AB-3362

Seat No.__________

M. Phil. Examination

April / May – 2003

Statistics (Compulsory) : Paper-II

(Theory of Distributions)

Time : **3** Hours] [Total Marks : **100**

Instructions : (i) Answer the following questions.
(ii) **Each** question carries **20** marks.
(iii) Scientific calculators and statistical tables can be used.

1 (a) What is intervened distribution ? Give some practical situations where intervened distributions may arise.

(b) Derive pmf of intervened Poission distribution. Obtain its mean and variance.

OR

1 (a) Define a spherical family of p-variate distributions. Show that p-variate normal distribution is a member of this family.

(b) Define Elliptical family of distributions.

Let $\underset{\sim}{x} \sim E_p\left(\underset{\sim}{\mu}, \Sigma\right)$. Obtain

(i) $E\left(\underset{\sim}{x}\right)$ and $V\left(\underset{\sim}{x}\right)$

(ii) $\phi_{\underset{\sim}{x}}\left(\underset{\sim}{t}\right) \forall \underset{\sim}{t} \in R^p$.

(c) Define Wishart distribution. Write its important properties. State and prove additive property of this distribution.

2 (a) Define intervened geometric distribution. Point out the real life situations where this distribution is recommended.

(b) For an IGD show that the k^{th} factorial moment $\mu'_{[k]}$ is given by

$$\mu'_{[k]} = \frac{(1-q)(1-\rho q)}{q(1-\rho)} \left[\frac{q^k}{(1-q)^{k+1}} - \frac{(\rho q)^k}{(1-\rho q)^{k+1}} \right]$$

for $k = 1, 2, 3........$ and q and ρ are the parameters of the distribution.

(c) Discuss the method of maximum likelihood to estimate the parameter of the IGD.

OR

2 (a) Define a generalised gamma distribution. Explain how it reduces to classical exponential and gamma distributions. How would you estimate the parameters of Generalised Gamma Distribution ?

(b) Define Generalised Possion Distribution (GPD). How will you obtain GPD as a particular case of modified power series distribution. Obtain the mean and variance of GPD.

3 (a) Define Lagrangian Beta and Gamma distributions.

(b) State and prove any two properties of Lagrangian gamma distribution.

(c) Define negative multinomial distribution. Obtain its moment generating function.

OR

3 (a) Define a mixture distribution. Prove N-S condition for the identifiability of a class of mixture distributions.

(b) State the different methods for estimating the parameters of a mixutre distribution and point out their merits and demerits.

(c) Using Rider's method obtain the estimate of the parameters of a mixture distribution with density.

$$f(x) = p_1 \mu_1^{-1} \exp(-x/\mu_1) + p_2\, \mu_2^{-1} \exp(-x/\mu_2)$$

$$x > 0,\ \mu_1 > 0\ (i = 1, 2);\ \ p_1 + p_2 = 1.$$

4 (a) Define a mixture of two binomial distributions. Explain the method of moments for estimating the parameters of the distribution. Also deduce the variance–covariance matrix of the estimates.

(b) Define a mixture of two Poission distributions. Give some applications of the distribution. Describe the method of maximum likelihood in estimating parameters of the distribution.

OR

4 (a) Define Dirichlet distribution. Write applications of Dirichlet integral. Using Dirichlet integral obtain volume and area of n-dimensional sphere.

(b) Le $F = \left\{ \underset{\sim}{x} : x_1 \in R;\ i = 1, 2.....,n,\ \sum_{i=1}^{n} x_i^2 \le w^2 \right\}$ If $\underset{\sim}{d}' = (d_1,\ d_2,.........,d_n)$ is a vector of scalars then show that

$$I_F = \int_F ....\int e^{\underset{\sim}{d}\underset{\sim}{x}}\, d\underset{\sim}{x} = \pi^{\frac{n}{2}} \sum_{r=0}^{\infty} \left(\frac{\underset{\sim}{d}'\underset{\sim}{d}}{2} \right)^r \frac{(w^2)^{\frac{n}{2}+r}}{r!\ 2^r\ \Gamma\left(\frac{n}{2} + r + 1\right)}.$$

5 (a) Write briefly about Frendu's bivariate exponential distribution.

(b) Write briefly about Gumble's bivariate exponential distribution.

OR

5 (a) Define truncated bivariate normal distribution. Obtain mean and variance of doubly truncated bivariate normal distribution (when the range of variable x is truncated from both sides and $-\infty < y < \infty$). Hence or otherwise obtain the mean and variance of singly truncated bivariate normal distribution.

$$\left(h < x < \infty, -\infty < y < \infty \text{ and } -\infty < x < k, -\infty < y < \infty\right)$$

(b) Define truncated trivariate normal distribution. Obtain mean and variance of this distribution.

(c) Define truncated multivariate normal distribution. Obtain mean and variance of this distribution.