DipIETE – ET/CS (NEW SCHEME) – Code: DE55/DC55

Subject: ENGINEERING MATHEMATICS - II

Time: 3 Hours **Max. Marks: 100**

DECEMBER 2009

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The value of the limit $\lim_{x\to 1}\left(\frac{x^3+3x-4}{2x^2+x-3}\right)$ is equal to

(A) $\frac{6}{5}$ **(B)** $\frac{4}{5}$
(C) 0 **(D)** -1

b. The value of definite integral $\int_{-a}^{a}|x|dx$ is equal to

(A) a **(B)** a^2
(C) 0 **(D)** 2a

c. The solution of $ye^y dx=\left(y^3+2xe^y\right)dy$ is

(A) $x^2+y^2e^{-y}=cy^2$ **(B)** $x-y^2e^{-y}=cy^2$
(C) $x+y^2e^{-y}=cy^2$ **(D)** None of these.

d. If z is a complex number with $|z|=1,\ \arg(z)=3\pi/4$ then the value of z is

(A) $(1+i)/\sqrt{2}$ **(B)** $(-1+i)/\sqrt{2}$
(C) $(1-i)/\sqrt{2}$ **(D)** $(-1-i)/\sqrt{2}$

e. How many seconds a clock would lose per day if the length of its pendulum were increased in the ratio 900 : 901?

(A) 48 **(B)** 45
(C) 40 **(D)** 44

f. Laplace transform of $te^{at}\sin(at),\ t>0$ is

(A) $\frac{(s-a)}{(s-a)^2+a^2}$ **(B)** $\frac{a(s-a)}{(s-a)^2+a^2}$
(C) $\frac{2a(s-a)}{\left[(s-a)^2+a^2\right]^2}$ **(D)** $\frac{(s-a)^2}{(s-a)^2+a^2}$

g. $L^{-1}\left(\frac{1}{s\left(s^2+1\right)}\right)$ is

(A) $1+\sin t$ **(B)** $1-\sin t$

(C) $1+\cos t$ **(D)** $1-\cos t$

h. If $f(x) = x \sin x, (-\pi, \pi)$ then the value of b_n is

(A) $-\pi$ **(B)** 0
(C) π **(D)** 2π

i. The volume of the parallelopiped whose three coterminous edges are given by $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \vec{b} = -3\hat{i} + 7\hat{j} - 3\hat{k}, \vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$ is

(A) 282 **(B)** -272
(C) 262 **(D)** 252

j. If the admittance and current of a circuit are given by the complex numbers $7+5j, 17-6j$ respectively, then the voltage of the circuit is

(A) $\frac{89}{74} + j\frac{127}{74}$ **(B)** $-\frac{89}{74} + j\frac{127}{74}$
(C) $\frac{89}{74} - j\frac{127}{74}$ **(D)** $-\frac{89}{74} - j\frac{127}{74}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Evaluate $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$ **(8)**

b. Find the volume formed by the revolution of the loop of the curve $y^2(a+x) = x^2(a-x)$ about x-axis. **(8)**

Q.3 a. Separate $\tan^{-1}(a+ib)$ into real and imaginary parts. **(8)**

b. Find the moment about the point M(-2,4,-6) of the force represented in magnitude and position by $\overrightarrow{AB}$ where the point A and B have the coordinates (1,2,-3) and (3,-4,2) respectively. **(8)**

Q.4 a. In a condenser discharging electricity the Voltage V satisfies the equation $K\frac{dv}{dt} + V = 0$ where K is a constant and t is the time measured in seconds. Given K=50, find the time t in which V decreases to one tenth of its original value. **(8)**

b. Find the Fourier series of the function $f(x) = \begin{cases} -1, & -\pi < x < -\pi/2 \\ 0, & -\pi/2 < x < \pi/2 \\ 1, & \pi/2 < x < \pi \end{cases}$ **(8)**

Q.5 a. Find the Laplace transform of $\frac{1-\cos t}{t^2}$ **(10)**

b. Evaluate $L^{-1}\left[\frac{1}{S(S-1)}\right]$ **(6)**

Q.6 a. Verify Cauchy's mean-value theorem for the functions e^x and e^{-x} in the interval (a, b). **(8)**

b. Find the Laplace transform of the function $f(t)=\begin{cases}\sin wt & \text{for } 0<t<\frac{\pi}{w}\\ 0 & \text{for } \frac{\pi}{w}<t<\frac{2\pi}{w}\end{cases}$ **(8)**

Q.7 a. Solve $\frac{d^2x}{dt^2}+9x=\cos 2t,\ if\ \ x(0)=1,\ x(\pi/2)=-1$ **(8)**

b. Solve $x\sin x\frac{dy}{dx}+(x\cos x+\sin x)y=\sin x$ **(8)**

Q.8 a. Given that $f(x)=x+x^2\ for -\pi<x<\pi$, find the Fourier expansion of *f(x)*. Deduce that $\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+.....$ **(8)**

b. Two circuits of impedances 2+4j ohms and 3+4j ohms are connected in parallel and a.c. voltage of 100 volts is applied across the parallel combination. Calculate the magnitude of the current as well as power factor for each circuit and the magnitude of the total current for the parallel combination and its power factor. **(8)**

Q.9 a. If $\left|\vec{A}+\vec{B}\right|=60,\ \left|\vec{A}-\vec{B}\right|=40,\ \left|\vec{B}\right|=46,$ find $\left|\vec{A}\right|$ **(8)**

b. If $U_n=\int_0^{\pi/2}x(\sin^n x)dx\ (n>1)$, then prove that $U_n=\frac{n-1}{n}U_{n-2}+\frac{1}{n^2}$. Deduce that $U_5=\frac{149}{225}$ **(8)**