Actuarial Society of India

EXAMINATIONS

1st November 2006

Subject CT4– Models

Time allowed: Three Hours (10.30 am – 1.30 pm)

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor

103 Questions

A1) A continuous time Markov sickness and death model has four states:

H : Healthy
S : Sick
T : Terminally Ill
D : Dead

From a healthy state the transitions are possible to states S and D, each at the rate of 0.05 per year. A sick person recovers his health at the rate of 1.0 per year; other possible transitions are to D and T, each with a rate of 0.1 per year. Only one transition is possible from the terminally ill state; and that is to state D with transition rate of 0.4 per year.

(i) Let $P(t) = \{p_{ij}(t): i,j ? H, S, T, D\}$ where denotes $p_{ij}(t)$ the probability of being in *state j* at time t given that the individual was in *state i* at time 0. State the Kolmogrov Forward Equation satisfied by the matrix P(t), making sure that you specify the entries of the matrix A which appears as a part of the solution. **(5)**

(ii) Calculate the probability of being healthy for at least 10 uninterrupted years given that you are healthy now. **(2)**

(iii) Let d_j denote the probability that a life, which is currently in *state j*, will never suffer terminal illness. By considering the first transition from state H, show that $d_H ? \frac{1}{2} ? \frac{1}{2} d_S$ and deduce similarly that $d_S ? \frac{1}{12} ? \frac{5}{6} d_H$. Hence, evaluate d_H and d_S. **(8)**

(iv) Write down the expected duration of a terminal illness, starting from the moment of first transition in to state T. Use the result of part iii to deduce the expectation of the future time spent terminally ill by an individual who is currently healthy. **(5)**
[20]

A2) Anil is currently in level 1 and is collecting Pokemon cards, which are given away free with éclairs packets. There are 50 different cards in the series and each éclairs packet contains one card, with each card being equally likely.

(i) Describe how the situation can be modeled as a Markov Process **(3)**

(ii) Calculate the expected number of éclairs packet that Anil has to buy until a complete set of 50 cards is obtained.

Note that you may use the approximation: $\overset{n}{\underset{k?1}{?}} \frac{1}{k} ? \ln n ? 0.5771$ **(4)**

(iii) How would you modify the model in part (i) if each éclairs contained two <u>different</u> cards? **(3)**
[10]

A3) A motor insurer operates a no – claims discount system that has 5 levels. The percentage of the basic premium paid by the insured in each level is as follows:

Level	Percentage premium charged
5	100
4	90
3	80
2	70
1	60

Insured motorists move between levels depending on the number of claims in the previous year. For each policyholder, the number of claims per year follows a Poisson distribution with a mean of 0.25.

For those in levels 2, 3, 4 and 5 at the start of the previous year:

? If no claims are made during the previous year, the insured moves down one level (e.g. from Level 4 to level 3)

? If one claim is made during the previous year, the insured moves up one level (except those in level 5 at the start of the previous year, who will remain in Level 5)

? If two claims are made during the previous year, the insured moves up two levels (except those in Level 5 at the start of the previous year, who will remain in Level 5 and those in Level 4 will move to Level 5)

? If three of more claims are made during the previous year, the insured moves to Level 5

For those in Level 1 at the start of the previous year, a no claims discount protection policy applies whereby they remain in Level 1 if they make one claim. If they make two claims, they move to Level 2. If they make 3 or more claims, they move to Level 5. If they make no claims, they remain in Level 1.

(i) Determine the transition matrix for the no claims discount system (assuming that all motorists continue their policy) **(4)**

(ii) A policyholder is in Level 3 for the 1st year of the policy. Assuming that the policy is maintained, calculate the probability that at the start of the 3rd year the policyholder will be

a. In Level 1
b. In Level 3 **(4)**

(iii) Answer the following:

a. State the conditions under which the probability of being in a particular state after n years (as n ?   ?) is independent of the initial state
b. Verify the conditions are satisfied in this instance
c. Determine the ultimate probability that the insured will be in Level 1 **(10)**

(iv) The insurer suspects that the model used for its calculations may be too simplistic. Given annual data listing numbers of claims per policy, broken down by discount level, state which test would be the most appropriate to test the assumption that the distribution of the number of claims per policy per year is Poisson with mean 0.25. **(2)**

[20]

104 Questions

B1)

a) Define complete and curtate expectation of life and derive from first principles their algebraic expressions. (4)

b) For a particular population e_{50}= 25.35 and e_{51}=24.65 .Calculate q_{50} (2)

[6]

B2)

a) Define Type I and Type II censoring (2)

b) The following data relate to 12 patients who had an operation which was intended to correct a life threatening condition, where time 0 is the start of the period of investigation

Patient Number	Time of Operation (in weeks)	Time Observation ended (in weeks)	Reason Observation ended
1	0	120	Censored
2	0	68	Death
3	0	40	Death
4	4	120	Censored
5	5	35	Censored
6	10	40	Death
7	20	120	Censored
8	44	115	Death
9	50	90	Death
10	63	98	Death
11	70	120	Death
12	80	110	Death

You can assume that censoring was non-informative with regard to the survival of any individual patient.

(i) Compute the Nelson-Aalen estimate of the cumulative hazard function ? (t), where *t* is the time since having the operation (6)

(ii) Using the results of part (i), deduce an estimate of the survival function for patients who have had this operation (2)

(iii) Estimate the probability of a patient surviving for at least 70 weeks after undergoing the operation (1)

[11]

B3) A large life office is investigating the recent mortality experience of its term assurance policyholders. It has been decided to graduate the data by reference to a standard table using the formula:

$$q_x / q^s_x = ax+b$$

Where q^s_x is the rate for standard table.

a) Explain why it is necessary to graduate crude rates of mortality for practical use? (4)

b) Describe briefly how you would estimate a and b in the formula using (8)

i.) A weighted least squares method
ii.) A maximum likelihood method

[12]

B4)

(i) Given that $p_x = 0.9$, calculate ${}_{0.5}p_x$ and ${}_{0.5}p_{x+0.5}$ using the following assumptions about mortality between ages x and x+1:
(a) Uniform distribution of deaths
(b) Balducci assumption (4)

(ii) Comment on how appropriate you think each of these assumptions is (2)

[6]

B5) A mortality investigation has been carried out over the three calendar years: 2000, 2001 and 2002.
Deaths during the Period of Investigation , ?x have been classified by age *x* at the date of death , where
x = calendar year of death – calendar year of birth.

(i) State the principle of correspondence (1)

(ii) State the rate year implied by this classification and give the age range of the lives at the beginning of the rate year. (2)

(iii) Censuses of the numbers alive on 1 July 2000, 1 July 2001 and 1 July 2002 have been tabulated and denoted by Px(½), Px(1½) and Px(2½) respectively, where x is the age determined at the date of each census.
The force of mortality at age $x+f$ is to be estimated using the formula

$$\mu_{x+f} = \quad ?x/ (Px(½)+Px(1½) + Px(2½)$$

Where ?x is the number of deaths.

a.) Determine the appropriate age definition *x* in $P_x(t)$, t =½,1½,2½ if this formula is correct. (3)
b.) Determine the value of f stating clearly all the assumptions you have made. (3)

[9]

B6) A mortality table has been estimated for the ages 4 to 100 inclusive. The rates have been graduated fitting a mathematical formula to the crude estimates. The deviations of the observed number of deaths from the expected number of deaths at each age using the graduated mortality rates have been calculated. The results are:

Positive Deviations	57
Negative Deviations	40

Test this graduation using the Signs Test by:
a) stating the Null Hypothesis
b) stating the sampling distribution of the test statistics if the Null Hypothesis is true
c) completing the test and stating your conclusions **[6]**

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