ENGINEERING MATHEMATICS—2009

M-201/09

Time : 3 Hours | Full Marks : 70

Group – A

[Multiple Choice Type Questions]

1. Choose the correct alternatives for any ten of the following : $10 \times 1 = 10$

i) If $A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, then A^{100} is

(a) $\begin{bmatrix} 1 & 0 \\ -150 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ -50 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ -100 & 1 \end{bmatrix}$ (d) None of these

ii) The set of vectors {(2, 1, 1,), (1, 2, 2), (1, 1, 1,)} in R^3 is
(a) linearly dependent (b) linearly independent
(b) basis of R^3 (d) none of these

iii) The matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is
(a) an orthogonal matrix (b) a symmetric matrix
(c) an idempotent matrix (d) a null matrix

iv) The value of the determinant $\begin{vmatrix} 1 & 4 & 16 \\ 1^2 & 2^2 & 4^2 \\ 0 & 1 & 6 \end{vmatrix}$ is
(a) 0 (b) 1 (c) 4 (d) 22

v) The solution of a system of n linear equations with n unknowns is unique if and only if
(a) det A = 0 (b) det A > 0 (c) det A < 0 (d) det A ≠ 0

vi) The eigenvalues of the matrix $\begin{bmatrix} 1 & 4 \\ 4 & 1 \end{bmatrix}$ are
(a) –5, –3 (b) 5, –3 (c) 3, –5 (d) 5, 3

vii) The general solution of p = log (px - y) where $p = \frac{dy}{dx}$ is
(a) $y = cx - c$ (b) $y = cx - e^c$ (c) $y = c^2x - e^{-c}$ (d) none of these

viii) Which of the following is not true (the notations have their usual meanings)?
(a) $\Delta = E - 1$ (b) $\Delta,\nabla = \Delta - \nabla$ (c) $\frac{\Delta}{\nabla} = \Delta - \nabla$ (δ) $\nabla = 1 - E^{-1}$

ix) $\Delta^2 e^x$ is equal to (h = 1)
(a) $(e-1)^2 e^x$ (b) $(e-1)\, e^x$ (c) $e^{2x}(e-1)$ (d) e^{2x}

x) The value of $\int_0^\infty \frac{\sin t}{t}\,dt$ is equal to
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

xi) If S and T are two subspaces of a vector space V, then which one of the following is a subspace of V also?

(a) $S \cup T$ (b) $S \cap T$ (c) $S - T$ (d) $T - S$

xii) If $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$ is the characteristic equation of a square matrix A, then A^{-1} is equal to

(a) $A^2 - 6A + 9I$ (b) $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{2}I$ (c) $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}$ (d) $A^2 - 6A + 9$

xiii) Co-factor of –3 in the determinant $\begin{vmatrix} -2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{vmatrix}$ is

(a) 4 (b) – 4 (c) 0 (d) none of these

Ans.: 1.(i). $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = I - B$ where $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

The Characteristic polynomial is, $\begin{vmatrix} \lambda & 0 \\ 1 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 = 0$

$\therefore$ The characteristic equation of B is $B^2 = 0$.

$A^{100} = (I - B)^{100} = I^{100} - 100_{C_1} I^{99}B + 100_{C_2} I^{98}B^2 - \ldots\ldots + B^{100} = I - 100B$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 100\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -100 & 1 \end{pmatrix}$ **Ans.:** (c)

(ii) $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 2(2-2) - 1(1-2) + 1(1-2) = 0 + 1 - 1 = 0$

$\therefore$ The given set of vectors are linearly dependent. **Ans.:** (a)

(iii) $A = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$, $|A| = \frac{1}{2}(1+1) = 1$

$A^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}^T = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = A^T$

$\therefore$ A is or thogonal matrix. **Ans.: (a)**

(iv) $\because R_1 = R_2$ $\therefore$ The value of the determinant. **Ans.: (a)**

(v) det $A \neq 0$. **Ans.: (d)**

(vi) The characteristic equation is $\begin{vmatrix} 1-x & 4 \\ 4 & 1-x \end{vmatrix} = 0$.

$\Rightarrow e^p = px - y \quad \Rightarrow y = px - e^p$.

It is clairaut's equations.

$\therefore$ The general solution is $y = ex - e^e$. **Ans.:** (b)

(vii) $\frac{\Delta}{\nabla} = \frac{E-1}{1-E^{-1}} = \frac{E(E-1)}{E-1} = E$ $\therefore \frac{\Delta}{\nabla} = \Delta + \nabla$, is not true. **Ans.:** (c)

(viii) $\Delta e^x = e^{x+1} - e^x = e^x(e-1)$

$\Delta^2 e^x = \Delta(\Delta e^x) = \Delta[e^x(e-1)] = (e-1)\Delta e^x = (e-1)^2 e^x$ **Ans.:** (a)

(x) $\int_0^{\infty} \frac{\sin t}{t} \cdot e^{-st} dt = L\left\{\frac{\sin t}{t}\right\}$

$$L\left\{\frac{\sin t}{t}\right\} = \int_s^{\infty} L\{\sin t\} ds = \int_s^{\infty} \frac{1}{1+s^2} ds = \left[\tan^{-1} s\right]_s^{\infty} = \tan^{-1}\infty - \tan^{-1} s = \frac{\pi}{2} - \tan^{-1} s$$

$\therefore \int_0^{\infty} \frac{\sin t}{t} dt = \lim_{s \to 0} \int_0^{\infty} \frac{\sin t}{t} e^{-st} dt = \lim_{s \to 0} \frac{\pi}{2} - \tan^{-1} s = \frac{\pi}{2}$ **Ans.:** (d)

(xi) S and T are subspaces of a vector space V. S n T is also a subsapce of V. **Ans.:** (b)

(xii) $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$ is the characteristic equation.

$\therefore A^3 - 6A^2 + 9A - 4I = 0$

$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0 \quad \Rightarrow A^{-1} = \frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}I$ **Ans.:** (b)

(xiii) Cofactor of –3 is $-\begin{vmatrix} 1 & 1 \\ 0 & 4 \end{vmatrix} = -4$ **Ans.:** (b)

Group – B

[Short Answer Type Questions]

Answer any *three* of the following $3 \times 5 = 15$

2. If A be a skew symmetric and (I + A) be a non-singular matrix, then show that $B = (I - A)(I + A)^{-1}$ is orthogonal.

Ans.: $\because$ (I + A) is non singular. i. $(E + A)^{-1}$ exists.

Let $B = (I - A)(I + A)^{-1}$

Now $BB^T = \{(I - A)(I + A)^{-1}\}\{(I - A)(I + A)^{-1}\}^T$

$= (I - A)(I + A)^{-1} [(I + A)^{-1}]^T (I - A)^{-1}\}^T$

$= (I - A)(I + A)^{-1} (I + A^T)^{-1} (I - A^T)$

$= (I - A)(I + A)^{-1} (I - A)^{-1} (I + A)$ $[\because A^T = -A]$

$= (I - A) [(I - A)(I + A)]^{-1} (I + A)$

$= (I - A)(I + A - A - A^2)^{-1} (I + A)$

$= (I - A) [(I + A)(I - A)]^{-1} (I + A)$

$= (I - A)(I - A)^{-1} (I + A)^{-1} (I + A)$

$= [(I - A)(I - A)^{-1}] [(I + A)^{-1}(I + A)] = II = I$

$\therefore B = (I - A)(I + A)^{-1}$ is skew symmetric matrix.

3. Evaluate $L^{-1}\left\{\dfrac{1}{(s-1)^2(s-2)^3}\right\}$.

Ans.: $= e^t L^{-1}\left\{\dfrac{1}{s^2(s-1)^3}\right\} = e^t L^{-1}\left\{\dfrac{-3}{s} - \dfrac{1}{s^2} + \dfrac{1}{(s-1)^3} - \dfrac{2}{(s-1)^2} + \dfrac{3}{s-1}\right\}$

$$= e^t\left(-3 - t + \frac{e^t}{2}t^2 - 2e^t t + 3e^t\right) = e^t\left[e^t\left(\frac{t^2}{2} - 2t + 3\right) - (3+t)\right]$$

Using convolution Theorem.

Let $f(s) = \dfrac{1}{(s-1)^2}$ l. $F(t) = te^t$ and $g(s) = \dfrac{1}{(s-2)^3}$ $\therefore$ $G(t) = \dfrac{t^2 e^2 t}{2}$

$$\therefore \quad L^{-1}\left\{\frac{1}{(s-1)^2(s-2)^3}\right\} = L^{-1}\{f(s) * g(s)\} = \int_0^t G(u)F(t-u)du$$

$$= \int_0^t \frac{u^2 e^m}{2}(t-u)e^{t-u}du = \frac{e^t}{2}\int_0^t (t-u)u^2 e^u du$$

Now, $\int (t-u)u^2 e^u du$

$$= t\int u^2 e^u du - \int u^3 e^u du = t\int u^2 e^u du - u^3 e^u + 3\int u^2 e^u du$$

$$= (t+3)\int u^2 e^u du - u^3 e^u = (t+3)\left\{u^2 e^u - 2\int ue^u du\right\} - u^3 e^u = (t+3)\left\{u^2 e^u - 2ue^u + 2e^u\right\} - u^3 e^u$$

l. $\displaystyle\int_0^t (t-u)u^2 e^u du = \left[(t+3)\left\{u^2 e^u - 2ue^u + 2e^u\right\} - u^3 e^u\right]_0^t$

$$= (t+3)\left(t^2 e^t - 2te^t + 2e^t\right) - t^3 e^t - (t+3)(2) = e^t\left(t^3 - 2t^2 + 2t + 3t^2 - 6t + 6 - t^3\right) - 2(t+3)$$

$$= e^t\left(t^2 - 4t + 6\right) - 2(t+3)$$

l. $L^{-1}\left\{\dfrac{1}{(s-1)^2(s-2)^3}\right\} = \dfrac{e^t}{2}\displaystyle\int_0^t (t-u)u^2 e^u du = \dfrac{e^t}{2}\left[e^t\left(t^2 - 4t + 6\right) - 2(t+3)\right]$

4. Solve the differential equation $\dfrac{dy}{dx} + y = y^3$ (cos x – sin x).

$$\Rightarrow \frac{1}{y^3}\frac{dy}{dx} + \frac{1}{y^2} = \cos x - \sin x \text{ .... (i)}$$

Let $\frac{1}{y^2} = z \quad \Rightarrow -\frac{2}{y^3}\frac{dy}{dx} = \frac{dz}{dx} \quad \Rightarrow \frac{1}{y^3}\frac{dy}{dx} = -\frac{1}{2}\frac{dz}{dx}$

∴ From (1) we get, $-\frac{1}{2}\frac{dz}{dx} + z = \cos x - \sin x \quad \Rightarrow \frac{dz}{dx} - 2z = 2(\sin x - \cos x)$........(2)

Here I.F. $= e^{\int -2dx} = e^{-2x}$

Multiplying both sides of (2) by e^{-2x} and integrating we get,

$$ze^{-2x} = 2\int e^{-2x}(\sin x - \cos x)dx = 2\int e^{-2x}\sin x dx - 2\int e^{-2x}\cos x dx$$

$$= \frac{2}{5}e^{-2x}(-2\sin x - \cos x + 2\cos x - \sin x) + c = \frac{2e^{-2x}}{5}(\cos x - 3\sin x) + e$$

$$\Rightarrow \frac{1}{y^2} = \frac{2}{5}(\cos x - \sin x) + ee^{2x}$$

5. Evaluate the definite integral $\int_1^4 (x + x^3)\, dx$ by using Trapezoldal rule, taking five ordinates and calculate the error.

Ans.: Here a = 1, b = 4, n = 4 $\quad \therefore\ h = \frac{b-a}{n} = \frac{3}{4}$

x	f(x)	$x + x^3$
1	2	
$\frac{7}{4}$		$\frac{7}{4} + \frac{343}{64}$
$\frac{10}{4}$		$\frac{10}{4} + \frac{1000}{64}$
$\frac{13}{4}$		$\frac{13}{4} + \frac{2197}{64}$
4	68	
$\sum$	70	$\frac{15}{2} + \frac{885}{16}$

We know from Trapezoidal Rule, $I = \frac{h}{2}[y_0 + y_n + z(y_1 + y_2 + ... + y_{n-1})]$

$$= \frac{3}{8}\left[70 + 2\left(\frac{15}{2} + \frac{885}{16}\right)\right] = \frac{3}{8}\left[85 + \frac{885}{8}\right] = 73{\cdot}359375$$

$$\int_1^4 (x + x^3)dx = \left[\frac{x^2}{2} + \frac{x^4}{4}\right]_1^4 = 71{\cdot}25$$

∴ Error = 73·359375 − 71·25 = 2·109375.

6. If $A(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, then show that, $A(\theta)\,A(\phi) = A(\phi)\,.\,A(\theta) = A(\theta+\phi)$.

Ans.: Refer to the Q.no. 2,(vi) of 2006.

Group – C
[Long Answer Type Questions]

Answer any *three* of the following $3 \times 15 = 45$

7. a) If $A=\begin{bmatrix}1 & 2 & 1\\ 1 & -4 & 1\\ 3 & 0 & -3\end{bmatrix}$, $B=\begin{bmatrix}2 & 1 & 1\\ 1 & -1 & 0\\ 2 & 1 & -1\end{bmatrix}$, show that $Ab = 6I_3$. Utilise this result to solve the following system of equations :

$2x+y+z=5 \qquad x-y=0 \qquad 2x+y-z=1$

b) Solve : $(y-px)(p-1)=P$ and obtain the singular solution. Here $P=\frac{dy}{dx}$.

c) Construct the interpolation polynomial for the function $y=\sin\pi x$, taking the points $x_0=0, x_1=\frac{1}{6}, x_2=\frac{1}{2}$.

Hence find $f\left(\frac{1}{3}\right)$ where $y=f(x)$.

Ans.: (a). $AB=\begin{pmatrix}1 & 2 & 1\\ 1 & -4 & 1\\ 3 & 0 & -3\end{pmatrix}\begin{pmatrix}2 & 1 & 1\\ 1 & -1 & 0\\ 2 & 1 & -1\end{pmatrix}=\begin{pmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{pmatrix}=6I_3$

and $BA=\begin{pmatrix}2 & 1 & 1\\ 1 & -1 & 0\\ 2 & 1 & -1\end{pmatrix}\begin{pmatrix}1 & 2 & 1\\ 1 & -4 & 1\\ 3 & 0 & -3\end{pmatrix}=\begin{pmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{pmatrix}=6I_3 \qquad \therefore\ B^{-1}=\frac{1}{6}A$

We can written the system of linear equation ina matrix form as : $\begin{pmatrix}2 & 1 & 1\\ 1 & -1 & 0\\ 2 & 1 & -1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}5\\ 0\\ 1\end{pmatrix}$

$\Rightarrow BX=R$ where $R=\begin{pmatrix}5\\ 0\\ 1\end{pmatrix}$, $x=\begin{pmatrix}x\\ y\\ z\end{pmatrix} \Rightarrow x=B^{-1}R=\frac{1}{6}AR=\frac{1}{6}\begin{pmatrix}1 & 2 & 1\\ 1 & -4 & 1\\ 3 & 0 & -3\end{pmatrix}\begin{pmatrix}5\\ 0\\ 1\end{pmatrix}$

$\Rightarrow \begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{6}\begin{pmatrix}6\\ 6\\ 12\end{pmatrix}=\begin{pmatrix}1\\ 1\\ 2\end{pmatrix} \qquad \therefore\ x=1,\ y=1,\ z=2.$

Ans.: (b). $(y-px)(p-1)=p \quad \Rightarrow y=px+\frac{p}{p-1}$ .... (1)

diff. w.r. to x, we get, $p=p+x\frac{dp}{dx}+\frac{(p-1)-p}{(p-1)^2}\frac{dp}{dx}$

$$\Rightarrow \frac{dp}{dx}\left(x-\frac{1}{(p-1)^2}\right)=0 \quad \frac{dp}{dx}=0 \Rightarrow p=e$$

The general solution is $y = ex + \frac{e}{c-1}$ and $x-\frac{1}{(p-1)^2}=0$

$$\Rightarrow x=\frac{1}{(p-1)^2}, \quad \Rightarrow p=1+\frac{1}{\sqrt{x}} \text{ .... (2)}$$

From (1) and (2), we get $y=x\left(1+\frac{1}{\sqrt{x}}\right)+\frac{1+\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}=x+2\sqrt{x}+1 \Rightarrow (y-x-1)^2=4x$

Ans.: (c). The given points are,

x	0	1/6	1/2
y = sin πx	0	1/2	1

We know from Lagrange's miter polution formula, –

$$f(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\times y_0+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\times y_1+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\times y_2$$

$$=0+\frac{(x-0)(x-\frac{1}{2})}{\left(\frac{1}{6}-0\right)\left(\frac{1}{6}-\frac{1}{2}\right)}\times\frac{1}{2}+\frac{(x-0)(x-\frac{1}{6})}{\left(\frac{1}{2}-0\right)\left(\frac{1}{2}-\frac{1}{6}\right)}\times 1 = -9x = -9x\left(x-\frac{1}{2}\right)+6x\left(x-\frac{1}{6}\right)$$

$$=3x\left(-3x+\frac{3}{2}+2x-\frac{1}{3}\right)=3x\left(\frac{7}{6}-x\right)=\frac{7}{2}x-3x^2 \qquad \therefore\ f\left(\frac{1}{3}\right)=\frac{5}{6}.$$

8. a) Solve the differential equation, $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=x^2e^{3x}$.

b) Apply suitable interpolation formula to calculate f (9) correct up to two significant figures from the following data :

x :	2	4	6	8	10
f (x) :	5	10	17	29	49

c) Determine the conditions under which the system of equations
$x + y + z = 1 \quad x + 2y - z = b \quad 5x + 7y + az = b^2$
admits of :
(i) only one solution, (ii) no solution, (iii) many solutions.

Ans.: (a). $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=x^2e^{3x}$ or, $(D^2-5D+6)y=x^2e^{3x}$

Let e^{mx} be the trial solution. ∴ The auxiliary equation is, $x^2 - 5x + 6 = 0$.
$\Rightarrow m = 3, 2$.

∴ The C.F. is $y_c = e_1e^{3x}+e_2e^{2x}$

To find P.I. $y_p = \frac{1}{D^2-5D+6}x^2e^{3x}$

$$= \frac{1}{(D-3)(D-2)}x^2e^{3x} = e^{3x}\frac{1}{(D-3-3)(D+3-2)}x^2 = e^{3x}\frac{1}{D(D+1)}x^2$$

$$= e^{3x}\frac{1}{D}\cdot(1+D)^{-1}x^2 = e^{3x}\cdot\frac{1}{D}\left(1-D+D^2\right)x^2 = e^{3x}\frac{1}{D}\left(x^2-2x+2\right)$$

$$= e^{3x}\int\left(x^2-2x+2\right)dx = e^{3x}\left(\frac{x^3}{3}-x^2+2x\right)$$

l. The solution is, $y = e_1e^{3x}+e_2e^{2x}+e^{3x}\left(\frac{x^3}{3}-x^2+2x\right)$.

Ans. (b)

x	f(x)	∇f(x)	∇²f(x)	∇³f(x)	∇⁴f(x)
2	5				
4	10	5			
6	17	7	2	3	0
8	29	12	5	3	
10	49	20	8		

Here x = 9. x_n = 10, h = 2. $\therefore\ u = \frac{x-x_n}{h} = \frac{9-10}{2} = -0\cdot5$

We know from Newton Backward formula,

$$f(x) = f(x_n)+u\Delta f(x_n)+\frac{u(u+1)}{2}\nabla^2f(x_n)+\frac{u(u+1)(u+2)}{6}\nabla^3f(x_n)$$

$$= 49 - 0\cdot5\times20+\frac{-0\cdot5(-0\cdot5+1)}{2}\times8+\frac{-0\cdot5(-0\cdot5+1)(-0\cdot5+2)}{6}\times3$$

$$= 49-10-0\cdot19-0\cdot0486875 = 38\cdot7613125$$

Ans.: (c). The angmented matrix is,

$$\left[\begin{array}{ccc|c}1&1&1&1\\1&2&-1&b\\5&7&a&b^2\end{array}\right]\xrightarrow[R_3^1=R_3-5R_1]{R_2^1=R_2R_1}\left[\begin{array}{ccc|c}1&1&1&1\\0&1&-2&b-1\\0&2&a-5&b^2-5\end{array}\right]\xrightarrow{R_3^{11}=R_3-2R_2}\left[\begin{array}{ccc|c}1&1&1&1\\0&1&-2&b-1\\0&0&a-5&b^2-2b-3\end{array}\right]$$

(i) The system will bave only are solution if a – 1 ≠ 0 ⇒ a ≠ 1.

(ii) The system will have no solution if a – 1 = 0 but b2 – 2b – 3 ≠ 0.

i.e, a = 0 but b ≠ 3 and b ≠ –1.

(iii)The system will have many solution if a – 1 = 0 and b² – 2b – 3 = 0.

i.e, a = 1,b =3, or b = –1.

9. a) Prove that $P^T A P$ is a symmetric or a skew-symmetric matrix according as A is symmetric or skew-symmetric.

b) Find the eigenvalues and the eigenvectors of the matrix $\begin{bmatrix}4&6\\2&9\end{bmatrix}$.

c) Solve by Cramer's rule the following system of equations :

$3x + y + z = 4 \qquad x - y + 2z = 6 \qquad x + 2y - z = -3.$

Ans.: (a). Let A is symmetric i.e, $A^T = A$. Now $\left[P^T A P\right]^T = P^T A^T \left(P^T\right)^T = P^T AP$.

$\therefore$ $P^T Ap$ is symmetric matrix.

Let A is skew symmetric matrix, i.e., $A^T = A$

Now, $\left[P^T AP\right]^T = P^T A^T \left(P^T\right)^T = P^T(-A)P = -P^T AP$

$\therefore$ $P^T AP$ is skew symmetric matrix.

Ans.: (b). Refer to the Q.no. 7(b) of 2005.

Ans.: (c). Here $\Delta = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & 2 & -1 \end{vmatrix} = 3(1-4) - 1(-1-2) + 1(2+1) = -9 + 3 + 3 = -3$

$$\Delta_1 = \begin{vmatrix} 4 & 1 & 1 \\ 1 & -1 & 2 \\ -3 & 2 & -1 \end{vmatrix} = 4(1-4) - 1(-6+6) + 1(12-3) = -12 + 0 + 9 = -3$$

$$\Delta_2 = \begin{vmatrix} 3 & 4 & 1 \\ 1 & 6 & 2 \\ 1 & -3 & -1 \end{vmatrix} = 3(-6+6) - 4(-1-2) + 1(-3-6) = 0 + 12 - 9 = 3$$

$$\Delta_3 = \begin{vmatrix} 3 & 1 & 4 \\ 1 & -1 & 6 \\ 1 & 2 & -3 \end{vmatrix} = 3(3-12) - 1(-3-6) + 4(2+1) = -27 + 9 + 12 = -6$$

$$\therefore\ x = \frac{\Delta_1}{\Delta} = \frac{-3}{-3} = 1 \qquad y = \frac{\Delta_2}{\Delta} = \frac{3}{-3} = -1 \qquad z = \frac{\Delta_3}{\Delta} = \frac{-6}{-3} = 2$$

$\therefore$ Solution is $x = 1$, $y = -1$, $z = 2$.

10. a) What is meant by linear independence of a set of n-vectors ?

b) Solve by the method of variation of parameters the equation $\frac{d^2y}{dx^2} + 9y = \sec 3x$.

c) Prove that $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$.

Ans.: (a). Linear independence of n vectors :

Let the n vectors are $v_1, v_2, v_3, \ldots\ldots v_{10}$,

The vectors are linearly independent if–

$e_1 v_1 + e_2 v_2 + e_3 v_3 + \ldots\ldots\ldots + e_n v_n = 0$ and $c_1 = c_2 = c_3 = \ldots\ldots\ldots\ldots = c_n = 0$.

Ans.: (b). Refer to the Q.no. 3(a) of 2005.

Ans.: (c). Refer to the Q.no. 2(a) of 2004.