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SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act,1956)

Course & Branch :B.E - CSE
Title of the Paper :Discrete Mathematics — Max. Marks :80
Sub. Code :411503-611503 — Time : 3 Hours
Date :06/11/2009 — Session :FN

PART - A (10 x 2 = 20)
Answer ALL the Questions

1. Show that $P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$.

2. Show that $(x)\ (H(x) \rightarrow M(x)) \wedge H(s) \Rightarrow M(s)$.

3. Let f(x) = x + 2; g(x) = x - 2 for all x in R, where R is the set of real numbers find the f og.

4. Let A = {1,2,3,4,6}. R be the relation on A defined by “x divides y” Write R as a set of ordered pairs and find the inverse relation of R.

5. If H and K are subgroups of a group G, show that $H \cap K$ is a sub group of group G.

6. Q be the set of all rational numbers and let * be defined on Q by a* b = a + b – ab. Is (Q, *) a semi group.

7. Three persons enter into a car, where there are 5 seats. In how many ways can they take up their seats?

8. Let a,b,c be any elements in a Boolean algebra B, show that (i) a *a = a (ii) a * 0 = 0.

9. Use a binary tree to represent ((a+b) * c) + (d/e).

10. Draw the graph whose incidence matrix is given below:

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

PART – B (5 x 12 = 60)

Answer All the Questions

11. (a) Show that $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$. (5)
(b) Show that the following premises are inconsistent
(i) If jack misses many classes through illness, then he fails high school.
(ii) If jack fails high school, then he is uneducated.
(iii) If jack reads a lot of books, then he is not uneducated.
(iv) Jack misses many classes through illness and reads lot of books. (7)

(or)

12. (a) Use indirect method to prove $(x)\,(P(x) \vee Q(x)) \Rightarrow (x)\,P(x) \vee (\exists x)\,Q(x)$. (7)
(b) Find the principle disjunctive normal form of
$P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$. (5)

13. (a) Let Z be the set of all integers and m > 1 be an integer. Show that congruence modulo m is an equivalence relation on Z. (7)

(b) Given A = {1,2,3,4}. Consider the relation in A as defined below R = {(1,1); (2,2); (2,3);(3,2);(4,2); (4,4)} (i) Draw its directed graph (ii) Is R reflexive, symmetric and transitive.

(or)

14. (a) If f : A $\rightarrow$B and g:B $\rightarrow$ C are any two functions, show that (i) if f and g are one to one then g o f is one to one (ii) if f and g are onto functions then $g\ o\ f$ is an on to function. (7)

(b) If f: $r \rightarrow$ R is defined by $f(x)$ = 2x – 3. show that f is one to one and onto, also find f^{-1}.

15. State and prove Lagranges theorem.

(or)

16. (a) Let H be a subgroup and let K be a normal subgroup of a group G. Show that HK is a subgroup of G. (7)

(b) Consider the group G = {1,2,34,5,6} under multiplication modulo 7.

(i) Find the multiplication table of G.

(ii) find 4^{-1}, 5^{-1} (iii) Is G cyclic. (5)

17. (a) Using Karnaugh map simplify x'yz + x'yz't+ y'zt'+ xyzt' + xy'z't'. (7)

(b) Show that the relation $\subseteq$ of set inclusion is a partial ordering on any collection of sets. (5)

(or)

18. (a) Using induction prove $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ (7)

(b) How many number of 3 digits can be formed with the digits 1,2,3,4 and 5 if the digits in the same number are not repeated? How many such numbers are possible between 100 and 10000?

19. If G = (V,E) is a graph with e edges. Show that $\sum_{v \in V} \deg_G(v) = 2e$ and hence show that the maximum number of edges in a simple graph with n vertices is n(n-1)/2.

(or)

20. Find the number of edges in a k-regular graph on p vertices and hence show that

    (a) a 3 – regular graph on 14 vertices exists and

    (b) 3 – regular graph on 17 vertices do not exists.