P1-11-4-6

01466

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CODE 6

PAPER 1

Time : 3 Hours **Maximum Marks : 240**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. The **question paper CODE** is printed on the right hand top corner of this sheet and on the back page (page No. 36) of this booklet.
2. No additional sheets will be provided for rough work.
3. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets are NOT allowed.
4. Write your name and registration number in the space provided on the back page of this booklet.
5. The answer sheet, a machine-gradable Optical Response Sheet (ORS), is provided separately.
6. DO NOT TAMPER WITH/MUTILATE THE ORS OR THE BOOKLET.
7. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilators.
8. This Question Paper contains 36 pages having 69 questions.
9. On breaking the seals, please check that all the questions are legible.

B. Filling the Right Part of the ORS :

10. The ORS also has a CODE printed on its Left and Right parts.
11. Make sure the CODE on the ORS is the same as that on this booklet. **If the codes do not match, ask for a change of the booklet.**
12. Write your Name, Registration No. and the name of centre and sign **with pen** in the boxes provided. **Do not write them anywhere else.** Darken the appropriate bubble **UNDER** each digit of your Registration No. with a **good quality HB pencil**.

C. Question paper format and Marking Scheme:

13. The question paper consists of **3 parts** (Chemistry, Physics and Mathematics). Each part consists of **four sections.**
14. In **Section I** (Total Marks: 21), for each question you will be awarded **3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (-1)** mark will be awarded.
15. In **Section II** (Total Marks: 16), for each question you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. There are **no negative marks** in this section.
16. In **Section III** (Total Marks: 15), for each question you will be awarded **3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (-1)** mark will be awarded.
17. In **Section IV** (Total Marks: 28), for each question you will be awarded **4 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** otherwise. There are **no negative marks** in this section.

DO NOT BREAK THE SEALS WITHOUT BEING INSTRUCTED TO DO SO BY THE INVIGILATOR

Write your Name, registration number and sign in the space provided on the back page of this booklet

Useful Data

$R = 8.314\ JK^{-1}\ mol^{-1}$ or $8.206 \times 10^{-2}\ L\ atm\ K^{-1}\ mol^{-1}$

$1F = 96500\ C\ mol^{-1}$

$h = 6.626 \times 10^{-34}\ Js$

$1\ eV = 1.602 \times 10^{-19}\ J$

$c = 3.0 \times 10^{8}\ ms^{-1}$

$N_A = 6.022 \times 10^{23}$

PART I : CHEMISTRY

SECTION – I (Total Marks : 21)

(Single Correct Answer Type)

This section contains **7 multiple choice questions**. Each question has four choices (A) , (B), (C) and (D) out of which **ONLY ONE** is correct.

1. Among the following compounds, the most acidic is

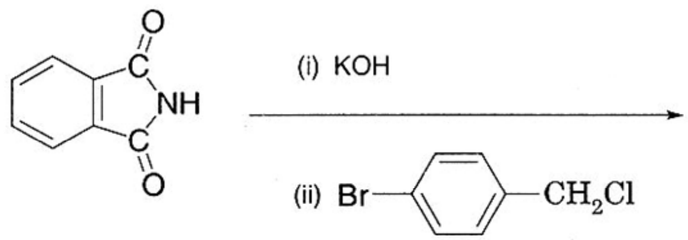
(A) p–nitrophenol (B) p-hydroxybenzoic acid

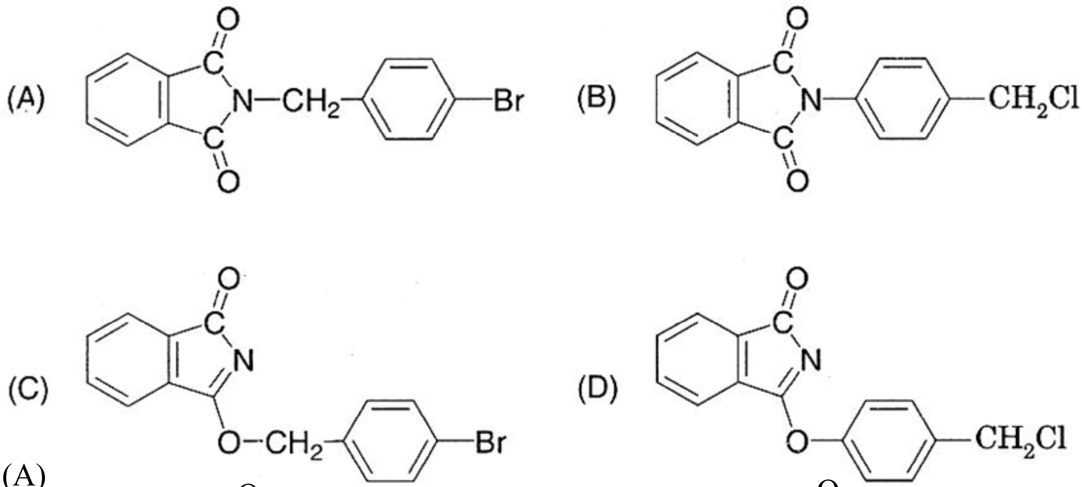
(C) o–hydroxybenzoic acid (D) p–toluic acid

1. (C)

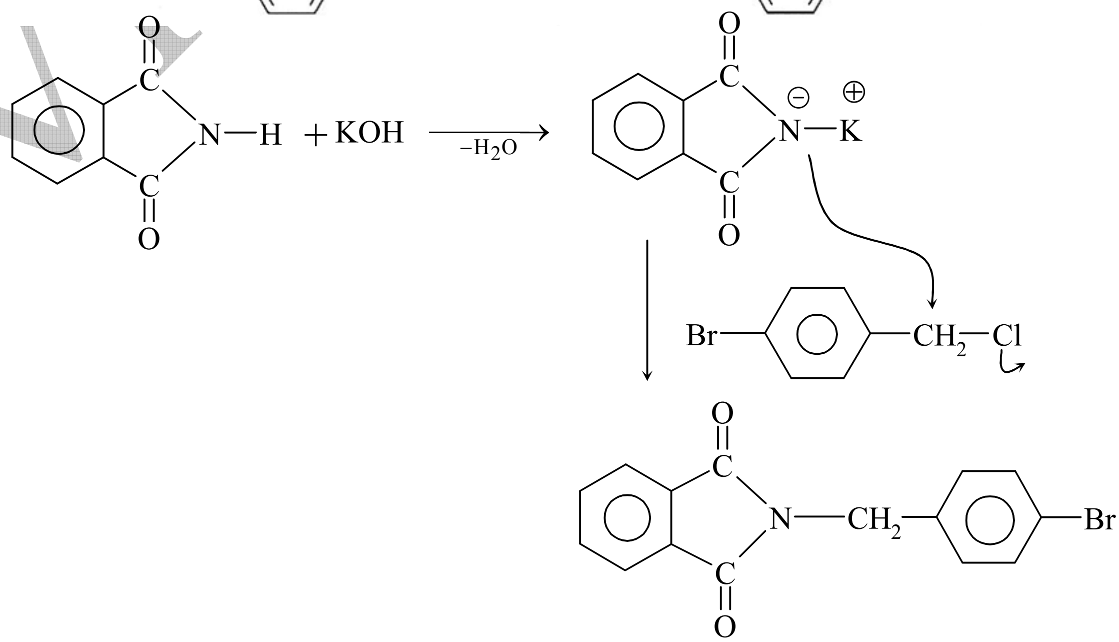
The most acidic compound is ortho–hydroxy benzoic acid due to ortho effect.

2. The major product of the following reaction is





2. (A)



3. Extra pure N_2 can be obtained by heating
(A) NH_3 with CuO (B) NH_4NO_3
(C) $(NH_4)_2Cr_2O_7$ (D) $Ba(N_3)_2$

3. (D)

$$(NH_4)_2Cr_2O_7 \rightarrow N_2 + 4H_2O + Cr_2O_3$$

$$NH_4NO_3 \rightarrow N_2O + H_2O$$

$$3CuO + 2NH_3 \rightarrow 3Cu + N_2 + 3H_2O$$

$$Ba\left(N_3\right)_2 \rightarrow 3N_2 + Ba \text{ (pure nitrogen)}$$

4. Geometrical shapes of the complexes formed by the reaction of Ni^{2+} with Cl^-, CN^- and H_2O, respectively, are
(A) octahedral, tetrahedral and square planar
(B) tetrahedral, square planar and octahedral
(C) square planar, tetrahedral and octahedral
(D) octahedral, square planar and octahedral

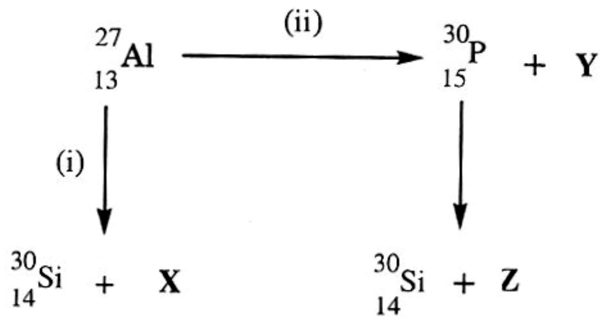
4. (B)

$$\left[NiCl_4\right]^{2-} \rightarrow sp^3 \text{ (tetrahedral)}$$

$$\left[Ni(CN)_4\right]^{2-} \rightarrow dsp^2 \text{ (square planar)}$$

$$\left[Ni(H_2O)_6\right]^{2+} \rightarrow sp^3d^2 \text{ (octahedral)}$$

5. Bombardment of aluminum by α–particle leads to its artificial disintegration in two ways, (i) and (ii) as shown. Products X, Y and Z respectively are,
(A) proton, neutron, positron
(B) neutron, positron, proton
(C) proton, positron, neutron
(D) positron, proton, neutron



5. (A)

$$\begin{array}{ccc} {}^{27}_{13}Al + {}^{4}_{2}He \rightarrow & {}^{30}_{15}P + {}^{1}_{0}n \\ \downarrow & \downarrow \\ {}^{39}_{14}Si + {}^{1}_{1}H & {}^{30}_{14}Si + {}^{0}_{1}e \end{array}$$

X – proton, Y – neutron, Z – positron

6. Dissolving 120 g of urea (mol. wt. 60) in 1000 g of water gave a solution of density 1.15 g/mL. The molarity of the solution is
(A) 1.78 M (B) 2.00 M (C) 2.05 M (D) 2.22 M

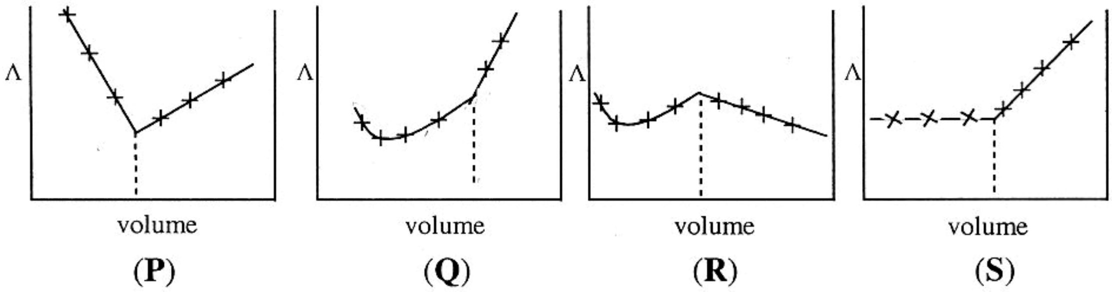
6. (C)

$$\text{Moles} = \frac{120}{60} = 2$$

$$\text{volume} = \frac{1000 + 120}{1.15} = \frac{1120}{1.15}\,\text{mL} = \frac{1.12}{1.15}\,\text{litre}$$

$$\text{Molarity} = \frac{\text{moles}}{\text{volume}} = 2.05 \text{ M}$$

7. $AgNO_3$(aq.) was added to an aqueous KCl solution gradually and the conductivity of the solution was measured. The plot of conductance (Λ) versus the volume of $AgNO_3$ is



(A) (P) (B) (Q) (C) (R) (D) (S)

7. (D)

SECTION – II (Total Marks : 16)

(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A) , (B) , (C) and (D) out of which **ONE or MORE** may be correct.

8. The correct statement(s) pertaining to the adsorption of a gas on a solid surface is (are)
(A) Adsorption is always exothermic.
(B) Physisorption may transform into chemisorption at high temerpature.
(C) Physisorption increases with increasing temperature but chemisorption decreases with increasing temperature.
(D) Chemisorption is more exothermic than physisorption, however it is very slow due to higher energy of activation.

8. (A), (B), (D)

9. Extraction of metal from the ore cassiterite involves
(A) carbon reduction of an oxide ore (B) self–reduction of a sulphide ore
(C) removal of copper impurity (D) removal of iron impurity

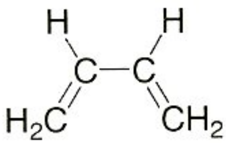
9. (A), (D)

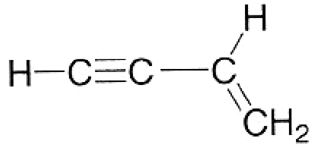
$$\underset{\text{(cassiterite)}}{SnO_2} + 2C \rightarrow Sn + 2CO$$

10. According to kinetic theory of gases
(A) collisions are always elastic.
(B) heavier molecules transfer more momentum to the wall of the container.
(C) only a small number of molecules have very high velocity.
(D) between collisions, the molecules move in straight lines with constant velocities.

10. (A), (C), (D)
Fact (A) gas molecules are perfectly elastic hence there is no loss in velocity of C. Small fraction of molecules have high velocity called R.M.S. velocity.

11. Amongst the given options, the compound(s) in which all the atoms are in one plane in all the possible conformations (if any), is (are)

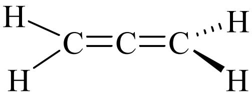
(A) 

(B) 

(C) $H_2C{=}C{=}O$

(D) $H_2C{=}C{=}CH_2$

11. (B), (C)


Non-planar compound

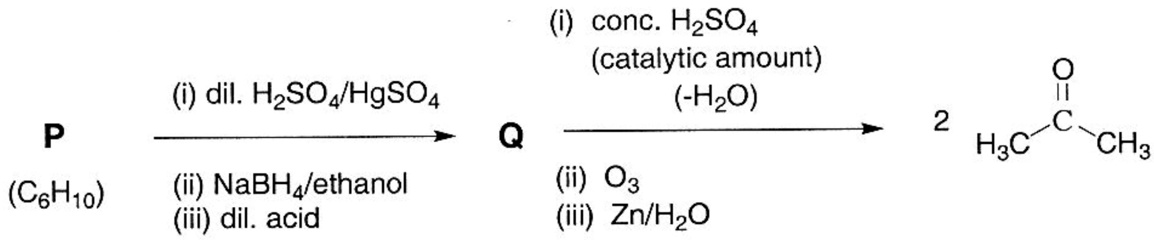
SECTION – III (Total Marks : 15)

(Paragraph Type)

This section contains **2 paragraphs**. Based upon one of the paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A) , (B) , (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 12 and 13

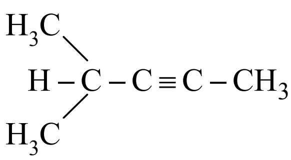
An acyclic hydrocarbon P, having molecular formula C_6H_{10}, gave acetone as the only organic product through the following sequence of reactions, in which Q is an intermediate organic compound.

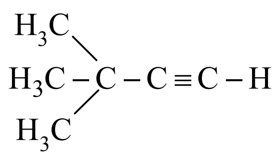


12. The structure of compound P is

(A) $CH_3CH_2CH_2CH_2-C\equiv C-H$

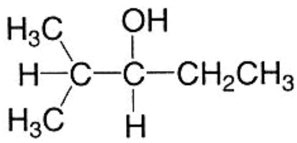
(B) $H_3CH_2C-C\equiv C-CH_2CH_3$

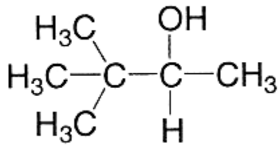
(C) 

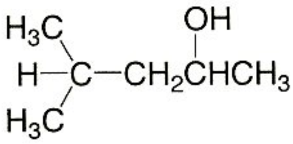
(D) 

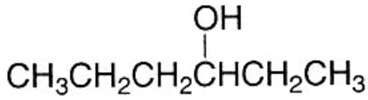
12. (D)

13. The structure of the compound Q is

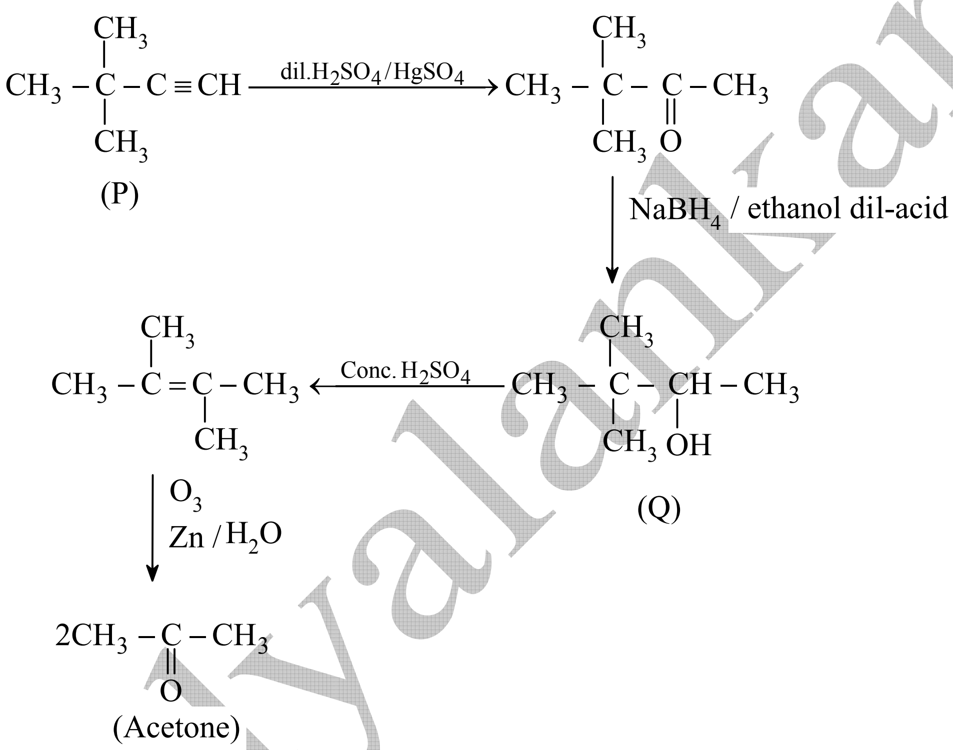
(A) 

(B) 

(C) 

(D) 

13. (B)



Paragraph for Question Nos. 14 to 16

When a metal rod M is dipped into an aquesous colourless concentrated solution of compound N, the solution turns light blue. Addition of aqueous NaCl to the blue solution gives a white precipitate O. Addition of aqueous NH_3 dissolves O and gives an intense blue solution.

14. The metal rod M is
(A) Fe (B) Cu (C) Ni (D) Co

14. (B)

15. The compound N is
(A) $AgNO_3$ (B) $Zn(NO_3)_2$ (C) $Al(NO_3)_3$ (D) $Pb(NO_3)_2$

15. (A)

16. The final solution contains

(A) $[Pb(NH_3)_4]^2$ and $[CoCl_4]^{2-}$
(B) $[Al(NH_3)_4]^{3+}$ and $[Cu(NH_3)_4]^{2+}$
(C) $[Ag(NH_3)_2]^+$ and $[Cu(NH_3)_4]^{2+}$
(D) $[Ag(NH_3)_2]^+$ and $[Ni(NH_3)_6]^{2+}$

16. (C)

Solution for 14 to 16

$$\underbrace{Cu + 2AgNO_3 \longrightarrow \underset{\text{light blue}}{Cu(NO_3)_2} + 2Ag}$$

$$\Big\downarrow 2NaCl$$

$$\underbrace{2AgCl\downarrow \text{ (white ppt)} + Cu^{+2} + 2NaNO_3}$$

$$\Big\downarrow$$

$$[Ag(NH_3)_2]^{+1} + \underset{\text{Deep blue}}{[Cu(NH_3)_4]^{+2}}$$

SECTION – IV (Total Marks : 28)

(Integer Answer Type)

This section contains **7 questions**. The answer to each of the questions is a **single–digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

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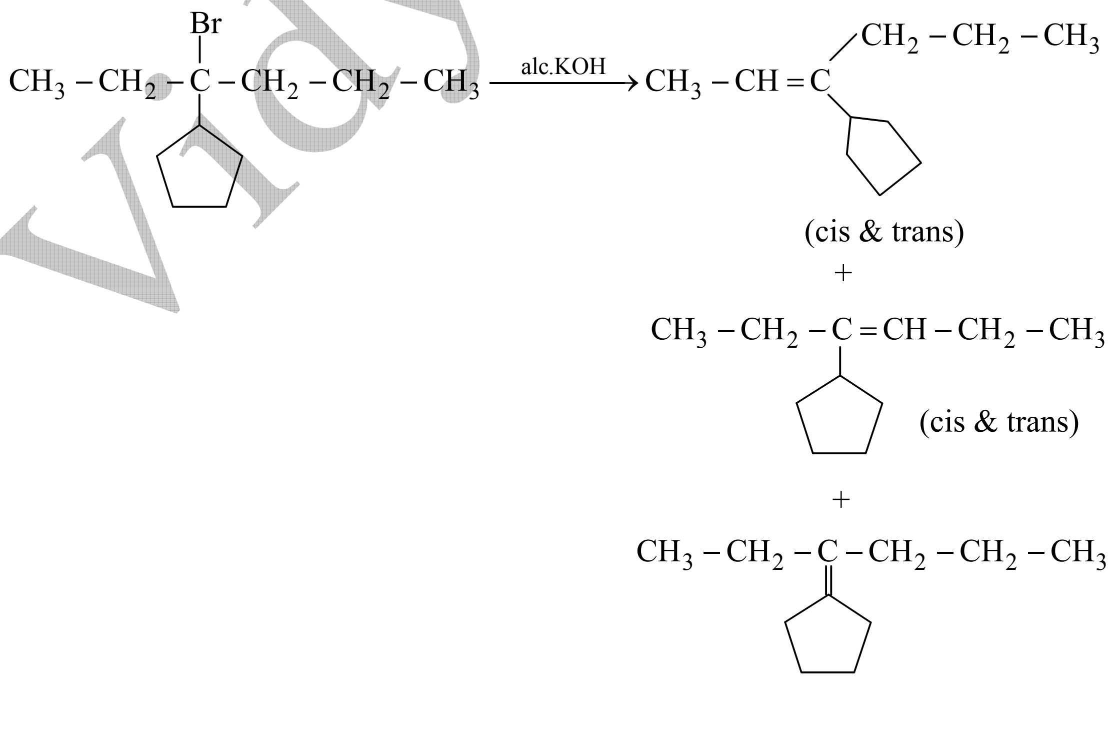
17. The total number of alkenes possible by dehydrobromination of 3–bromo–3–cyclopentylhexane using alcoholic KOH is

17. [5]



18. A decapeptide (Mol. Wt. 796) on complete hydrolysis gives glycine (Mol. Wt. 75), alanine and phenylalanine. Glycine contributes 47.0 % to the total weight of the hydrolysed products. The number of glycine units present in the decapeptide is

18. [6]

19. To an evacuated vessel with movable piston under external pressure of 1 atm., 0.1 mol of He and 1.0 mol of an unknown compound (vapour pressure 0.68 atm. at 0 °C) are introduced. Considering the ideal gas behaviour, the total volume (in litre) of the gases at 0 °C is close to

19. [7]

$P_{He} = 1 - 0.68 = 0.32$ atm

$P_{He}V = nRT$

$$V = \frac{0.1 \times 0.0821 \times 273}{0.32} = 7\,\text{litre}$$

20. The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

20. [4]

KE > 0 $\Rightarrow$ shows photoelectric effect.

$$KE = \frac{hc}{\lambda} - \phi$$

$$\text{Energy incident} = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{300 \times 10^{-9}} \times \frac{1}{(1.6 \times 10^{-19})} = 4.125\ \text{eV}$$

Kinetic energy are positive for Li, Na, K and Mg.

21. The maximum number of electrons that can have principal quantum number, n = 3 and spin quantum number, $m_s = -\frac{1}{2}$, is

21. [9]

The maximum number of electrons with spin ($m_S = -1/2$) = n^2 = 9

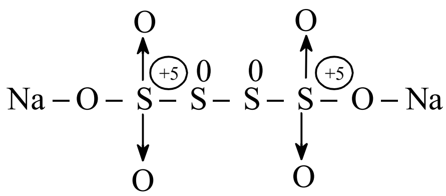
22. Reaction of Br_2 with Na_2CO_3 in aqueous solution gives sodium bromide and sodium bromate with evolution of CO_2 gas. The number of sodium bromide molecules involved in the balanced chemical equation is

22. [5]

$$3Br_2 + 3Na_2CO_3 \rightarrow 5NaBr + NaBrO_3 + 3CO_2$$

23. The difference in the oxidation numbers of the two types of sulphur atoms in $Na_2S_4O_6$ is

23. [5]



PART II : PHYSICS

SECTION–I (Total Marks : 21)

(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

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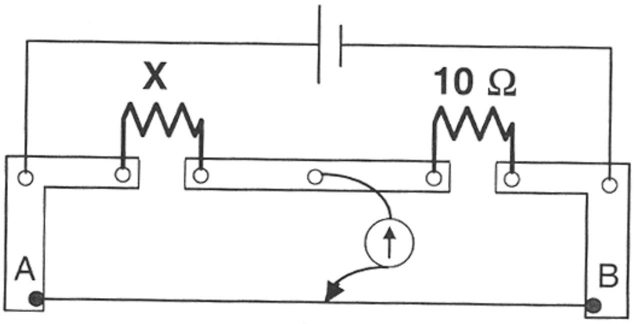
24. A meter bridge is set–up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is



(A) 10.2 ohm (B) 10.6 ohm (C) 10.8 ohm (D) 11.1 ohm

24. (B)

$\frac{X}{10} = \frac{l_1^1}{l_2^1}$

where l_1^1 and l_2^1 are the lengths with end corrections.

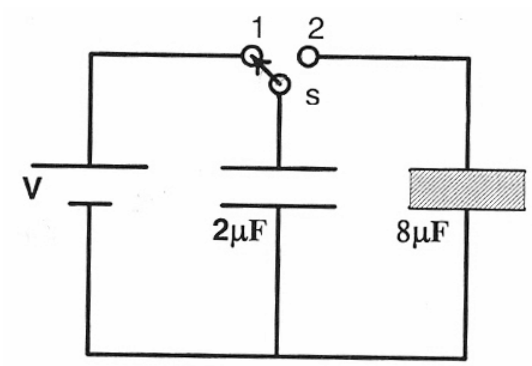
$l_1^1 = 52 + 1 = 53$ cm

$l_2^1 = 48 + 2 = 50$ cm

$\frac{X}{10} = \frac{53}{50} \rightarrow X = \frac{53}{50} \times 10 = \frac{53}{5}$

$\boxed{X = 10.6\,\Omega}$

25. A 2 μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



(A) 0%
(B) 20%
(C) 75%
(D) 80%

25. (D)

$C_1 = 2\mu F$, $C_2 = 8\ \mu F$

Initial Energy : $E_i = \frac{1}{2} C_1 V^2 = \frac{1}{2}\frac{Q^2}{C_1} = \frac{Q^2}{4}$

The initial charge : $Q = C_1 V$

The charge is distributed as Q_1 and Q_2 till the potentials are same.

$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{2\mu F}{8\mu F} = \frac{1}{4}$$

$\therefore \quad Q_1 = \frac{Q}{5}; \qquad Q_2 = \frac{4Q}{5}$

Find energy :

$$E_f = \frac{1}{2}\frac{Q_1^2}{C_1} + \frac{Q_2^2}{2C_2}$$

$$= \frac{Q_1^2}{4} + \frac{Q_2^2}{2\times 8} = \frac{Q_1^2}{4} + \frac{Q_2^2}{16} = \frac{Q^2}{25}\left[\frac{1}{4} + \frac{1}{16}\times 16\right] = \frac{Q^2}{20}$$

Energy dissipated : $\frac{Q^2}{4} - \frac{Q^2}{20} = \frac{4Q^2}{20} = \frac{Q^2}{5}$

% dissipated : $= \frac{Q^2/5}{Q^2/4}\times 100 = \frac{4}{5}\times 100 = 80$

26. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is

(A) 8.50 kHz (B) 8.25 kHz (C) 7.75 kHz (D) 7.50 kHz

26. (A)

Frequency received by the wall.

$$\nu_1 = \frac{v}{v - v_S}\nu_0$$

The frequency reflected by the wall is same as the frequency received by the wall.

The apparent reflected frequency

$$\nu_2 = \frac{v + v_0}{v}\nu_1$$

$$\nu_1 = \left(\frac{320}{320-10}\right)\nu_0 = \frac{32}{31}\times 8\,\text{kHz}.$$

$$\nu_2 = \frac{320+10}{320}\times \nu_1 = \frac{33}{32}\times\frac{32}{31}\times 8 = 8.5\text{ kHz}$$

27. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is

(A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$ (C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

27. (A)

$$W = \frac{P_1V_1 - P_2V_2}{r-1} = \frac{nR[T_1 - T_2]}{r-1}$$

PV^r = constant

TV^{r-1} = constant

$T_1V_1^{r-1} = T_2V_2^{r-1}$

$$r = \frac{5}{3}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{r-1} = \left(\frac{56}{7}\right)^{2/3} = 8^{2/3} = 4$$

$$W = \frac{nR[T_1 - T_2]}{2/3} = \frac{nR[T_1 - 4T_1]}{2/3} = \frac{3}{2}nR\times -3T_1 = -\frac{9}{2}nRT_1$$

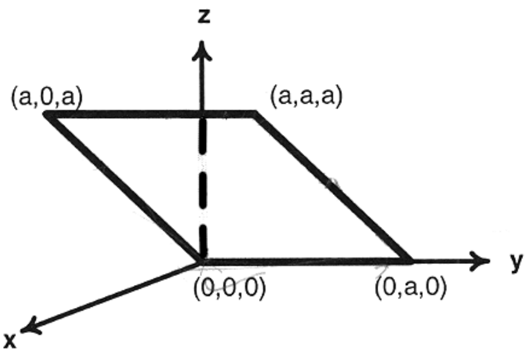
22.4 litre → 1 mole.
5.6 litre → 1/4 mole.

$W = -\frac{9}{2}\times\frac{1}{4}\ R\,T_1 = -\frac{9}{8}\ R\,T_1$

Work done by the gas is W.

Work done on the gas is $-W = \frac{9}{8} R\,T_1$

28. Consider an electric field $\vec{E} = E_0\,\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is



(A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$

(C) E_0a^2 (D) $\frac{E_0a^2}{\sqrt{2}}$

28. (C)

$\vec{E} = E_0\hat{x}$

$\vec{\ell} = a\hat{j}$

$\vec{b} = (a\hat{i}+a\hat{k})$

$\vec{\ell} = a\hat{j}$

$\vec{b} = a\hat{i}+a\hat{k}$

$\vec{A} = \vec{\ell}\times\vec{b} = a\hat{j}\times(a\hat{i}+a\hat{k})$

$= a^2(\hat{j}\times(\hat{i}+\hat{k})) = a^2(-\hat{k}+\hat{i}) = a^2(\hat{i}-\hat{k})$

$\phi = (\vec{E}\cdot\vec{A}) = E_0\hat{i}\cdot a^2(\hat{i}-\hat{k}) = (Ea^2)$

29. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of single-ionized helium atom is

(A) 1215 Å (B) 1640 Å (C) 2430 Å (D) 4687 Å

29. (A)

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{m^2}-\frac{1}{n^2}\right)$$

for Balmer series

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{4}-\frac{1}{n^2}\right)$$

$$\frac{1}{\lambda_1} = R\left(\frac{1}{4}-\frac{1}{9}\right)$$

$$\frac{1}{\lambda_1} = \frac{5R}{36}$$

$$\Rightarrow \quad \lambda_1 = \frac{36}{5R}$$

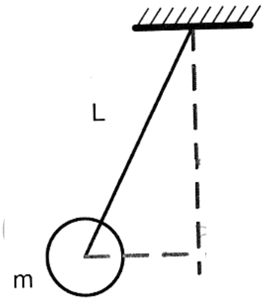
$$\frac{1}{\lambda_2} = R(4)\left(\frac{1}{4}-\frac{1}{16}\right) = 4R\left(\frac{4-1}{16}\right) = \frac{3R}{4}$$

$$\Rightarrow \quad \lambda_2 = \frac{4}{3R}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{\frac{4}{3}}{\frac{36}{5}} = \frac{20}{3 \times 36} = \frac{5}{27}$$

$$\lambda_2 = \frac{5\lambda_1}{27} = \frac{5(6561)}{27} = 1215 \text{ Å}$$

30. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is



(A) 9
(B) 18
(C) 27
(D) 36

30. (D)

$$T \cos\theta = mg$$

$$T \sin\theta = m\,\omega^2 \ell \sin\theta$$

$$T = m\,\omega^2 \ell$$

$$\omega^2 = \frac{T}{m\ell} = \frac{324}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = 4 \times 324$$

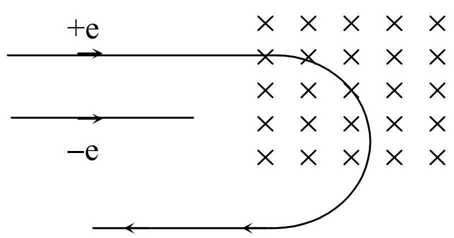
$$\omega = \sqrt{4 \times 324} = 36 \text{ rad/sec}$$

SECTION–II (Total Marks : 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

31. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to be velocity. Which of the following statement(s) is/are true?
(A) They will never come out of the magnetic field region.
(B) They will come out traveling along parallel paths.
(C) They will come out at the same time.
(D) They will come out at different times.

31. (B), (D)



$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(\frac{mv}{qB}\right)}{v} = \frac{2\pi m}{qB} = \left(\frac{2\pi}{B}\right)\left(\frac{m}{q}\right)$$

$\frac{m}{q}$ for both will be different so time taken will be different.

32. A spherical metal A of radius R_A and a solid metal sphere B of radius R_B ($< R_A$) are kept far apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then

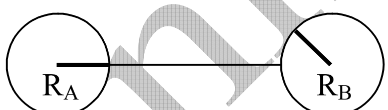
(A) $E_A^{\text{inside}} = 0$

(B) $Q_A > Q_B$

(C) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

(D) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

32. (A), (B), (C), (D)



$E_{\text{inside}} = 0$ for metallic shell

$$\frac{Q_A}{4\pi\varepsilon_0 r_A} = \frac{Q_B}{4\pi\varepsilon_0 r_B}$$

$$\frac{Q_A}{r_A} = \frac{Q_B}{r_B}$$

$$\frac{Q_A}{Q_B} = \frac{r_A}{r_B}$$

$$Q \propto r$$

$$R_A > R_B$$

$$Q_A > Q_B$$

$$\frac{Q_A 4\pi r_A}{4\pi r_A^{\ 2}} = \frac{Q_B 4\pi r_B}{4\pi r_B^{\ 2}}$$

$$\sigma_A r_A = \sigma_B r_B$$

$$\frac{\sigma_A}{\sigma_B} = \frac{r_B}{r_A}$$

$$\frac{E_A}{E_B} = \frac{\sigma_A}{\sigma_B} = \frac{r_B}{r_A} < 1$$

$$\Rightarrow \frac{E_A}{E_B} < 1 \Rightarrow E_A < E_B$$

32. Which of the following statement(s) is/are correct?

(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.

(B) The Gauss law can be used to calculate the field distribution around an electric dipole.

(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

(D) The work done by the external force is moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

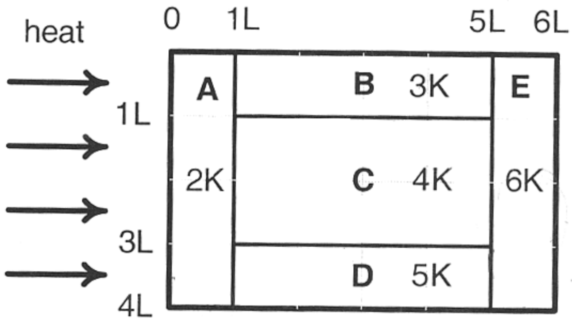
32. (C), (D)

Gauss law tells $\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$ [i.e., constant w.r.t. distance from a point charge]

$\Rightarrow$ Electric flux from a point charge $\propto \frac{1}{\sqrt{r}}$ [in new condition & that is not constant]

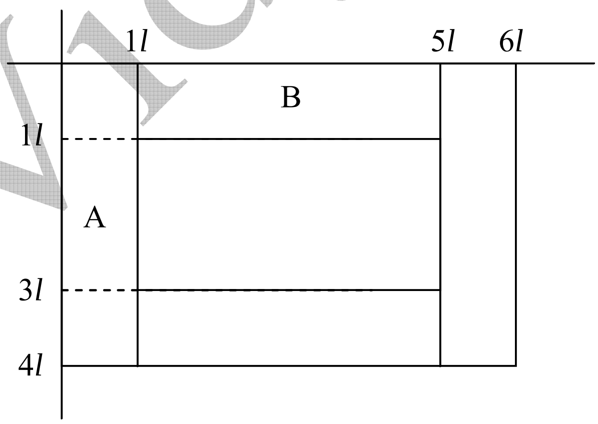
Hence (A) is incorrect.
Gauss' law is used to calculate amount of flux but not the field distribution.
Hence (B) is incorrect.
If there are only two charges then (C) is correct.
By definition, (D) is correct.

33. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state



(A) heat flow through A and E slabs are same.
(B) heat flow through slab E is maximum.
(C) temperature difference across slab E is smallest.
(D) heat flow through C = heat flow through B + heat flow through D.

33. (A), (C), (D)
(A) is correct.
[$\therefore$ B is wrong, as flow will be same in (A) and (E) in steady state]



Let width is x

$A_A = 4l\,(x)$	$= 4A$	$l_A = l$
$A_B = l\,(x)$	$= A$	$l_C = l_B = l_D = 4l$
$A_C = (2l)\,(x)$	$= 2A$	$l_E = l$
$A_D = l\,(x)$	$= A$	
$A_E = 4l\,(x)$	$= 4A$	

$$G_A = \frac{2K(4A)}{l} = \frac{8KA}{l} = 8G$$

$$G_B = \frac{3KA}{4l} = \frac{3}{4}G$$

$$G_C = \frac{4K\,3(2A)}{4l} = \frac{2KA}{l} = 2G$$

$$G_E = \frac{6K(4A)}{l} = \frac{24KA}{l} = 24G$$

$$G_D = \frac{(5K)A}{4l} = \frac{5KA}{4l} = \frac{5}{4}G$$

$$G_{BCD(eq)} = \frac{5KA}{4l} + \frac{3KA}{4l} + \frac{2KA}{l} = \left(\frac{5}{4} + \frac{3}{4} + 2\right)G = 4G$$

(C) $(\Delta T)_A = \dfrac{i}{8G}$

$$(\Delta T)_{BCD} = \frac{i}{4G} = \frac{i}{4G}$$

$$(\Delta T)_E = \frac{i}{24G}$$

$(\Delta T)_E < (\Delta T)_A < (\Delta T)_{BCD}$

(C) is correct.

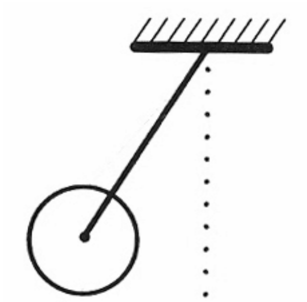
(D) $i_B = \dfrac{3}{4}G(\Delta T)_{BCD}$

$$i_C = 2G\,(\Delta T)_{BCD}$$

$$i_D = \frac{5}{4}G(\Delta T)_{BCD} \quad \Rightarrow \quad i_C = i_B + i_D$$

(D) is correct.

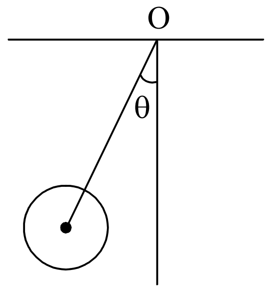
34. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (<L) is attached at its center to the free end of the rod. Consider two ways the disc is attached : (case A) The disc is not free to rotate about its center and (case B) the disc is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true?



(A) Restoring torque in case A = Restoring torque in case B.
(B) Restoring torque in case A < Resorting torque in case B.
(C) Angular frequency for case A > Angular frequency for case B.
(D) Angular frequency for case A < Angular frequency for case B.

34. (A), (C)

τ for both is = $(M + m)\, x_c\, g \sin\theta$ same is both case in A.
(x_c is position of C.M from O)
In both cases, ΔU is same



$$K_A = \frac{1}{2}(M+m)(V_C{}^2)_A \quad \text{(at lowest point)}$$

$$K_B = \frac{1}{2}(M+m)(V_C)_B{}^2 + \frac{1}{2}I_C\omega_C{}^2$$

$\because \quad K_A = K_B$

$\Rightarrow \quad (V_C)_B < (V_C)_A \Rightarrow \left(\omega_{C_0}\right)_B < \left(\omega_{C_0}\right)_A$

$\omega_{C_0} = f\,\theta_0$

Angular velocity of center of mass about O (pivot) and f is angular frequency

θ_0 is same for both $f \propto V_C$

$f_B < f_A$

(C) is also correct

SECTION–III (Total Marks : 15)

(Paragraph Type)

This section contains **2 paragraphs.** Based upon one of the paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 35 and 36

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency 'ω_p', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

35. Taking the electronic charge as 'e' and the permittivity as 'ε_0', use dimensional analysis to determine the correct expression for ω_p.

(A) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$ (C) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

35. (C)

Using dimensional analysis

$N \Rightarrow [L^{-3}]$

$e \Rightarrow [A\,T]$

$m \Rightarrow [M]$

$\varepsilon_0 \Rightarrow [M^{-1}\,L^{-3}\,T^4\,A^2]$

$$\therefore \sqrt{\frac{Ne^2}{m\varepsilon_0}} = \sqrt{\frac{L^{-3}\,.\,A^2T^2}{M\,.\,M^{-1}\,L^{-3}\,T^4\,A^2}}$$

$$\omega_p \Rightarrow \sqrt{\frac{1}{T^2}} \Rightarrow \frac{1}{T} = [T^{-1}]$$

36. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}\ m^{-3}$. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.

(A) 800 nm (B) 600 nm (C) 300 nm (D) 200 nm

36. (B)

$$\nu = \frac{C}{\lambda}$$

$$\nu = \frac{\omega}{2\pi}$$

$$\therefore \frac{\omega_p}{2\pi} = \frac{C}{\lambda}$$

$$\lambda = \frac{2\pi C}{\omega_p}$$

$$\omega_p = \sqrt{\frac{Ne^2}{m\varepsilon_0}} = \sqrt{\frac{4\times10^{27}\times(1.67\times10^{-19})^2}{10^{-30}\times10^{-11}}} \approx 3.14 \times 10^{15}\ \text{sec}^{-1}$$

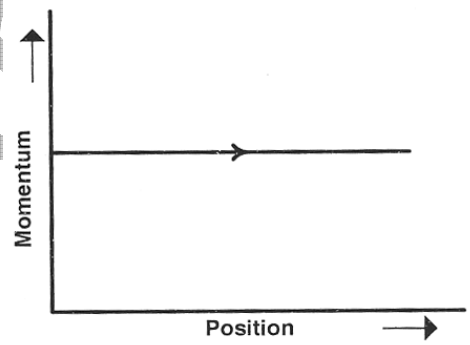
$$\therefore \lambda = \frac{2\pi C}{\omega_p} = \frac{2\times3.14\times3\times10^8}{3.14\times10^{15}} = 6 \times 10^{-7}\ \text{m}$$

$$= 600 \times 10^{-9}\ \text{m}$$

$$\lambda = 600\ \text{nm}$$

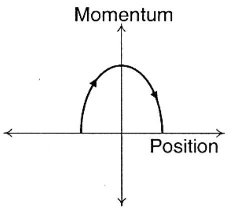
Paragraph for Question Nos. 37 to 39

Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.

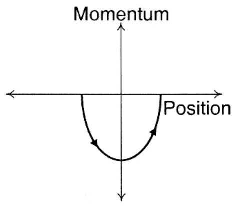


37. The phase space diagram for a ball thrown vertically up from ground is

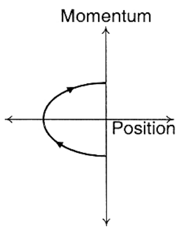
(A)



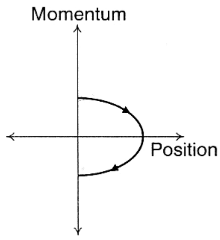
(B)



(C)



(D)

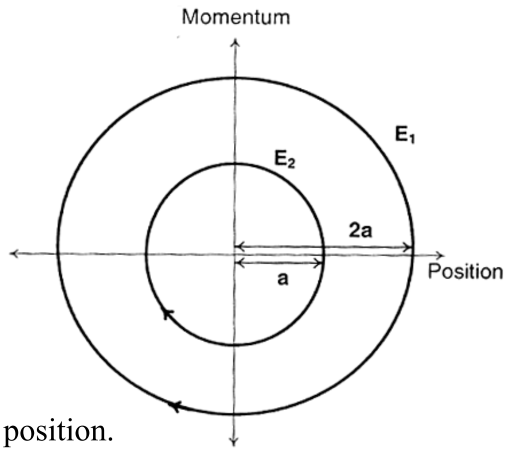


37. (D)

As the body is thrown upward its initial momentum is maximum. As time increases the momentum decreases (the body going up), become 0 at maximum height.

And then comes back. Therefore, the momentum becomes negative (change in direction).

38. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then



(A) $E_1 = \sqrt{2}E_2$ (B) $E_1 = 2\ E_2$
(C) $E_1 = 4\ E_2$ (D) $E_1 = 16\ E_2$

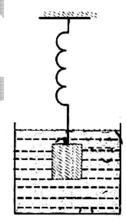
38. (C)
Consider both the particles initially at their extreme position.
Therefore, the particle begins from maximum position and momentum is 0.

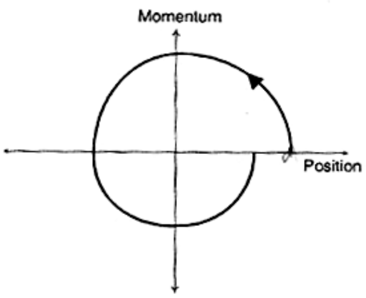
$$E \propto (\text{position})^2$$

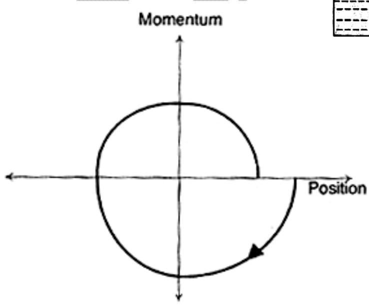
$$\frac{E_1}{E_2} = \left(\frac{2a}{a}\right)^2 = 4$$

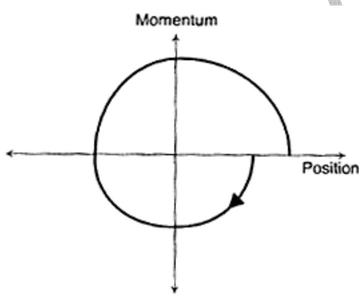
$$E_1 = 4\ E_2$$

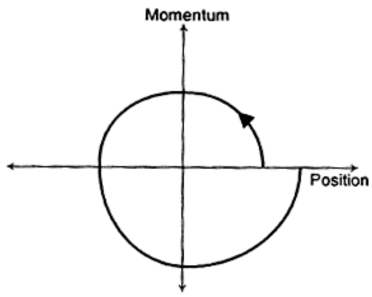
39. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(A)


(B)


(C)


(D)


39. (B)
Looking at sign convention,
take upward motion as +ve and downward motion as negative.
take upward extreme as +ve and downward extreme as negative.

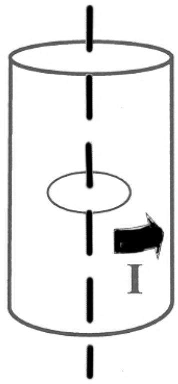
i) Beginning from the upward extreme
The particle has a downward velocity which increases with time.
Hence at mean position the momentum is maximum negative.
ii) The particle moves ahead in negative but momentum decreases in same direction. Then momentum become 0 and position becomes negative extreme.
iii) Now the momentum changes direction to +ve and position is till in negative, till it reaches mean position where momentum becomes maximum in +ve direction.
iv) It then comes back to its +ve extreme with momentum decreasing but in + direction.
v) In this whole cycle, some amount of energy is absorbed due to resistance offered by liquid and hence the final position will be less than initial position.

Therefore, the correct representation will be (B).

SECTION–IV (Total Marks : 28)
(Integer Answer Type)

This section contains **7 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

40. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos(300\ t)$ where I_0 is constant. If the magnetic moment of the loop is $N\ \mu_0\ I_0 \sin(300\ t)$, then 'N' is



40. [6]

The magnetic field at the centre of cylinder $= \dfrac{\mu_0 I}{L}$

$\therefore$ Flux through the ring $= \dfrac{\mu . I \pi r^2}{L}$ (r = radius of ring)

$\therefore$ EMF $= \dfrac{\left(\mu_0 \pi r^2\right)}{L} \dfrac{dI}{dt}$

Let resistance of loop be, R = 0.005 Ω

$\therefore$ current $= \dfrac{\mu_0 \pi r^2}{RL} . \dfrac{dI}{dt}$

$$\text{Magnetic moment} = \frac{\mu_0}{RL}\left(\pi r^2\right)^2 \frac{dI}{dt} = \frac{\mu_0}{RL}\left(\pi r^2\right)^2 300.I_0 \sin(300t)$$

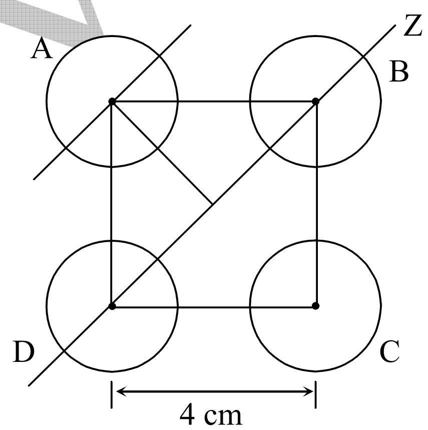
$$= \frac{300}{RL}\left(\pi r^2\right)^2 \pi.I.\sin(300\,t)$$

$$N = \frac{300}{0.005 \times 10}\left(\pi \times 10^{-2}\right)^2$$

$$= \frac{300}{0.005 \times 10} \times \pi^2 \times 10^{-4} = \frac{\pi^2 \times 3 \times 10^{-2}}{5 \times 10^{-3} \times 10} \approx 6$$

41. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is

41. [9]



a = 4 cm

$R = \left(\dfrac{\sqrt{5}}{2}\right)$

Moment of inertia about I is required

$$\text{M.I. of A about I} = \frac{2}{5}mR^2 + m\left(\frac{a}{\sqrt{2}}\right)^2$$

$$\therefore \text{ Total M.I. of the system} = 2\times\frac{2}{5}mR^2 + 2\left(\frac{2}{5}mR^2 + \frac{ma^2}{2}\right)$$

$$= \frac{4}{5}\times\frac{1}{2}\times\left(\frac{\sqrt{5}}{2}\right)^2 + 2\left[\frac{2}{5}\times\frac{1}{5}\times\left(\frac{\sqrt{5}}{2}\right)^2 + \frac{1}{2}.\frac{1}{2}\times 4^2\right] \text{kg cm}^3$$

$$= \left(\frac{1}{2} + 2\left[\frac{1}{4} + 4\right]\right)\text{kg cm}^2 = 1 + 8 = 9 \text{ kg cm}^2 = 9 \times 10^{-4} \text{ kg m}^2$$

42. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is

42. [1]

$A = \lambda N$

N = No. of radioactive nuclei

$$t_a = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{t_a}$$

$$\therefore N = \frac{A}{\lambda} = A \times t_a$$

Total mass of sample = mN [m = mass of one nucleus]

$$= mA\,t_a = 10^{-2}\,\text{kg} \times 10^{10} \times 10^9 = 10^{-6}\,\text{kg}$$

43. Steel wire of length 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L >> diameter of the wire. Then the value of 'm' in kg is nearly

43. [3]

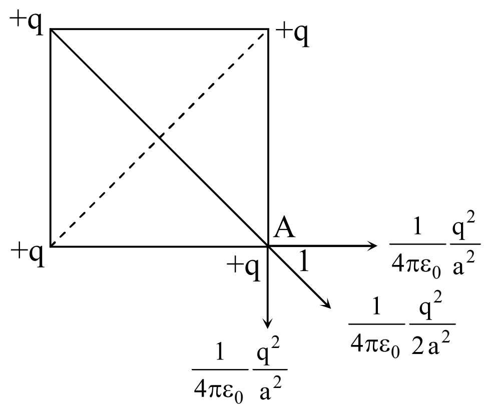
$$\text{Strain} = \frac{\text{stress}}{y} = \frac{mg}{\pi r^2 y} = \frac{\Delta\ell}{L} \Rightarrow \Delta\ell = \frac{mgL}{\pi r^2 y}$$

$$\Delta\ell = L\,\alpha\,\Delta T = \frac{mg\,L}{\pi r^2 y}$$

$$\Rightarrow m = \frac{\pi r^2 y\alpha\Delta T}{g} = \frac{22}{7}\times\frac{10^{-6}\times10^{11}\times10^{-5}\times10}{10} \approx 3 \text{ kg}$$

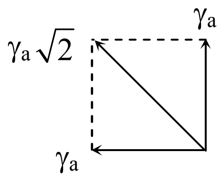
44. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where 'k' is a constant. Then N is

44. [3]



Net electrostatic force on A $= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left[\sqrt{2}+\frac{1}{2}\right]$

Force due to surface tension T



By equilibrium

$\gamma_a\sqrt{2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left[\sqrt{2}+\frac{1}{2}\right] \quad \therefore \; a^3 = K\left(\frac{q^2}{a}\right) \quad \therefore \; N = 3$

45. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N = 10 μ, then N is

45. [5]

$F = -\mu mg\cos\theta + mg\sin\theta \quad \ldots(1)$

$3F = mg\sin\theta + \mu mg\cos\theta \quad \ldots(2)$

$4F = 2\,mg\sin\theta$

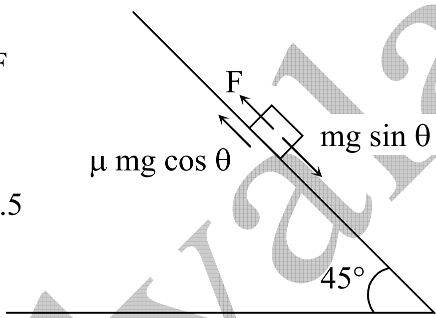
$2F = mg\cos\theta$

$F = (1-\mu)\,2F$

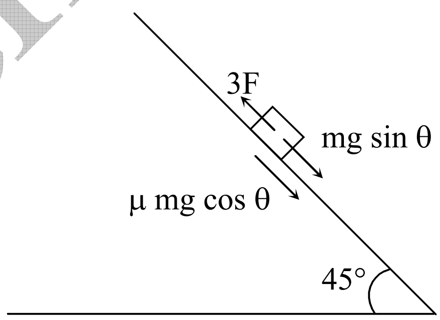
$\frac{1}{2} = 1-\mu$

$\mu = 1-\frac{1}{2} = 0.5$

$10\,\mu = 5$

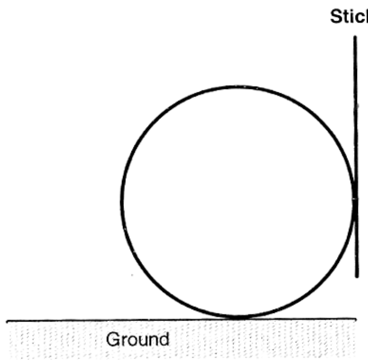


For Eq (1)

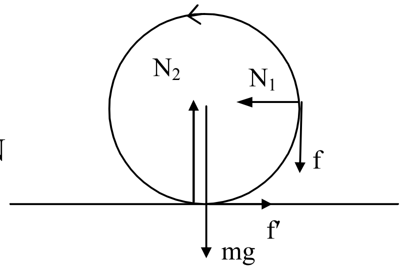


For Eq (2)

46. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is



46. [4]



$N_1 = 2$ N

$N_2 = mg + f = mg + \mu N_1$

$N_1 - f' = 2\text{kg} \times 0.3\ \text{ms}^{-2} = 0.6$ N

$(f' - f)R = mR^2 \cdot \alpha$

$\Rightarrow f' - f = ma = 0.6$ N

$\therefore N_1 - f = 1.2$ N

$\Rightarrow N_1(1-\mu) = 1.2$ N

$\Rightarrow (1-\mu) = \frac{1.2\text{N}}{2} = 0.6$

$\Rightarrow \mu = 0.4$

PART III - MATHEMATICS

SECTION – I (Total Marks : 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has four choices (A), (B), (C), and (D) out of which **ONLY ONE** is correct.

47. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is

(A) 1 (B) 2 (C) 3 (D) 4

47. (C)

$\alpha + \beta = 6 \qquad \alpha\beta = -2$

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{(\alpha^{10} - \beta^{10}) + \alpha\beta(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{(\alpha^{10} + \alpha^9\beta) - (\beta^{10} + \alpha\beta^9)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^9(\alpha+\beta) - \beta^9(\alpha+\beta)}{2(\alpha^9 - \beta^9)} = \frac{\alpha+\beta}{2} = \frac{6}{2} = 3$$

48. A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x–axis, then the equation of L is

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

48. (B)

Let slope of required line m.

So, $\tan 60° = \left|\frac{m - (-\sqrt{3})}{1 + m(-\sqrt{3})}\right|$

As slope of line $\sqrt{3}x + y = 1$ is $-\sqrt{3}$

$\Rightarrow \quad \sqrt{3} = \pm \frac{m + \sqrt{3}}{1 - \sqrt{3}m}$

$\oplus$ sign, $\sqrt{3} - 3m = m + \sqrt{3}$

$\Rightarrow m = 0$

Not possible as not intersect x–axis.

$\ominus$ sign, $\sqrt{3} - 3m = -m - \sqrt{3}$

$\Rightarrow 2\sqrt{3} = 2m$

$\Rightarrow \boxed{m = \sqrt{3}}$

So, equation of required line is

$y - (-2) = \sqrt{3}(x - 3)$

$\Rightarrow \quad y + 2 = \sqrt{3}x - 3\sqrt{3}$

$\Rightarrow \quad y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

49. Let (x_0, y_0) be the solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$
$$3^{\ln x} = 2^{\ln y}$$

Then x_0 is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

49. (C)

Given $(2x)^{\ell n2} = (3y)^{\ell n3}$

$\Rightarrow \quad \ell n2 \,.\ell n2x = \ell n3\ \ell n(3y) \qquad \ldots\ldots(1)$

and $\quad 3^{\ell n x} = 2^{\ell n y}$

$\Rightarrow \quad \ell n x\ \ell n 3 = \ell n y\ \ell n 2 \qquad \ldots\ldots(2)$

From (1) & (2) eliminating y we get,

$$\ell n 2\ \ell n 2x = \ell n\ 3\left(\ell n3 + \frac{\ell n x\, \ell n3}{\ell n 2}\right)$$

$\Rightarrow \quad (\ell n2)^2\ \ell n2x = \ell n3(\ell n3\ell n2 + \ell nx\, \ell n3)$

$\Rightarrow \quad (\ell n2)^2\, \ell n2x = (\ell n3)^2\, \ell n2 + (\ell n3)^2\, \ell nx$

$\Rightarrow \quad (\ell n2)^2\, \ell n2 + (\ell n2)^2\, \ell n\, x = (\ell n3)^2\, \ell n2 + (\ell n3)^2\, \ell nx$

$\Rightarrow \quad \ell n\, x\left[(\ell n\, 2)^2 - (\ell n\, 3)^2\right] = \ell n\, 2\left[(\ell n\, 3)^2 - (\ell n\, 2)^2\right]$

$\Rightarrow \quad \ell n\, x = -\ell n\, 2$

$\Rightarrow \quad \ell n\ x = \ \ell n\ 2^{-1}$

$\Rightarrow \quad x = \frac{1}{2}.$

50. The value of $\displaystyle\int_{\sqrt{\text{In}2}}^{\sqrt{\text{In}3}} \frac{x \sin x^2}{\sin x^2 + \sin\left(\text{In}6 - x^2\right)} dx$ is

(A) $\frac{1}{4}\text{In}\frac{3}{2}$ (B) $\frac{1}{2}\text{In}\frac{3}{2}$ (C) $\text{In}\frac{3}{2}$ (D) $\frac{1}{6}\text{In}\frac{3}{2}$

50. (A)

$$\int_{\sqrt{\ell n 2}}^{\sqrt{\ell n3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ell n6 - x^2)}\, dx$$

Put, $\quad x = \sqrt{t}$

$\Rightarrow \quad x^2 = t$

$\Rightarrow \quad 2x\ dx = dt$

$\Rightarrow \quad x\ dx = \frac{1}{2}dt$

x	$\sqrt{\ell n2}$	$\sqrt{\ell n3}$
t	$\ell n2$	$\ell n3$

$$I = \frac{1}{2}\int_{\ell n2}^{\ell n3} \frac{\sin t\, dt}{\sin t + \sin(\ell n6 - t)}$$

$$= \frac{1}{2}\int_{\ell n2}^{\ell n3} \frac{\sin(\ell n3 + \ell n2 - t)}{\sin(\ell n3 + \ell n2 - t) + \sin(\ell n6 - (\ell n3 + \ell n2 - t))} dt$$

$$= \frac{1}{2}\int_{\ell n2}^{\ell n3} \frac{\sin(\ell n6 - t)}{\sin(\ell n6 - t) + \sin t} dt$$

$$2I = \frac{1}{2}\int_{\ell n2}^{\ell n3} dt = \frac{1}{2}[\ell n3 - \ell n2] \qquad \Rightarrow I = \frac{1}{4}\ell n\frac{3}{2}.$$

51. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by

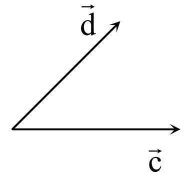
(A) $\hat{i} - 3\hat{j} + 3\hat{k}$ (B) $-3\hat{i} - 3\hat{j} - \hat{k}$ (C) $3\hat{i} - \hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

51. (C)

Let $\vec{d}$ in the plane of $\vec{a}$ and $\vec{b}$

$\vec{d} = \alpha\vec{a} + \beta\vec{b}$

$\vec{d} = \hat{i}(\alpha + \beta) + \hat{j}(\alpha - \beta) + \hat{k}(\alpha + \beta)$



$$\frac{\vec{d}\cdot\vec{c}}{|c|} = \frac{1}{\sqrt{3}}$$

$$\frac{\alpha + \beta - (\alpha - \beta) - (\alpha + \beta)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{\beta - \alpha}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\alpha - \beta = -1$$

Let $\alpha = 1$, $\beta = 2$

$\vec{d} = \hat{i}(3) + \hat{j}(-1) + \hat{k}(3) = 3\hat{i} - \hat{j} + 3\hat{k}$

52. Let P = $\{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and Q = $\{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \varnothing$ (B) $Q \not\subset P$

(C) $P \not\subset Q$ (D) P = Q

52. (D)

P = $\{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\} = \{\theta : \sin\theta = (\sqrt{2} + 1)\cos\theta\}$

Q = $\{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

= $\{\theta : \sin\theta(\sqrt{2} - 1) = \cos\theta\} = \{\theta : \sin\theta = (\sqrt{2} + 1)\cos\theta\}$

"Multiplying by $\sqrt{2} - 1$ on both sides".

$\therefore$ P = Q

53. Let the straight line x = b divide the area enclosed by $y = (1 - x)^2$, y = 0, and x = 0 into two parts R_1 $(0 \le x \le b)$ and R_2 $(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

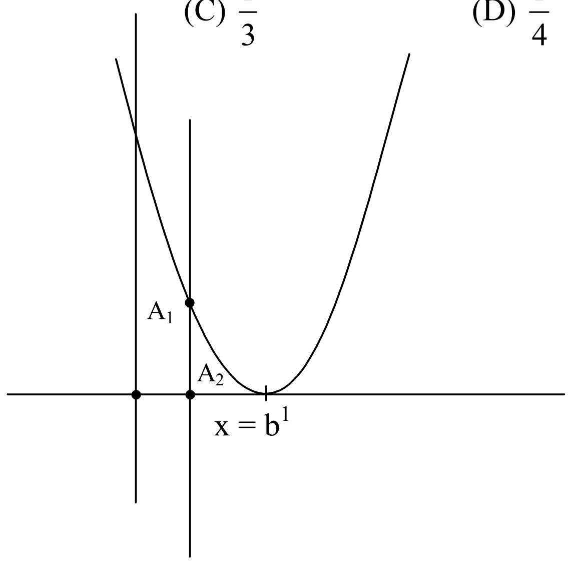
(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

53. (B)

$$A_1 + A_2 = \int_0^1 (x-1)^2 .dx$$

$$A_1 + A_2 = \left.\frac{(x-1)^3}{3}\right|_0^1$$

$$\boxed{A_1 + A_2 = \frac{1}{3}}$$



$A_1 = \int_0^b (x-1)^2 .dx$

$\boxed{A_1 = \frac{(b-1)^3}{3} + \frac{1}{3}}$

$A_1 - A_2 = \frac{1}{4}$

$A_1 + A_2 = \frac{1}{3}$

$2A_1 = \frac{7}{12}$ $\qquad$ $A_1 = \frac{7}{24} = \frac{(b-1)^3}{3} + \frac{1}{3}$

$\Rightarrow \frac{(b-1)^3}{3} = -\frac{1}{24}$

$\Rightarrow b-1 = -\frac{1}{2}$ $\qquad$ $\Rightarrow \boxed{b = \frac{1}{2}}$

Section – II (Total Marks : 16)

(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE** or **MORE** may be correct.

54. Let M and N be two 3 × 3 non−singular skew−symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2 N^2 (M^T N)^{-1} (MN^{-1})^T$ is equal to
(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

54. There seems to be an ambiguity in the question since 3 × 3 skew−symmetric matrices can't be non−singular.
[Property : Determinant of an odd order skew−symmetric matrix is always zero]

55. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are
(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

55. (A), (D)
Let vectors $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$
As $\vec{r}$ is coplanar with $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$

$$\Rightarrow \begin{vmatrix} x & y & z \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow -3x + y + z = 0$ …(1)
Also given $\vec{r}$ is $\perp^r$ to $\hat{i}+\hat{j}+\hat{k}$
$\Rightarrow \vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) = 0$
$\Rightarrow x + y + z = 0$ …(2)
from (1) and (2) by cross multiplication

$$\frac{x}{0} = \frac{y}{4} = \frac{z}{-4}$$

$$\frac{x}{0} = \frac{y}{1} = \frac{z}{-1}$$

$\Rightarrow \vec{r} = \hat{j} - \hat{k}$ or $\vec{r} = -\hat{j} + \hat{k}$

Aliter

A vector perpendicular to $\vec{c}$ and in the plane of $\vec{a}$ and $\vec{b}$ is

$\lambda\left(\vec{c} \times (\vec{a} \times \vec{b})\right)$

$\vec{c} \times \left(\vec{a} \times \vec{b}\right) = 4\left(-\hat{j} + \hat{k}\right)$

For $\lambda = \pm\frac{1}{4}$ Option (A) and (D) are correct.

56. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(A) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(B) a focus of the hyperbola is (2, 0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2 - 3y^2 = 3$

56. (B), (D)

$$\frac{x^2}{4} + \frac{y^2}{1} = 1 \qquad \ldots(1)$$

Eccentricity of ellipse (1) is $e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

Focus of ellipse $S = (\pm\, ae, 0) \equiv (\pm\sqrt{3}, 0)$

by given condition,

$$\sqrt{1 + \frac{b^2}{a^2}} = \frac{1}{\frac{\sqrt{3}}{2}} \quad \Rightarrow 1 + \frac{b^2}{a^2} = \frac{4}{3}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{1}{3} \qquad \ldots(2)$$

So, eccentricity of hyperbola

$$e' = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}}$$

As given hyperbola passes focus $(\pm\sqrt{3}, 0)$

$\Rightarrow \frac{3}{a^2} - \frac{0}{b^2} = 1 \quad \Rightarrow a^2 = 3$

$\Rightarrow a = \pm\sqrt{3}$

So, from (2) b = 1

$\therefore$ equation of hyperbola is $\frac{x^2}{3} - \frac{y^2}{1} = 1$

$\Rightarrow x^2 - 3y^2 = 3$

Focus is $(\pm ae, 0) \equiv \left(\pm\sqrt{3}\cdot\left(\frac{2}{\sqrt{3}}\right), 0\right)$

$\equiv (\pm 2, 0)$

57. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y), \forall x, y \in \mathbb{R}$.
If f(x) is differentiable at x = 0, then
(A) f(x) is differentiable only in a finite interval containing zero
(B) f(x) is continuous $\forall\, x \in \mathbb{R}$.
(C) $f'(x)$ is constant $\forall\, x \in \mathbb{R}$
(D) f(x) is differentiable except at finitely many points

57. (B), (C)
Given

$$f(x + h) = f(x) + f(h)$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(h)}{h} = f'(0)$$

= given it exists (using L′ Hospital rule)

$\therefore$ $f'(x)$ exists $\forall\, x\, \varepsilon\, \mathbb{R}$ $\therefore f'(0)$ exist (given)
(B) and (C) are correct

Section – III (Total Marks : 15)
(Paragraph Type)

This section contains **2 paragraphs**. Based upon one of the paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question nos. 58 and 59

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

58. The probability of the drawn ball from U_2 being white is
(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

58. (B)
Required probability

$$=\frac{1}{2}\left[\frac{{}^3C_1}{{}^5C_1}\times 1+\frac{{}^2C_1}{{}^5C_1}\times\frac{1}{2}\right]+\frac{1}{2}\left[\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}+\frac{{}^3C_2}{{}^5C_2}\times 1+\frac{{}^3C_1\times{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right]=\frac{1}{2}\times\frac{12}{15}+\frac{1}{2}\times\frac{11}{15}=\frac{23}{30}$$

59. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is
(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

59. (D)
Applying Baye's Theorem,

Required probability = $\dfrac{\frac{12}{30}}{\frac{12}{30}+\frac{11}{30}} = \dfrac{12}{23}$

Paragraph for Question Nos. 60 to 62

Let a, b and c be three real numbers satisfying

$$[a\ \ b\ \ c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0\ \ 0\ \ 0] \qquad \ldots\ldots. \text{(E)}$$

60. If the point P(a, b, c), with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

(A) 0 (B) 12 (C) 7 (D) 6

60. (D)

By equation(E),

$a + 8b + 7c = 0$

$9a + 2b + 3c = 0$

$7a + 7b + 7c = 0$

This system of equations has no unique solution as $\Delta = 0$

$\therefore$ We can find a : b : c

By solving simultaneously $\dfrac{a}{1} = \dfrac{b}{6} = \dfrac{c}{-7} = K$

If (a, b, c) lies on $2x + y + z = 1$, then $2a + b + c = 1$.

$\therefore 2K + 6K - 7K = 1 \Rightarrow K = 1$.

$\therefore 7a + b + c = 7 + 6 - 7 = 6$

61. Let ω be a solution of $x^3 - 1 = 0$ with $\text{Im}(\omega) > 0$. If $a = 2$ with b and c satisfying (E), then the value of $\dfrac{3}{\omega^a} + \dfrac{1}{\omega^b} + \dfrac{3}{\omega^c}$ is equal to

(A) −2 (B) 2 (C) 3 (D) −3

61. (A)

$\omega = -\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}$.

If $a = 2$, then $b = 12$, $c = -14$.

$\therefore \dfrac{3}{\omega^a} + \dfrac{1}{\omega^b} + \dfrac{3}{\omega^c} = \dfrac{3}{\omega^2} + \dfrac{1}{\omega^{12}} + \dfrac{3}{\omega^{-14}}$

$= 3\omega + 1 + 3\omega^2 = 3(-1) + 1 = -2$

62. Let $b = 6$, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ is

(A) 6 (B) 7 (C) 6/7 (D) ∞

62. (B)

If $b = 6$, then $a = 1$, $c = -7$.

$$\sum_{n=0}^{\infty}\left(\frac{\beta+\alpha}{\alpha\beta}\right)^n = \sum_{n=0}^{\infty}\left(-\frac{b}{c}\right)^n = \left(\frac{6}{7}\right)^0 + \left(\frac{6}{7}\right)^1 + \ldots\ldots\infty$$

$$= \frac{1}{1-\frac{6}{7}} = 7.$$

Section – IV (Total Marks : 28)
Integer Answer Type

This section contains **7 questions**. The answer to each of the questions is a **single–digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to the darkened in the ORS.

63. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is

63. [8]

AM $\geq$ GM

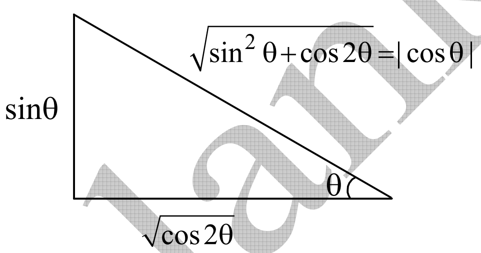
$$\frac{a^{-5}+a^{-4}+a^{-3}+a^{-3}+a^{-3}+1+a^{8}+a^{10}}{8} \geq \left(a^{-5}\times a^{-4}\times\left(a^{-3}\right)^{3}\times a^{8}\times a^{10}\right)^{\frac{1}{8}}$$

$$a^{-5}+a^{-4}+a^{-3}+a^{-3}+a^{-3}+1+a^{8}+a^{10} \geq 8$$

64. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

64. [1]

$$f(\theta)=\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$$



$$f(\theta)=\sin\left(\sin^{-1}\left(\frac{\sin\theta}{|\cos\theta|}\right)\right)$$

$$\frac{-\pi}{4}<\theta<\frac{\pi}{4}$$

$\cos\theta > 0$

$$f(\theta)=\sin\left(\sin^{-1}\left(\tan\theta\right)\right)$$

$$\frac{-\pi}{4}<\theta<\frac{\pi}{4}$$

$-1 < \tan\theta < 1$

$\sin^{-1}\sin(\tan\theta) = \tan\theta$

$f(\theta) = \tan\theta$

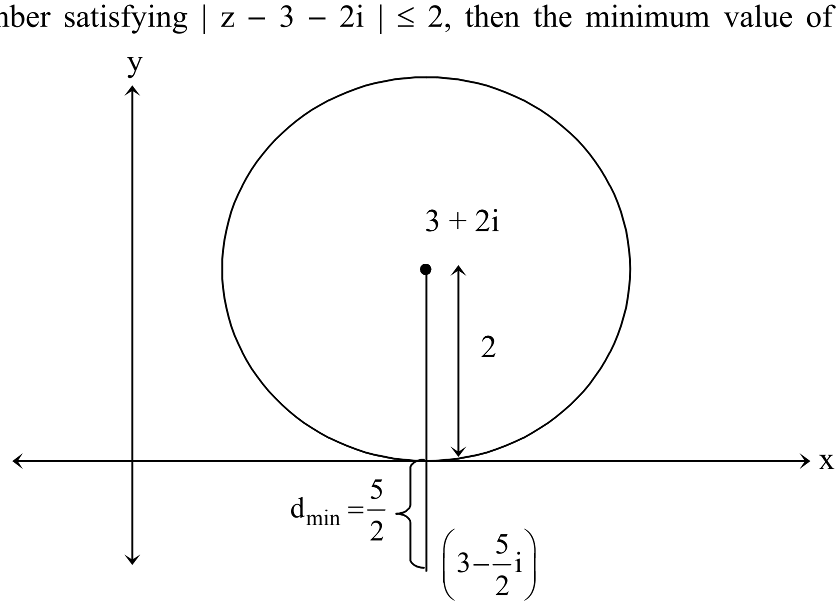
$$\frac{df(\theta)}{d(\tan\theta)}=1.$$

65. If z is any complex number satisfying $|z - 3 - 2i| \leq 2$, then the minimum value of $|2z - 6 + 5i|$ is

65. [5]

$$|2z-6+5i|=2\left|z-\left(3-\frac{5}{2}i\right)\right|$$

$2\,d_{min} = 5$



66. Let f : [1, ∞) → [2, ∞) be a differentiable function such that f(1) = 2. If $6\int_1^x f(t)dt = 3xf(x) - x^3$ for all x ≥ 1, then the value of f(2) is

66. [6]

Differentiating. w.r.t. to x.

$6\ f(x) = 3x\ f'(x) + 3f(x) - 3x^2$

$f(x) = xf'(x) - x^2$

$x\dfrac{dy}{dx} - y = x^2$

$\dfrac{dy}{dx} - \dfrac{y}{x} = x$

$I.\ F. = e^{\int -\frac{1}{x}\cdot dx} = \dfrac{1}{x}$

$\dfrac{1}{x} \times y = \int \dfrac{1}{x} \times x.dx$

$\dfrac{y}{x} = x + c$

At x = 1, y = 2,

$\therefore c = 1$

$\dfrac{y}{x} = x + 1$

at x = 2

y = 6

67. The positive integer value of n > 3 satisfying the equation $\dfrac{1}{\sin\left(\frac{\pi}{n}\right)} = \dfrac{1}{\sin\left(\frac{2\pi}{n}\right)} + \dfrac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

67. [7]

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$$

$$\Rightarrow \sin\left(\frac{2\pi}{n}\right)\sin\left(\frac{3\pi}{n}\right) = \sin\left(\frac{\pi}{n}\right)\sin\left(\frac{3\pi}{n}\right) + \sin\left(\frac{\pi}{n}\right)\sin\left(\frac{2\pi}{n}\right)$$

$$\Rightarrow \sin\left(\frac{2\pi}{n}\right)\left[\sin\left(\frac{3\pi}{n}\right) - \sin\left(\frac{\pi}{n}\right)\right] = \sin\left(\frac{\pi}{n}\right)\cdot\sin\left(\frac{3\pi}{n}\right)$$

$$\Rightarrow \sin\left(\frac{2\pi}{n}\right) \times 2\cos\left(\frac{2\pi}{n}\right)\sin\left(\frac{\pi}{n}\right) = \sin\left(\frac{3\pi}{n}\right)\sin\left(\frac{\pi}{n}\right)$$

$$\Rightarrow \sin\left(\frac{4\pi}{n}\right)\sin\left(\frac{\pi}{n}\right) = \sin\left(\frac{3\pi}{n}\right)\sin\left(\frac{\pi}{n}\right)$$

$$\Rightarrow \sin\left(\frac{4\pi}{n}\right) = \sin\left(\frac{3\pi}{n}\right) \qquad \left(\because \sin\left(\frac{\pi}{n}\right) \neq 0\right)$$

$$\Rightarrow n = 7 \qquad \left(\because \sin\left(\frac{3\pi}{7}\right) = \sin\left(\frac{4\pi}{7}\right)\right)$$

68. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is

68. [9]

$S_p = \frac{p}{2}\left(2a_1 + (p-1)d\right)$

$d = a_2 - a_1$

$S_m = \frac{m}{2}\left(2a_1 + (m-1)d\right)$

$S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$

$$\frac{S_m}{S_n} = \frac{m}{n}\left(\frac{2a_1 + (m-1)d}{2a_1 + (n-1)d}\right) = 5\left(\frac{2a_1 + (5n-1)d}{2a_1 + (n-1)d}\right)$$

$$\frac{S_m}{S_n} = 5\frac{\left((2a_1 - d) + 5nd\right)}{(2a_1 - d) + nd}$$

$\frac{S_m}{S_n}$ is independent of n if $2a_1 - d = 0$ or $d = 0$

if $2a_1 - d = 0$

$d = 2a_1 = 6$

hence, $a_2 = 3 + 6 = 9$

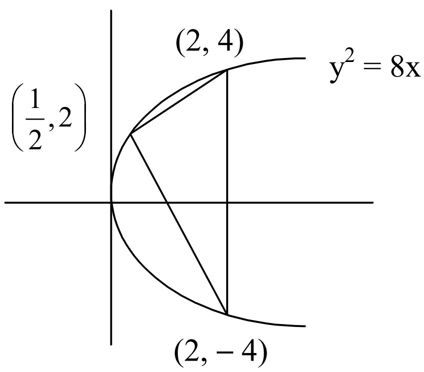
if $d = 0$ $\quad a_2 = a_1 = 3$

69. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is

69. [2]

We know that the area of the triangle inscribed in a parabola is twice the area of the triangle formed by the tangents at the vertices of the triangle.

So tangents at P $\left(\frac{1}{2}, 2\right)$ and end points of latus rectum form triangle (Δ_1) is half of area of triangle formed by the points P and end points of latus rectum (Δ_2). So $\frac{\Delta_1}{\Delta_2} = 2$



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