

Fifth Semester Examination - 2008

ELECTROMAGNETIC THEORY

Full Marks - 70

Time : 3 Hours

Answer Question No. 1 which is compulsory
and any five from the rest.

The figures in the right-hand margin
indicate marks.

1. Explain the following : 2×10

- (a) How is the position vector of a point in cylindrical coordinates related to its position vector in spherical coordinates ?
- (b) If a vector field is solenoidal at a given point in space, does it necessarily follow that the vector field is zero at that point?

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- (c) What is dielectric polarization ?
- (d) What is the difference between the conduction and convection currents ?
- (e) Why cannot the \mathbf{B} field of an infinitely long, straight, current carrying conductor have a component in the direction of the current ?
- (f) What is the relation between vector magnetic potential \mathbf{A} and the magnetic flux through a given area ?
- (g) Under what circumstances is the net voltage around a closed loop equal to zero ?
- (h) How does β of a low-loss dielectric medium compare to that of a lossless medium ?
- (i) What are the physical length of a half wave dipole operating at 100 MHz and 1 GHz ?

(j) Why antennas are used in the form of an array ?

2. (a) Consider a vector field in a rectangular coordinate system defined by $\mathbf{F}(x, y, z) = z \mathbf{a}_y$. Determine the flux of the vector field out of a closed cylinder of length 2 m and radius 2 m centered on the z axis and extended from $z=0$ to $z=2$. 4

(b) Two points in a cylindrical coordinate system are $P_1(2, 60^\circ, 5)$ and $P_2(1, 30^\circ, 3)$. Determine an expression for the vector \mathbf{D} directed from P_1 to P_2 . Determine the distance between the two points by

- (i) obtaining $|\mathbf{D}|$ and
- (ii) direct integration.

Also find an expression for unit vectors at P_1 and P_2 , which are pointed in the direction of \mathbf{D} . 6

3. (a) Consider a uniformly charged disk of radius r . Determine the electric field by first obtaining the potential function. 5

(b) A coaxial cable has two concentric cylinders of radii a and b , $a < b$. The space between the two cylinders is filled with dielectric of relative permittivities ϵ_{r1} for $a < r < c$ and ϵ_{r2} for $c < r < b$. If the inner cylinder is held at a potential V_0 with respect to the outer sphere, determine \mathbf{E} and \mathbf{P} in the two regions. 5

4. (a) Determine the resistance R of a spherical capacitor of inner sphere radius a and outer sphere radius b that is filled with a lossy dielectric having material parameters σ and ϵ . 5

(b) Given $H = \frac{I r}{2\pi a^2} \mathbf{a}_\phi$ within a very long solid conductor of radius a . Determine the current density within the conductor. Also, find the magnetic field external to the conductor. 5

5. (a) A material having a conductivity σ and radius r carrying a direct current I . Determine the net power entering a wire of length L by integrating the Poynting Vector over the wire surface. Show directly that this is the power dissipated by the wire resistance. 6

(b) A material having a conductivity σ and permittivity ϵ is placed in a sinusoidal, time varying electric field having a frequency ω . At what frequency will the conduction current equal the displacement current? If $\sigma = 10^{-2} \text{ S/m}$ and $\epsilon = 3\epsilon_0$, determine the frequency. 4

6/ (a) Show that the exact expressions for the attenuation and phase constant can be written explicitly as

$$\alpha = \frac{\beta'}{\sqrt{2}} \left[\sqrt{1 + \tan^2 \delta} - 1 \right]^{1/2} \text{ Np/m}$$

$$\beta = \frac{\beta'}{\sqrt{2}} \left[\sqrt{1 + \tan^2 \delta} + 1 \right]^{1/2} \text{ rad/m}$$

Where $\beta' = \omega \sqrt{\mu \epsilon}$ and $\tan \delta = \frac{\sigma}{\omega \epsilon}$.

5

(b) A 100 MHz uniform plane wave travelling in a lossy dielectric ($\mu_r \approx 1$) has the following phasor expression for the magnetic field intensity vector $H = (1\mathbf{a}_y + j2\mathbf{a}_z) e^{-\alpha x} e^{j2x}$. Write complete time domain expressions for the electric and magnetic field vectors. 5

7. (a) Consider a dipole antenna of total length 1 that is driven by a voltage source at its input. Assume the antenna is sufficiently

short so that the current decreases uniformly from its maximum at the centre to zero at the end points. Derive an expression for the far field electric field in terms of the input current. 6

(b) Determine the directive gain and directivity of the elementary magnetic dipole. 4

8. The conductivity of a homogeneous conducting medium, bounded by $10 \text{ cm} \leq r \leq 20 \text{ cm}$, $30^\circ \leq \theta \leq 45^\circ$ and $30^\circ \leq \phi \leq 60^\circ$ is 0.4 S/m . The surface at $\theta = 45^\circ$ is at a ground potential, and the surface at $\theta = 30^\circ$ is at 100 V . Using Laplace's equation, determine the resistance of the medium, neglecting the edge effects. 10