q.

Consider the tree



(a) $$q_1 = \frac{60e^r - 50}{70 - 50} = 0.65$$ [1]

$$q_2 = \frac{40e^r - 30}{50 - 30} = 0.6$$ [1]

$$q_3 = \frac{50e^r - 30}{60 - 40} = 0.625$$ [1]

(b) Need to show that

$$E_Q[e^{-r.n}\, s_{t+n} | F_t] = s_t \qquad n \geq 0$$ [2]

consider $t = 1$ and $s_1 = 60$

$$e^{-0.04879}[70q_1 + 50(1 - q_1)] = 60 = s_1$$ [1]

If $t = 1$ and $s_1 = 40$ then

$$e^{-0.04879}[50q_2 + 30(1 - q_2)] = 40$$ [1]

If $t = 0$

$$e^{-0.04879}[60q_3 + 60(1 - q_3)] = 50$$ [1]

Finally $s_0 = 50$

$$E_Q[e^{-2r}s_2 | F_0] = E_Q[E_Q[e^{-r}s_2 | F_1] | F_0]$$

$$= e^{-0.04879}[60q_3 + 40(1 - q_3)]$$ [1]

$$= 50$$

(c) At $t = 1$ hedging strategy requires holding

stock of $\phi \quad = \frac{f(50) - f(30)}{50 - 30} = \frac{5 - 25}{20} = -1$ [1]

$$\text{cash/bond } \psi = e^{-r}(f(50) - \phi\, 50)$$
$$= e^{-0.04879}(5 - (-1)\,.\,50) = 52.38 \qquad [1]$$

Value of hedge at $t = 1$ (when stock price = 40)

$= (-1)\ .\ 40 + 52.38$

$= 12.38$ [1]

OR $f(40) = e^{-r}(q_2 \times 5 + (1 - q_2)\, 25) = 12.38$

[Check claim is replicated at $t = 2$

if $s = 50$ thus $f(50) = 52.38 \times 1.05 - 1 \times 50 = 5$

if $s = 30$ thus $f(30) = 52.38 \times 1.05 - 30 = 25$]

$f(60) = e^{-r}(q_1 \times 0 + (1 - q_1) \times 5)$

$= 1.667$ [1]

$f(50 \text{ at } t = 0) = e^{-r}(q_3 + 1.667 + (1 - q_3) \times 12.38)$

$= 5.4138$ [1]

at $t = 0$ $\phi = \dfrac{f(60) - f(40)}{60 - 40} = \dfrac{1.667 - 12.38}{20}$

$= -0.5355$ [1]

$\psi = e^{-0.04879}(f(60) - \phi\,.\,60)$

$= 32.1873$ [1]

if stock goes down to 40 at $t = 1$

value at $t = 1 = -0.5355 \times 40 + 32.1873 \times e^{0.04879}$

$= \underline{\underline{12.38}}$ [2]

This is value of hedge at $t = 1$, hence self-financing.