Subject: ENGINEERING MATHEMATICS - I

Time: 3 Hours **DECEMBER 2010** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x \sin x}$ is :

(A) $\dfrac{1}{2}$ **(B)** $\dfrac{1}{3}$

(C) $\dfrac{1}{4}$ **(D)** $\dfrac{2}{3}$

b. If $y = (x+1)(x+2)$, then $\dfrac{dy}{dx}$ is

(A) $2x-3$ **(B)** $3x+2$

(C) $2x+3$ **(D)** $3x-2$

c. $\int x \cot^{-1} x dx$ is

(A) $\dfrac{x^2}{2}\cot^{-1} x + \dfrac{1}{2}\left[x - \tan^{-1} x\right] + C$ **(B)** $x \cot^{-1} x + 2\left[x + \tan^{-1} x\right] + C$

(C) $x \cot^{-1} x - 2\left[x - \tan^{-1} x\right] + C$ **(D)** $\dfrac{x^2}{2}\cot^{-1} x - \dfrac{1}{2}\left[x + \tan^{-1} x\right] + C$

d. If $\Delta = \begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$, then value of x is:

(A) $\dfrac{1}{3}, -2$ **(B)** $\dfrac{1}{2}, 3$

(C) $\dfrac{1}{3}, 2$ **(D)** $\dfrac{1}{2}, -3$

e. If $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & zw \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$ then x, y, z and w is equal to

(A) 4,3,2,1 **(B)** 1,2,3,4
(C) 2,4,1,3 **(D)** 3,2,4,1

f. The order and degree of differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \sin x$ is

(A) 0 = 2, D = 1 **(B)** 0 = 2 D = 2
(C) 0 = 1, D = 1 **(D)** 0 =1, D = 2

g. The eighth term from the beginning in the expansion of $\left(x + \frac{1}{y}\right)^{11}$ is

(A) $\frac{333x^4}{y^4}$ **(B)** $\frac{33x^4}{y^4}$
(C) $\frac{30x^4}{y^4}$ **(D)** $\frac{330x^4}{y^4}$

h. The value of $2\cos\frac{\pi}{13}\cdot\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$ is

(A) 1 **(B)** 2
(C) 0 **(D)** 3

i. The Cartesian co-ordinate $\left(2, 2\sqrt{3}\right)$ is equal to the polar co-ordinate is

(A) $4, \pi/3$ **(B)** $2, \pi/2$
(C) $1, \pi/3$ **(D)** $2, \pi/3$

j. The area of a triangle whose vertices are (2,7), (3,-1), (-5,6) is

(A) 28 sq. unit **(B)** 28.5 sq. unit
(C) 25 sq. unit **(D)** 25.5 sq. unit

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $\sin y = x\sin(a+y)$, then prove that, $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$. **(8)**

b. Find the maximum and minimum values of the function $f(x) = \frac{4}{x+2} + x$. **(8)**

Q.3 a. Evaluate $\int \sec^3 x dx$. **(8)**

b. Evaluate $\int_0^{\pi/2} \sin 2x \cdot \log(\tan x)\, dx$. **(8)**

Q.4 a. Show that, $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}^{-1}$. **(8)**

b. Using Cramer's method solve the following system of linear equations for x, y, z

$$x + y + z = -1$$
$$x + 2y + 3z = -4$$
$$x + 3y + 4z = -6$$

(8)

Q.5 a. Solve $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$. **(8)**

b. Solve $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$ **(8)**

Q.6 a. Find the term independent of x in the expansion of $\left(3x^2 - \frac{1}{2x^3}\right)^{10}$. **(8)**

b. The sum of first three terms of a G.P. is 16 and the sum of the next three term is 128. Find the sum of nth terms of G.P. **(8)**

Q.7 a. Prove that, $\frac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta} = \tan 6\theta$. **(8)**

b. In any triangle ABC prove that **(8)**

$$\frac{a^2 \sin(B - C)}{\sin A} + \frac{b^2 \sin(C - A)}{\sin B} + \frac{c^2 \sin(A - B)}{\sin C} = 0$$

Q.8 a. Show that the equation of the straight line through the origin making angle ϕ with line y = mx + b is $\frac{y}{x} = \frac{m \pm \tan\phi}{1 \mp m\tan\phi}$. **(8)**

b. If the lines y = 3x+1 and 2y = x+3 are equally inclined to the line y = mx+4. Find the value of m. **(8)**

Q.9 a. Find the equation of the ellipse having its Centre at the point (2, -3) and one focus at (3, -3) and one vertex at (4, -3). **(8)**

b. Find the co-ordinate of foci, eccentricity, length of the latus-rectum of the ellipse $25x^2 + 4y^2 = 100$. **(8)**