Subject: ENGINEERING MATHEMATICS - II

Time: 3 Hours **DECEMBER 2010** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The value of the limit $\underset{x\to 0}{\text{lt}} \dfrac{8^x - 2^x}{x}$ is equal to

(**A**) log 4 (**B**) log 2
(**C**) log 3 (**D**) log 5

b. The value of definite integral $\int_0^{\pi} |\cos x| dx$ is equal to

(**A**) 0 (**B**) 2
(**C**) 1 (**D**) ∞

c. The solution of (1+cos x) dy = (1-cos x) dx is

(**A**) $2 \tan x/2 - 2x + c$ (**B**) $2 \tan x/2 - x/2 + c$
(**C**) $2 \tan x/2 - x + c$ (**D**) $2 \tan x/2 + x + c$

d. If $|z + i| = |z - i|$, then the value of z is equal to

(**A**) 1 (**B**) 0
(**C**) ∞ (**D**) x

e. The power factor is equal to

(**A**) I.R (**B**) $\dfrac{V}{R}$
(**C**) $\dfrac{|Z|}{R}$ (**D**) $\dfrac{R}{|Z|}$

f. The Laplace transform of e^{-3t}. (cos4t + 3sin4t) is

(A) $\frac{s+4}{s^2+2s+4}$ **(B)** $\frac{s+12}{s^2+3s+6}$

(C) $\frac{s+15}{s^2+6s+25}$ **(D)** $\frac{s+15}{s^2+6s+15}$

g. The $L^{-1}\left(\frac{s}{(s^2-1)^2}\right)$ is equal to

(A) $\frac{t}{2}\cosh$ **(B)** $\frac{t}{2}\sinh t$

(C) $2t\sinh$ **(D)** $2t\cosh t$

h. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3$ and $\vec{a}\,.\,\vec{b} = 3$, then the angle between the vectors is equal to

(A) 30° **(B)** 45°

(C) 60° **(D)** 90°

i. The area of parallelogram. Whose adjacent sides are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$, is

(A) $5\sqrt{6}$ sq.unit **(B)** $2\sqrt{3}$ sq.unit

(C) $3\sqrt{2}$ sq.unit **(D)** None of above

j. If the voltage and current of a circuit are given by the complex numbers 70+20j and 20-6j respectively then the admittance in the form of complex number is equal to

(A) 3.56 + 2.23J **(B)** 2.35 + 1.25j

(C) 1.57 + 2.56j **(D)** 2.94 + 1.88j

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Evaluate $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$ **(8)**

b. The loop of the curve $ay^2 = x\,(x-a)^2$ revolves about the x-axis. Find the volume of the solid so generated. **(8)**

Q.3 a. Separate $\sin^{-1}(\alpha + i\beta)$ into real and imaginary parts. **(8)**

b. The forces $2\hat{i}+7\hat{j}, 2\hat{i}-5\hat{j}+6\hat{k}, -\hat{i}+2\hat{j}-\hat{k}$ act on a point P whose position vector is $4\hat{i}-3\hat{j}-2\hat{k}$. Find the vector moment of the resultants of three forces acting at P about the point Q, whose position vector is 6i +j -3k. **(8)**

Q.4 a. A condensor of the capacity c is discharged through an inductance L and a resistance R, in the series and the charge Q at the time t satisfies the equation $L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{c}=0$ given that L = 0.25 H, R = 250 ohms, and $c = 2\times10^{-6}$ farad and that when t = 0, the charge Q is 0.002 coulomb and the current $\frac{dQ}{dt}=0$, obtain the value of Q in the terms of t. **(8)**

b. Find the Fourier series of the function **(8)**

$$f(t)=\begin{cases}0 & \text{when} \quad -2<t<-1\\ k & \text{when} \quad -1<t<1\\ 0 & \text{when} \quad 1<t<2\end{cases}$$

Q.5 a. Find the Laplace transform of $\frac{1-\cos 2t}{t}$. **(6)**

b. Evaluate $L^{-1}\left[\frac{s+4}{s(s-1)(s^2+4)}\right]$ **(10)**

Q.6 a. Verify Rolle's Theorem for the function f(x) = x (x+2) $e^{-x/2}$ in the interval (-2,0) **(8)**

b. Find the Laplace Transform of the periodic function (saw tooth wave)

$f(t)=\frac{kt}{t}$ for $0<t<T, f(t+T)=f(t)$ **(8)**

Q.7 a. Solve the equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+2y=5\sin t$

if $y(0) = y'(0)=0$ **(8)**

b. Solve the equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\sin 2x$ **(8)**

Q.8 a. Find the Fourier series representing,

$f(x)=x, \quad 0<x<2\pi$ **(8)**

b. A resistance of 20 ohms, an inductance of 0.2 H and a capacitance of 100 micro farad are connected in series across 220 volts, 50 cycles/sec mains. Determine (i) Impedance (ii) Current (iii) Voltage across L, R and C. **(8)**

Q.9 a. Find the area of the triangle formed by the points whose position vectors are 3i+j, 5i +2j +k, i-2j +3k. **(8)**

b. Verify Langrage's Mean-value theorem for $f(x) = \log_e x$ in the interval $[1, e]$ **(8)**