ROLL NO. _____

DipIETE – ET/CS (NEW SCHEME) – Code: DE51 / DC51

Subject: ENGINEERING MATHEMATICS - I

Time: 3 Hours **Max. Marks: 100**

DECEMBER 2011

NOTE: There are 9 Questions in all.

- **Please write your Roll No. at the space provided on each page immediately after receiving the Question Paper.**
- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. $\lim\limits_{x\to 0} \dfrac{\sin 5x + 2x}{3x + \sin 3x}$ is :

(A) $\dfrac{7}{6}$ **(B)** $\dfrac{6}{7}$

(C) $\dfrac{5}{6}$ **(D)** $\dfrac{6}{5}$

b. If $y = \dfrac{x^3 \cos x}{\sin x}$, then $\dfrac{dy}{dx}$ is

(A) $x^3 \operatorname{cosec}^2 x + 3x^2 \cot x$ **(B)** $3x^3 \operatorname{cosec}^2 x - x^2 \cot x$

(C) $-x^3 \operatorname{cosec}^2 x - 3x^2 \cot x$ **(D)** $-x^3 \operatorname{cosec}^2 x + 3x^2 \cot x$

c. $\int \dfrac{x^2}{a^6 - x^6} dx$ is

(A) $\dfrac{1}{6a^3} \log \left| \dfrac{a^3 + x^3}{x^3 - a^3} \right| + C$ **(B)** $\dfrac{1}{6a^3} \log \left| \dfrac{a^3 + x^3}{a^3 - x^3} \right| + C$

(C) $\dfrac{1}{6a^3} \log \left| \dfrac{a^3 - x^3}{a^3 + x^3} \right| + C$ **(D)** $\dfrac{1}{6a^3} \left| \dfrac{x^3 - a^3}{x^3 + a^3} \right| + C$

d. If $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, then X & Y is

(A) $X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}, Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ **(B)** $X = \begin{bmatrix} 5 & 1 \\ 4 & 0 \end{bmatrix}, Y = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

(C) $X = \begin{bmatrix} 0 & 5 \\ 1 & 4 \end{bmatrix}, Y = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$ **(D)** $X = \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

e. If $\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ and ω is a cube root of unity, then the value of Δ is

(A) 1 **(B)** 3
(C) 0 **(D)** 2

f. The coefficient of x^5 in the expansion of $\left(x + \frac{1}{x^3}\right)^{17}$ is

(A) 630 **(B)** 650
(C) 670 **(D)** 680

g. The solution of the differential equation $\frac{dy}{dx} = e^{y+x} + e^{y-x}$ is

(A) $y = e^x(x+1)$ **(B)** $y = e^x(x+1)+1$
(C) $y = e^x(x-1)+1$ **(D)** none of these

h. If $A + B = 45^\circ$, then the value of $(\cot A - 1)(\cot B - 1)$ is

(A) 1 **(B)** -2
(C) 2 **(D)** -1

i. The area of the quadrilateral whose vertices, taken in order, are $(1,1)$, $(3,4)$, $(5,-2)$ & $(4,-7)$ is

(A) $\frac{43}{2}$sq.units **(B)** $\frac{41}{2}$sq.units
(C) $\frac{45}{2}$sq.units **(D)** $\frac{47}{2}$sq.units

j. Three consecutive vertices of a parallelogram ABCD are $A(3,0)$, $B(5,2)$, $C(-2,6)$. Then the fourth vertex D is

(A) $(4,-4)$ **(B)** $(4,4)$
(C) $(3,-4)$ **(D)** $(-4,4)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $y = x^{(x^x)}$, then find $\frac{dy}{dx}$ **(8)**

b. Find the equations of the tangent and the normal to the curve $y^2 = 4ax$ at $(at^2, 2at)$. **(8)**

Q.3 a. $\int e^{2x} \sin 5x dx$ **(8)**

b. $\int_1^3 \frac{1}{(x+1)(x+2)(x+3)}dx$ **(8)**

Q.4 a. Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $x^2 - 3x - 7 = 0$. Thus, find A^{-1}. **(8)**

b. Whether the system is consistent or inconsistent also find the solution, by Cramer's Rule, if exists

$x - y + 3z = 6$

$x + 3y - 3z = -4$

$5x + 3y + 3z = 10$ **(8)**

Q.5 a. Solve $(x^2 - y^2)dx = 2xydy$ **(8)**

b. Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$ **(8)**

Q.6 a. Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5.....(2n-1)}{n!}2^n.x^n$ **(8)**

b. Find four numbers in A.P. whose sum is 20 and the sum of whose squares is 120. **(8)**

Q.7 a. If A, B, C are the angles of a triangle, then prove that :
sin 2A + sin 2B + sin 2C= 4 sin A sin B sin C **(8)**

b. Prove that

$\sin^2 A + \sin^2(60^\circ + A) + \sin^2(60^\circ - A) = \frac{3}{2}$ **(8)**

Q.8 a. Find the equation of a straight line passing through the point (3, 4) and inclined to positive direction of x-axis at an angle of $\frac{3\pi}{4}$. Find also the co-ordinates of two points on it, on opposite side of (3, 4) and at a distance of $\sqrt{2}$ from it. **(8)**

b. Find the distance between the lines

$9x + 40y - 20 = 0$ and $9x + 40y + 21 = 0$ **(8)**

Q.9 a. Find the equation of the circle of radius 5 whose centre lies on y- axis passes through (3, 2). **(8)**

b. Find the vertex, focus and directrix of the parabola $4y^2 + 12x - 12y + 39 = 0$ **(8)**