SATHYABAMA UNIVERSITY

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Course & Branch :B.E - CSE
Title of the Paper :Discrete Mathematics Max. Marks :80
Sub. Code :411503/611503 Time : 3 Hours
Date :26/04/2010 Session :AN

PART - A (10 x 2 = 20)
Answer ALL the Questions

1. What do you mean by indirect method of proof?
2. Negate the statement $(\forall x) p(x) \wedge (\exists y) p(y)$.
3. Define equivalence relation.
4. Give an example for a bijective mapping.
5. Prove that a subgroup H of a group is normal iff each left coset of H in G is
   equal to the right coset of H in G.
6. Define Cyclic group.
7. Obtain the number of permutations of all letters of the word "ENGINEERING"
8. Solve the recurrence relation $a_n - 2a_{n-1} = 3^n$, $a_1 = 5$.
9. Prove that the sum of the degrees of the vertices of a group G is equal to twice the number of edges in G.
10. Show that the maximum number of edges in a simple graph with "n" vertices is $\frac{n(n-1)}{2}$.

PART – B (5 x 12 = 60)
Answer ALL the Questions

11. (a) Show that $\rceil$P is a valid conclusion from the premises
   $R \rightarrow \rceil Q$, $R \vee S$, $S \rightarrow \rceil Q$ and $P \rightarrow Q$
   (b) Obtain the principal disjunctive normal form for
   $(p \wedge q) \vee (\rceil p \wedge r) \vee (q \wedge r)$.

(or)

12. (a) Show that the following set of premises is inconsistent. If the contract is valid, then John is liable for penalty. If John is liable for penalty, he will go bankrupt. If the bank will loan him money, he will not go bankrupt. As a matter of fact, the contract is valid and the bank will loan him money.
    (b) Use indirect method to show that
    $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$

13. (a) Let $f : A \rightarrow B,\ g: B \rightarrow C$ be bijection. Prove that $g_o f: A \rightarrow C$ is also a bijection.
    (b) Let S_{30} be the set of positive divisors of 30. If $\leq$ is the relation of divisibility, prove that $(S_{30}, \leq)$ is poset. Draw the Hasse diagram of a poset.

(or)

14. (a) If $f : A \rightarrow B,\ g : B \rightarrow C$ are invertible functions, then $g_o f : A \rightarrow C$ is also invertible and $(g_o f)^{-1} = f^{-1} {}_o\, g^{-1}$.
    (b) If A,B,C and D are sets, prove that $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$.
    Give an example to support this result.

15. (a) If $a,b \subset G$ then prove that $(a*b)^{-1} = b^{-1}*a^{-1}$.
    (b) State and prove Lagrange's theorem.

(or)

16. (a) Prove that the intersectio0n of two normal subgroup is a normal subgroup.
    (b) Define a Cyclic group. Prove that any group of prime order is cyclic.

17. Solve the recurrence relation $S(k) - 4\ S(k-1) + 4\ S\ (k-2) = 3k+2^K$ with $S(0) = 1$, $S(1) = 1$

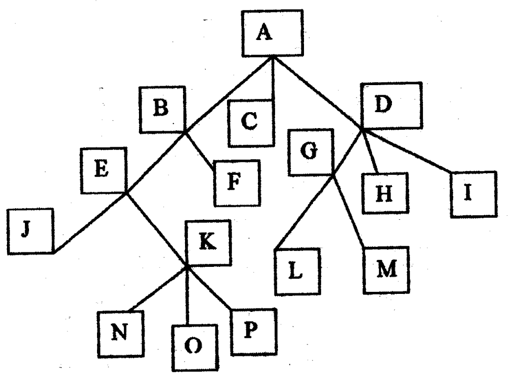
(or)

18. (a) Show that a lattice is distributive iff

$$(a*b)\oplus(b*c)\oplus(c*a) = (a \oplus b)*(b \oplus c)*(c \oplus a).$$

(b) Use mathematical induction to show that
$1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$.

19. In which order does (i) a preorder, (ii) inorder (iii) a post order traversal visit the vertices of the ordered rooted tree given in figure.



20. Represent the postfix expression ab + cd * ef *1*- - *a** as a binary tree and write also the corresponding infix and prefix forms.