SATHYABAMA UNIVERSITY

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Course & Branch: B.E - CSE
Title of the paper: Discrete Mathematics
Semester: V Max. Marks: 80
Sub.Code: 411503/511503/611503 Time: 3 Hours
Date: 04-11-2008 Session: FN

PART – A (10 x 2 = 20)

Answer All the Questions

1. Show that $(P \rightarrow Q) \rightarrow Q \Rightarrow PVQ$

2. Show that $\{7, \wedge\}$ is functionally complete.

3. What is a symmetric relation? Give an example for a relation which is neither symmetric nor antisymmetric.

4. If f: $R \rightarrow R$ be given by $f(x) = e^{-x}$. What is the inverse of the function f?

5. If $(G,*)$ is an abilian group, prove that $(a*b)^2 = a^2*b^2$ a, b E G.

6. Find the left cosets of H = {[0], [4], [8]} in the group $(Z_{12}, +_{12})$.

7. Define distributive and modular lattices.

8. In any Boolean algebra, show that $(a + b)(a^! + c) = a^! b + ac = bc$

9. What is the maximum number of edges in a sample digraph with n verticals?

10. Draw the directed tree corresponding to $x^3 + 1$.

PART – B (5 x 12 = 60)

Answer All the Questions

11. (a) Obtain the principle conjunctive normal form for

$$(7P \rightarrow R) \wedge (Q \Leftrightarrow P)$$

(b) Show that the following premises are inconsistent.
$P \rightarrow Q$, $Q \rightarrow S$, $R \rightarrow 7S$ and $P \wedge R$.

(or)

12. (a) Show that the formula
$[(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)]$ tautologically implies the formula R.

(b) Discuss the validity of the argument. "All educated persons are well behaved. Ram is educated. No well behaved person is quarrelsome. Therefore Ram is not quarrelsome.

13. (a) Let f: $R \rightarrow R$ and g: $R \rightarrow R$ be functions given by

$$F(x) = x^3 - 4, \quad g(x) = x + 5.$$

Find $f \circ g$, $g \circ f$, f^{-1} and state whether the functions f and g are injective.

(b) Let x = {2, 3, 6, 12, 14, 36}. Let s be the relation defined by s = {(x, y)/x divides y, where x, y, E x}. Is S a partial order relation? If so, draw the Hasse diagram.

(or)

14. (a) Let Z denote the set of integers. The relation R is defined by
R = {<x, y>/(x-y) is divisible by 5, where x, y E Z}

(b) If f: $x \rightarrow y$ and g: $y \rightarrow Z$ are both onto, show that $g \circ f$ is also onto. Is $(g \circ f)$ one to one if both g and f are one to one?

15. (a) Show that the kernel of a group Homomorphism form G to H is a normal subgroup of G.

(b) State and prove Calley's Theorem.

(or)

16. (a) Show that every subgroup of a cyclic group is cyclic.
(b) State and prove Lagrange's Theorem.

17. (a) Using generating function, solve the recurrence equation.
$Y_{x+2} - Y_{x+1} - 6Y_x = 0$, $Y_1 = 1$, $Y_0 = 2$

(b) Show that a lattice $(L, \wedge, \vee)$ is distributive if and only if for all a, b, c, in L, $(a \vee b) \wedge c \leq a \vee (a \wedge c)$

(or)

18. (a) Using the principle of mathematical induction, prove that
$1^2 + 2^2 + \ldots\ldots\ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

(b) Show that $(B_n, *, \oplus, ', O_n, 1_n)$ is a Boolean algebra.

19. (a) Give a directed tree representation of the following formula
$(PV\ (7P \wedge Q)) \wedge ((7P \vee Q) \wedge 7R)$

(b) Show by means of an example that a simple digraph in which exactly one node has in degree O and every other node has in degree 1 is not necessarily a directed tree.

(or)

20. (a) Define a strong component of a simple digraph with an example. Prove that every node of a simple diagraph lies in exactly one strong component.

(b) Show that in a completer binary tree, the total number of Edges is given by $2\ (n_i - 1)$ where n_i is the number of terminal nodes.