Sample Question Paper - I

MATHEMATICS

Class XII

Time Allowed : 3 hours **Max. Marks : 100**

General Instructions

(i) The question paper consists of three parts A, B and C. Part A is compulsory for all students. In addition to part A, every student has to attmpt either Part B or Part C.

(ii) **For Part A**

Question numbers 1 to 8 are of 3 marks each.

Question numbers 9 to 15 are of 4 marks each.

Question numbers 16 to 18 are of 6 marks each.

(iii) **For Part B/Part C**

Question numbers 19 to 22 are of 3 marks each.

Question numbers 23 to 25 are of 4 marks each.

Question number 26 is of 6 marks.

(iv) All questions are compulsory.

(v) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(vi) Use of calculator is not permitted. However, you may ask for logarithmic and statistical tables, if required.

SECTION–A

1. If $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Find x and y such that $A^2 = xA + yI$.

2. Using properties of determinants show that

$$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(x-z)^2$$

3. A bag contains 3 red, 4 black and 2 green balls. Two balls are drawn at random from the bag. Find the probability that both balls are of different colours.

4. A pair of dice is rolled. Find the probability of getting a doublet or sum of numbers to be atleast 10.

5. Evaluate $\int \sqrt{1+2\tan x(\sec x+\tan x)}\,d x$.

6. Evaluate $\int \frac{1}{\sqrt{3+4x-2x^2}}\,dx$.

7. Form a differential equation of family of all circles having centres on X-axix and radius 2 units.

OR

Show that $y=\cos(\cos x)$ is a solution of the differential equation

$$\frac{d^2y}{d^2x}-\cot x\frac{dy}{dx}+y.\sin^2 x=0$$

8. Solve the differential equation

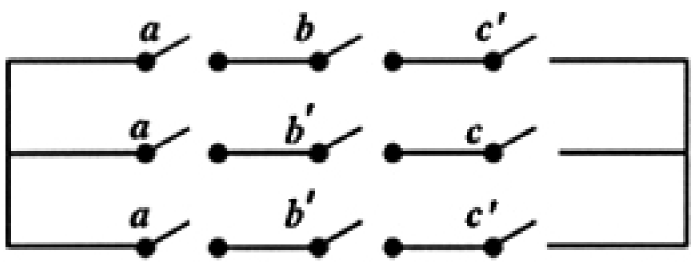
$$x\frac{dy}{dx}+y=x\cos x+\sin x,\ \text{given that}\ \ y(\pi/2)=1$$

9. Using properties of Boolean algebra prove that if

$x+y=x+z$ and $x'+y=x'+z$ then $y=z$

OR

Write the boolean expression for the following circuit



Simplify the expression and construct the switching circuit for the simplified expression.

10. Evaluate $\lim_{x\to 0} \frac{1-\cos x\sqrt{\cos 2x}}{\left(e^{x^2}-1\right)}$

11. Differentiate sec $(2x-1)$ w.r.t. x using first principle.

12. If $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \pi/2$

13. Water is leaking from a conical funnel at the rate of $5 cm^3/sec$. If the radius of the base of funnel is 5cm and height 10 cm find the rate at which is water level the dropping when it 2.5 cm from the top.

14. Evaluate : $\int \frac{1}{x^4 - 5x^2 + 16} dx$

15. Evaluate : $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

OR

Evaluate : $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x} + \sqrt{4-x}} dx$

16. If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$, Find A^{-1} and use it to solve the system of equations :-

$x + y + 2z = 0$

$x + 2y - z = 9$

$x - 3y + 3z = -14$

17. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.

OR

A rectangle is inscribed in a semi circle of radius 'a' with one of its sides on the diameter of semi circle. Find the dimensions of the rectangle so that its area is maximum. Find also the area.

18. Find the area of smaller region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the straight line $\frac{x}{4} + \frac{y}{3} = 1$.

OR

Evaluate $\int_1^3 (2x^2+3x+5)\,dx$ as limit of a sum.

SECTION–B

19. Find the value of p so that the vectors $\hat{i}+2\hat{j}-3\hat{k}$, $p\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}-4\hat{j}+5\hat{k}$ are coplanar.

20. If $\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{o}$ then prove that $\vec{a}+\vec{b}=k\vec{c}$ where k is a scalar.

21. A locomotive driver travelling at 72km/hr. finds a signal 210 metres ahead of him indicating he should stop. He instantly applies brakes to stop the train. The train retards uniformly and stops 10 metres before the signal post. What time did he take to stop the train?

22. A ball projected vertically upwards takes t seconds to reach a height h metres. If t' seconds is the time taken by the ball to reach from this point to the ground, prove that

$h=\frac{1}{2}gtt'$ and that the maximum height reached is $\frac{1}{8}g(t+t')^2$

OR

A man rows across a flowing river in time t_1, and rows an equal distance down the stream in time t_2. If v be the velocity of man in still water and u that of the streem, show that $t_1:t_2=\sqrt{v+u}:\sqrt{v-u}$.

23. Find the distance of the point (1, –2, 3) from the plane $x-y+z=5$ measured parallel to the line $\frac{x+1}{2}=\frac{y+3}{3}=\frac{z+1}{-6}$.

24. Two unlike parallel forces $\vec{P}$, $\vec{Q}$ (P>Q) act at two points c units apart. If the direction of $\vec{Q}$ is reversed, then prove that the resultant is displaced through the distance $\frac{2PQ}{P^2-Q^2}c$ units.

OR

A body of weight 25N is suspended by two strings of length 30 cm and 40 cm. respectively. The other ends of the strings are fastened to two points in the same horizontal line 50 cm apart. Find the tensions in the strings.

25. Two forces each of magnitude $20\sqrt{3}$ units form a couple. If one of the forces acts at the origin inclined at 60^0 to the positive direction of x – axis, find where the line of action of the other force cuts x-axis, given that the moment of the couple is –60 units.

26. Prove that the plane x + 2y –z = 4 cuts the sphere $x^2+y^2+z^2-x+z=3$ in a circle. Also find the centre and the radius of the circle.

SECTION – C

19. There are two Bags I and II. Bag I contains 3 white and 4 black balls and Bag II contains 5 white and 6 black balls. One ball is drawn at random from one of the bags and is found to be white find the probability that it was drawn from Bag I.

20. The mean and variance of a binomial distribution are 10 and $\frac{5}{3}$ respectively.

Find $P(X \geq 1)$.

OR

Suppose 10% of people in a town are post graduates. Using Poisson distribution, find the probability that in a sample of 20 people, not more than 2 are post-graduates Take $e^{-2}=0.135$]

21. Solve the following linear programming problem graphically :

Maximise z = 6x + 5y

Subject to $3x+5y\leq 15$

$5x+2y\leq 10$

$x,\ y\geq 0$

22. An aeroplane can carry a maximum of 250 passengers. A profit of Rs 500 is made on each first class ticket and a profit of Rs 350 on each economy class ticket. The airline reserves at least 25 seats for first class. However at least 3 times as many passengers prefer to travel by economy class than first class. Form a L.P.P. to determine how many tickets of each type must be sold in order to maximise profit for the airline.

23 A, B and C are partners investing Rs 70000, Rs 42000 and Rs 35000 respectively with the understanding that after allowing $\frac{1}{8}$th of the profit to C as a manager, the remaining profit is divided amongst the three in proportion to the amount of capital invested by each. At the end of the year, C received Rs 6400. What was the total profit and how much profit did A and B receive?

OR

A, B and C start a business by investing capitals in the ratio of 20:15:12. A withdraws half of his capital at the end of six months and $\frac{2}{3}$ of the remaining after another 3 months. B withdraws one-fourth of his capital after 9 months. Find the share of each in a profit of Rs 18910 at the end of the year. Profit is to be divided in the ratio of their adjusted (effective) capitals.

24. A machine, being used by a company, is estimated to have a life of 15 years. At that time, the new machine would cost Rs 74400 and the scrap of the old machine would yield Rs 4600 only. A sinking fund is created for replacing the machine at the end of its life. What sum should be invested by the company at the end of each year to accumulate at 6% per annum.

25. The marginal cost of producing x units of a product is given by M.C. = $2x\sqrt{x+5}$. The cost of producting 4 units of the product is Rs 314.40. Find the cost function and the average cost function.

26. A man holds bills of Rs 10000 and Rs 12000 which are due on March 15, 2003 and April 20, 2003 respectively. Both the bills are presented to a banker for discounting on January 1, 2003. If the difference between two discounts is Rs 96, find the rate percent at which the discounts are calculated.

SAMPLE QUESTION PAPER – II

MATHEMATICS

CLASS XII

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(ii) **For Part A :**
Question numbers 1 to 8 are of 3 marks each.
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(v) Internal choice has been provided in some questions. You have to attempt only one of the choices in such questions.

(vi) Use of calculator is not permitted. You may ask for logarithmic and statistical tables, if required.

SECTION–A

1. If $A = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$, verify that $A^2 - 4A + I = 0$. Hence find A^{-1}.

OR

If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$, using principle of mathematical induction show that

$A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$, for all $n \in N$.

2. Using properties of determinants, show that :

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

3. There are two bags I and II. Bag I contains 3 white and 2 red balls, Bag II contains 2 white and 4 red balls. A ball is transferred from bag I to Bag II (without seeing its colour) and then a ball is drawn from bag II. Find the probability of getting a red ball.

4. Two cards are drawn successively (without replacement) from a well shuffled pack of playing cards. Find the probability distribution of number of spades.

5. Evaluate : $\int \frac{\cos x - \sin x}{1 + \sin 2x} dx$.

6. Evaluate : $\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$.

7. Solve the differential equation : $xdy-ydx=\sqrt{x^2+y^2}\,dx$.

8. Solve the differential equation : $ye^y\,dx=(y^3+2x\,e^y)\,dy$.

9. Simplify the boolean expression : $x(x+y)+(y'+x)\,y)'$.

10. Examine the continuity of the function :

$$f(x)=\begin{cases}\frac{|\sin x|}{x}, & x\neq 0\\ 1, & x=0\end{cases}\text{ at } x=0.$$

11. $y=x^{\sin x}+(\sin x)^x$, find $\frac{dy}{dx}$.

12. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

13. Find the intervals in which the function f given by $f(x)=\sin x-\cos x$, $0\leq x\leq 2\pi$ is (i) increasing (ii) decreasing.

OR

It is given that for the function f defined by

$f(x)=x^3+bx^2+ax$, $x\in[1,3]$, Rolle's theorem holds with $c=2+\frac{1}{\sqrt{3}}$.

Find the values of a and b.

14. Evaluate : $\int\frac{3x-2}{(x+3)(x+1)^2}dx$.

15. Evaluate : $\int_3^6(|x-3|+|x-4|+|x-5|)\,dx$

16. Using determinants, solve the following system of equations :

$$x-y+3z=6$$
$$x+3y-3z=-4$$
$$5x+3y+3z=10$$

OR

Solve the following system of equations :

$$x+y+z=1$$
$$ax+by+cz=d$$
$$a^2x+b^2y+c^2z=d^2$$

17. An open box with a square base is to be made out of a given quantity of card board of area a^2 square units. Find the dimensions of the box so that the volume of the box is maximum. Also find the maximum volume.

OR

Find the equation of tangent and normal to the curve $x = a\cos t + at\sin t$, $y = a\sin t - at\cos t$, at any pint 't'. Also show that the normal to the curve is at a constant distance from origin.

18. Make a rough sketch and find the area of the region : (using integration)
$\{(x, y) : x^2 + y^2 \le 2ax ; y^2 \ge ax, x \ge 0, y \ge 0\}$.

SECTION–B

19. If $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, find $\left(2\vec{a} - \vec{b}\right) \times \left(\vec{a} + 2\vec{b}\right)$.

20. Using vectors prove that the altitudes of a triangle are concurrent.

OR

Prove that $\left[\vec{b} \times \vec{c},\ \vec{c} \times \vec{a},\ \vec{a} \times \vec{b}\right] = \left[\vec{a},\ \vec{b},\ \vec{c}\right]^2$

21. Find the shortest distance between the lines
$\vec{r} = (1 + 2\lambda)\,\hat{i} + (2 + 3\lambda)\,\hat{j} + (3 + 4\lambda)\,\hat{k}$ and
$\vec{r} = (2 + 3\mu)\,\hat{i} + 4(1 + \mu)\,\hat{j} + 5(1 + \mu)\,\hat{k}$.

22. Find the value of k for which the plane $x + y + z - \sqrt{3}\,k = 0$ touches the sphere $x^2 + y^2 + z^2 = 9$.

23. Show that the lines $\dfrac{x - 1}{2} = \dfrac{y - 3}{4} = -z$ and $\dfrac{x - 4}{3} = \dfrac{1 - y}{2} = z - 1$ are coplanar. Also find the equation of plane containing the lines.

24. The resultant of two forces $\vec{P}$ and $\vec{Q}$ acting at a point is of magnitude $\sqrt{3}Q$ and its direction makes an angle of 30° with the direction of $\vec{P}$. Show that either $P = Q$ or $P = 2Q$.

25. A body of weight 20 N hangs by a string from a fixed point. The string is drawn out of the vertical by applying a force of 10 N to the weight. In what direction must this force be applied in order that, in equilibrium, the deflection of the string from the vertical may be of 30° ? Also find the tension in the string.

OR

$\vec{P}$ and $\vec{Q}$ are two unlike parallel forces acting at two different points of a rigid body. When the magnitude of $\vec{P}$ is doubled, it is found that the line of action of $\vec{Q}$ is mid- way between the lines of action of the new and the original resultants. Find the ratio of P and Q.

26. A bullet is fired from the top of a tower 210 meters high with a velocity of 280 m/s at an angle of projection of 30°. Find :
   (i) in how many seconds, the bullet reaches the ground.
   (ii) how far beyond the point of release, the bullet strikes the ground.
   (iii) magnitude and direction of its velocity when it hits the ground. [Take $g = 9.8\ m/s^2$].

SECTION–C

19. A bill of Rs 35000 drawn on April 19, 2002 at 6 months, was discounted on a certain date at 5% per annum and the proceeds were Rs 34300. When was the bill discounted ?

20. If the banker's gain on a bill is $\frac{1}{7}$th. of the banker's discount at 10% per annum. Find the period for which the bill was discounted.

21. A die is thrown 10 times. If getting a prime number is considered a success, find the probability of getting not more than 8 successess.

OR

If the variance of the Poisson distribution is 2, find the probabilities for $r = 1, 2, 3$ and 4.

22. A man is known to speak truth 4 out of 5 times. He throws a pair of dice and reports that it is a doublet. Find the probability that it is actually a doublet.

23. A starts business with Rs 1,50,000. After sometime B joins with a capital of Rs 4,00,000. At the end of the year, the profit is divided in the ratio 1 : 2. If the profit is divided in the ratio of their adjusted (effective) capitals, when did B join ?

24. A buys a house for Rs 15,88,600, for which he pays Rs 4,00,000 cash down and the balance in 10 annual equal instalments paid at the end of each year. If the rate of interest is 5% p.a. compounded annually, how much money has he to pay every year ? [Take $(1.056)^{-10} = 0.6138$]

25. If the cost function C (x) of a firm is given by $C(x) = \frac{1}{3}x^3 - 5x^2 + 30x + 10$, where x is the output, find :

    (i) Marginal Cost Function (MC)

    (ii) Average Cost Function (AC).

    Also, show that

$$\text{Slope of Average Cost function} = \frac{x\,(\text{Marginal Cost function}) - \text{Cost function}}{x^2}$$

OR

A manufacturer finds that he can sell X products per week at Rs p each, where $p = 2\left(100 - \frac{x}{4}\right)$. If his cost of x products is given by $C(x) = 120x + \frac{x^2}{2}$, find, how many products per week he should manufacture so that his profit is maximum. Also find the maximum profit per week.

26. A retired person has Rs 70,000 to invest and two types of bonds are available in the market for investment. First type of bonds yields an annual income of 8% on the amount invested and the second type of bonds yields 10 % per annum. As per norms, he has to invest a minimum of Rs 10,000 in the first type and not more than Rs 30,000 in the second type. How should he plan his investment, so as to get maximum return, after one year of investment ?