Section – II

56. $V = 7^{3n} - 3^{5n}$, where n is a natural number. Which of the following would always divide V?
a. 100 b. 150 c. 225 d. None of these

57. In a ΔPQR, $\angle P = 60^\circ$ and the radius of its circumcircle is 2. The altitude from Q to PR bisects PR. The radius of its incircle is
a. $\frac{2}{3}$ b. 1 c. $\frac{1}{2}$ d. $\frac{3}{2}$

58. Find the number of rational numbers $\frac{a}{b}$ such that
I. $0 < \frac{a}{b} < 1$,
II. a and b are co-prime natural numbers and
III. Product of a and b is 15!
a. 2^7 b. 2^6 c. 2^5 d. 2^{15}

59. If $\log_{30} 3 = x$ and $\log_{30} 5 = y$, then $\log_8 30$ is equal to
a. $3(1 - x - y)$ b. $\frac{1}{3(1-x-y)}$ c. $\frac{3}{(1-x-y)}$ d. $\frac{1-x-y}{3}$

60. Two prime numbers P_1, P_2 ($P_1 < P_2$) are called twin primes if they differ by 2. (e.g. 11, 13 or 41, 43 etc.) If P_1 and P_2 are twin primes with $P_2 > 23$ then which of the following numbers would always divide $P_1 + P_2$?
a. 12 b. 8 c. 24 d. None of these

61. Let T be a point on the side PS of a square PQRS of area 256 sq cm. The perpendicular to the line TR at R meets the line segment PQ extended at U. If the area of the ΔRUT is 200 sq cm, then the length of QU is
a. 12 cm b. 4 cm c. 8 cm d. 9 cm

62. If $1000^7 - 17$ is written in decimal notations, sum of its digits would be
a. 177 b. 181 c. 179 d. 182

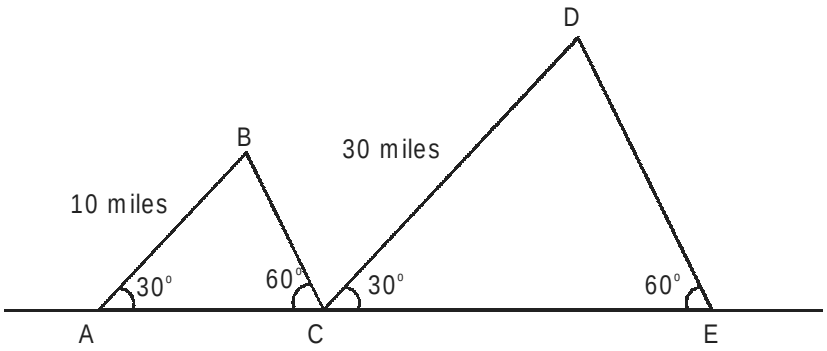
63. What is the remainder when the sum $1^5 + 2^5 + 3^5 + 4^5 + 5^5 + ... + 3000^5$ is divided by 4?
a. 3 b. 2 c. 1 d. 0

64. The equation $4x + \frac{5}{x+3} = \frac{10}{2x+6} - 12$ has
a. One root only b. Two roots only c. Three roots only d. No root

65. If in the expansion of $(a + b)^n$ the coefficients of the 7th and 18th terms are equal, then the roots of the equation $x^2 + (n + 1)x + n$ are
a. 25 and 1 b. 23 and 1 c. –24 and –1 d. –23 and –1

66. A class with n students is organized into four Groups keeping the following conditions :
I. Each student belongs to exactly two groups and
II. Each pair of groups has exactly one student in common.
Then
a. n = 12 b. n = 6 c. n = 8 d. n = 18

Directions for questions 67 and 68: Answer the questions based on the following information.



Johnny is driving from A to E over two mountains (assume on straight paths). The two mountains are triangular and similar in shape as shown in the figure. Johnny travel uphill at 20 miles/hr and downhill at 40 miles/hr.

67. What is the total time taken to travel from A to E (approximately)?
a. 3 hours 12 min b. 2 hours 21 min c. 2 hours 35 min d. Data insufficient

68. What is the distance between the peaks B and D as the crow flies?
a. $10\sqrt{\frac{28}{3}}$ miles b. $10\sqrt{\frac{7}{3}}$ miles c. $\frac{10}{3}$ miles d. None of these

69. Find the maximum value of $Y = 5 - |X+1| - |X-3|$.
a. 1 b. 5 c. 4 d. ∞

70. Four people are standing in a cinema ticket counter. The ticket is worth Re 1. Two of these people have a coin Rs. 2 only and two of them have a coin of Re. 1 only. In how many ways can they be arranged such that the man at the counter does not have a problem in giving them the change, if it is known that he had no money to start with ?
a. 4! b. $\frac{4!}{2!\,2!}$ c. 8 d. 2

71. Two natural numbers x and y when divided by another natural number k leave positive remainders a, b respectively. When x + y is divided by k, the remainder is c ($\neq 0$). Then which of the following statements is necessarily true?
a. c = a + b
b. k can be uniquely identified, if a, b, c are known
c. c = a + b or c = (a + b) – k
d. All statement are false

72. In which base system can the decimal number 19600 be written as 111100?
a. Base 2 b. Base 5 c. Base 7 d. Base 14

73. If x is a 2 digit natural number and $x! - \sum x$ is divisible by x, then how many values can x take?
($\sum x$ indicates sum of all natural numbers form 1 to x)
a. 90 b. 50 c. 45 d. 40

74. When 34369 and 31513 are divided by a certain three digit number, the remainders are equal. The remainder is
a. 79 b. 97 c. 87 d. Cannot be determined

75. p, q, r, s are any four positive real numbers, the minimum value of $\frac{p}{q}+\frac{q}{r}+\frac{r}{s}+\frac{s}{p}$ is
a. 1 b. 2 c. $2\sqrt{2}$ d. 4

76. Let VK and KJ be perpendicular line segments, each of length 6cm. Suppose P and Q are the mid-points of VK and KJ respectively. If R is the point of intersection of VQ and PJ, then the area of triangle RQJ is
a. 6 sq. cm b. 3 sq. cm c. $3\sqrt{2}$ sq. cm d. $6\sqrt{2}$ sq. cm

77. PQRS is a fixed rectangle with PQ = 4cm and QR = 8 cm. ABCD is a rectangle such that P, Q, R and S lie on AB, BC, CD and DA respectively. Then the maximum possible area of ABCD is
a. 64 sq. cm b. 72 sq. cm c. 128 sq. cm d. 144 sq. cm

78. What is the sum to n terms of the series
1, 3 , 6, 10, 15, 21, ... so on.
a. $\frac{n(n+1)}{2}$ b. $\frac{n(n+1)(n+2)}{6}$ c. $\frac{n(n+1)(n+2)}{3}$ d. $\frac{n^2+3n+4}{4}$

79. V is an integer. For how many different values of V, the number $V^4 - 20V^2 + 4$ is a prime number?
a. One b. Two c. Three d. None of these

80. If V_1, V_2, V_3 ... V_x are any x consecutive three digit numbers such that at least one of these numbers (from V_1 to V_x) is divisible by the sum of its own digits. Then what should be the minimum value of x?
a. 3 b. 6 c. 9 d. 18

81. A regular octagon is inscribed in a circle of radius 1, the product of the distances from a fixed vertex to the other seven vertices is
a. 16 b. 8 c. 12 d. 6

Direction for questions 82 and 83: Answer the questions based on the following information.
The distance between towns A and B is less than 100 miles. Pinky starts from A and after 10 hours comes across a milestone indicating the distance from A to that point. She proceeds further and 2 hours later comes to another milestone having the same digits but reverse in order.

82. The distance covered by Pinky when she reaches the second milestone is
a. 63 miles b. 54 miles c. 72 miles d. Data insufficient

83. What is Pinky's speed?
a. 9 miles/hr b. 4.5 miles/hr c. 3 miles/hr d. Data insufficient

84. If 20 items of A are sold at a profit of x% and 30 items of B are sold at a profit of 2x%. What is the net profit per cent?
a. 1.4x% b. 1.6x% c. 1.5x% d. Data insufficient.

85. If X is the smallest number that is divisible by both 6 and 5, than the power of 10 that would completely divide the product of the first 20 multiples of X is:
a. 4 b. 24 c. 27 d. None of these

86. A four digit number 5775 is written 8000 times (i.e. a 32000 digit number) side by side. What is the remainder when this number is divided by 18?
a. 1 b. 3 c. 5 d. 12

87. Let V be a set of real numbers such that if p is any real number in the set then there exists two numbers in V whose average is p. Then
a. V is a finite set
b. V is a set containing all real numbers
c. V is a set of all numbers in the interval (2, 3).
d. None of these

88. Through the centroid of an equilateral triangle, a line parallel to the base is drawn. On this line, an arbitrary point V is taken inside the triangle. Let d denote the distance of V from the base of the triangle. Let d_1 and d_2 be the distances of V from the other two sides of the triangle. Then

a. $d = \sqrt{d_1 d_2}$ b. $d = \frac{2d_1 d_2}{d_1 + d_2}$ c. $d = \frac{d_1 + d_2}{2}$ d. $d = \frac{d_1 d_2}{d_1 + d_2}$

89. If V > 1 and $V + \frac{1}{V} < \sqrt{5}$, then

a. $\frac{\sqrt{5}-1}{2} < V < \frac{\sqrt{5}+1}{2}$ b. $\frac{\sqrt{5}-1}{2} < V < 1$

c. $1 < V < \frac{\sqrt{5}+1}{2}$ d. None of these

90. The minimum value of $3x + 3y + z$ subject to the condition $xyz = 24$, where x, y and z are all positive real numbers, is

a. $14 \times 3^{\frac{1}{3}}$ b. 18 c. 216 d. 12

91. The largest integer that always divides $n^5 - 5n^3 + 4n$, is (where 'n' is natural number)
a. 40 b. 60 c. 24 d. 120

92. In a quadrilateral PQRS, $\angle P = 120°$, $\angle Q = 90°$, $\angle R = 60°$, PQ = 13 cm and PS = 46 cm. What is the length of PR?
a. 31cm b. 62 cm c. 68 cm d. 34 cm

Directions for questions 93 to 95: Answer the questions based on the following information.
Consider the sequence:
$V_1 = 101$, $V_2 = 10101$, $V_3 = 1010101$, $V_4 = 101010101$, and so on

93. V_n is prime for (n is a natural numbers)
a. Only one value of n b. Exactly two values of n
c. Exactly three values of n d. Infinitely many values of n

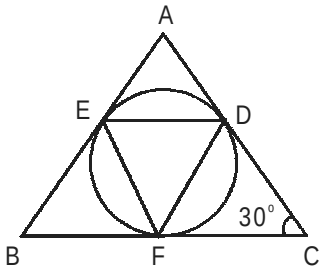
94. If n is odd then V_n is always divisible by (n > 1)
a. 101 b. 101 and 99 c. 101 and 11 d. 101 and 7

95. V_4 is divisible by
I. 11111
II. 9091
a. I and II both b. Only I c. Only II d. Neither I nor II

96. A vessel contains V litres of a 3 : 2 milk to water solution. If 10 litres of water is added and the concentration of milk in the resultant solution lies between 50% and 40%, find the range of values of V?
a. 50 litres $\leq$ V $\leq$ 60 litres b. 40 litres $\leq$ V $\leq$ 50 litres
c. 30 litres $\leq$ V $\leq$ 40 litres d. None of these

97. Two guys A and B are walking on a moving escalator, A in the same direction as the escalator and B in the direction opposite to it. A's speed is 5 steps/second and B's step is 10 steps/second. If they started at either ends of the escalator simultaneously and meet 10 s after they started, how many steps are there in the escalator?
a. 15 steps b. 150 steps c. 50 steps d. Data insufficient

98. ΔABC is a triangle whose sides are tangential to an inscribed circle at DEF. If $\angle C = 30^o$, then $\angle DEF$ is



a. 60^o b. 150^o c. 75^o d. None of these

99. If 2 and 4 are two roots of the expression $x^4 + 2x^3 + ax^2 + bx + 3$ (a and b are constants) then what is the value of 2a + 3b?
a. 106 b. –106 c. 53 d. –53

100. If $p + q + r = 1$, $p^2 + q^2 + r^2 = 9$ and $p^3 + q^3 + r^3 = 1$, the value of $\frac{1}{p} + \frac{1}{q} + \frac{1}{r}$ will be
a. 1 b. –4 c. –1 d. –8

101. If three prime numbers all greater than 100 are in A.P, then their common difference
a. must be divisible by 6.
b. must be greater than 600.
c. must be divisible by 2 but not necessarily by 3.
d. Need not be divisible by either of 2 or 3.

102. If $\log_k V = 6$, and $\log_{25k}(8V) = 3$, then k is
a. 12.5 b. 2.5 c. $\frac{5^{\frac{2}{3}}}{2}$ d. $(12.5)^{\frac{2}{3}}$

103. If a and c are both positive odd integers then roots of the equation $ax^2 + x + c = 0$ can never be
a. irrational numbers b. rational number
c. positive d. both (b) and (c)

104. Let PQRS be a rectangle. How many circles in the plane of PQRS have a diameter whose end points are the vertices of PQRS?
a. 1 b. 5 c. 6 d. 9

105. X is a number whose value is equal to $10^3a + 10^2b + 10c + 2$. Which of the following statements are false (a, b, c are all natural numbers) ?
a. If X when divided by "a" leaves a remainder 2, there are more than one possible value of "a".
b. X is not a perfect square.
c. X is a 4 digit number.
d. Only (a) is false.

106. If n! is a perfect square, how many values of n exist?
a. One b. Two c. Three d. Infinite

107. Let $\{V_n\}$ be a sequence such that
$V_1 = 2$, $V_2 = 1$ and $2V_n - 3V_{n-1} + V_{n-2} = 0$ for $n > 2$, then V_9 is equal to
a. $\frac{1}{128}$ b. $\frac{1}{256}$ c. $\frac{1}{64}$ d. $\frac{1}{32}$

108. What is the remainder, when (P – 2) ! is divided by P? (P is a prime number)
a. 1 b. P – 1 c. P – 6 d. Cannot be determined

109. The sides of a rectangular field are "x" m and "y" m, where x and y are natural numbers. The perimeter of the field is 78 m. The field is divided into 13 regions of area V sq m each. What is the value of V if it is a natural number?
a. 104 b. 52 c. 26 d. 65

110. The number of integers lying between 2000 and 9000 (excluding 2000 and 9000) which have at least two digits equal is
a. 3471 b. 3470 c. 3472 d. None of these