Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.Tech/SEM-2/M-201/2010

2010

MATHEMATICS

Time Allotted : 3 Hours | Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{100} is

a) $2^{99} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

b) $2^{101} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

c) $2^{100} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

d) none of these.

ii) If λ is a eigenvalue of A, then λ^4 is an eigenvalue of

a) A^4 b) A^3

c) A d) none of these.

iii) The rank of $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ is

a) 0 b) 2

c) 1 d) none of these.

iv) Which of the following sets is linearly independent ?

a) $\{(1, 2), (2, 4)\}$

b) $\{(1, 2, 3), (2, 4, 6), (1, 1, 1)\}$

c) $\{(2, 0, 0), (0, 3, 0), 0, 0, 4)\}$

d) none of these.

v) $L\{e^{-2t} \cos t\}$ is equal to

a) $\frac{p}{p^2 + 4p + 5}$

b) $\frac{p - 1}{p^2 + 4p + 5}$

c) $\frac{p + 1}{p^2 + 4p + 5}$

d) $\frac{p + 2}{p^2 + 4p + 5}$.

vi) Which of the following is not true ?

a) $\Delta \equiv E - 1$ b) $\Delta \nabla \equiv \Delta - \nabla$

c) $\Delta \frac{1}{\nabla} \equiv \Delta + \nabla$ d) $\Delta \equiv 1 - E^{-1}$.

vii) If the true and approximate value of a quantity are x_t and x_a respectively, then the relative error is given by

a) $\left|\frac{x_t - x_a}{x_t}\right|$ b) $\left|\frac{x_a - x_t}{x_a}\right|$

c) $\left|\frac{x_a - x_t}{x_t - x_a}\right|$ d) $|x_t - x_a|$.

viii) The sum of the eigenvalues of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$ is

a) 4 b) 5

c) 2 d) none of these.

ix) The value of the determinant $\begin{vmatrix} 17 & 58 & 97 \\ 19 & 60 & 99 \\ 18 & 59 & 98 \end{vmatrix}$ is

a) 1 b) 2

c) 3 d) 0.

x) The value of λ for which the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 3 & -2 & \lambda \end{bmatrix}$ is singular, is

a) 3/2 b) 2

c) 1 d) 1/3.

xi) $\frac{1}{D-1}x^2$ is equal to

a) $x^2 + 2x + 2$

b) $-\left(x^2 + 2x + 2\right)$

c) $2x - x^2$

d) $-\left(2x - x^2\right)$.

xii) The norm of the vector $\alpha = (-1, 2, 3)$ in R^3 with standard inner product is

a) $\sqrt{12}$ b) $\sqrt{14}$

c) $\sqrt{3}$ d) $\sqrt{2}$.

xiii) The degree and order of the differential equation $\left(\frac{d^2y}{dx^2} + 2\right)^{3/2} = x\frac{dy}{dx}$ are respectively

a) $\frac{3}{2}$, 2 b) 2, 3

c) 3, 2 d) 1, 4.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the following system of equations with the help of Gauss' Elimination method :

$$x_1 + x_2 + 4x_3 = 6$$

$$3x_1 + 2x_2 - 2x_3 = 9$$

$$5x_1 + x_2 + 2x_3 = 13.$$

3. Prove that $\Delta \equiv e^{hD} - 1$.

(The notations have their usual meanings).

4. Expand by Laplace's method to prove that

$$\begin{vmatrix} o & a & b & c \\ -a & o & d & e \\ -b & -d & o & f \\ -c & -e & -f & o \end{vmatrix} = (af - be + cd)^2 .$$

5. Solve $\frac{dy}{dx} + y = y^3 (\cos x - \sin x)$.

6. Evaluate $\int_0^{\pi/6} \sqrt{1 + \sin x}\, dx$ using Simpson's one-third rule by taking five ordinates.

GROUP - C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that (3, 1, - 2), (2, 1, 4) and (1, - 1, 2) form a basis of R^3 .

b) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{bmatrix}$$

c) Solve by Cramer's rule :

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x - y + z = 2.$$

8. a) Solve the differential equaion by Laplace Transformation :

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 3y = t\cos t$$

$$y(0) = 0,\ y'(0) = 0.$$

b) Solve by the method of variation of parameters :

$$\frac{d^2y}{dx^2} + a^2y = \sec ax\ .$$

c) Find the particular integral of

$$(D^2 + 4)\,y = x\sin^2 x\ .$$

9. a) Estimate the missing term from the table :

x	2	4	6	8	10
y	5	13	*	53	85

b) The values of a function $f(x)$ are given for certain values of x as follows :

x :	4	5	6	8
$f(x)$:	3·11	2·96	2·85	2·7

Obtain the value of $f(5{\cdot}5)$ using Lagrange's interpolation formula.

c) Compute $\int_1^2 \frac{\sin x}{\sqrt{x}}\,dx$ using Simpson's one-third rule taking $h = \frac{1}{6}$.

10. a) Prove that for two invertible matrices A and B of the same order $(AB)^{-1} = B^{-1}A^{-1}$.

b) Reduce the following matrix to a row-reduced echelon form and hence find its rank :

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{bmatrix}$$

c) Solve $(D^2 - 5D + 6)\, y = x^2 e^{3x}$, $D \equiv \frac{d}{dx}$.