Section – I

1. N is an even number when written in the decimal system. If S is the sum of the digits of N, when it is written in base 7, we can surely say that S would always be
   a. even b. odd c. composite d. Both (a) and (c)

2. There are X boxes so that the first box contains 7 balls, the second box contains 9 balls, and in general, the ith box contains i balls more than the (i – 1)th box for i > 1. The total number of balls in X boxes are
   a. $\frac{X}{6}\left(X^2 + 3X + 38\right)$ b. $\frac{X}{12}\left(X^2 + 3X + 38\right)$
   c. $\frac{X}{6}\left(X^2 + 3X + 36\right)$ d. $\frac{X}{6}\left(X^2 + 5X + 36\right)$

3. One of the angles of a triangle is 60° and length of its two sides is 6 cm and 7 cm. The length of the third side of the triangle is
   a. $\sqrt{43}$ b. $3 + \sqrt{22}$ c. $\sqrt{43}$ or $3 + \sqrt{22}$ d. None of these

4. If a, b and c are all positive numbers and $ab^3c^4 = 6912$, what can be the minimum value of a + b + c?
   a. $12^{\frac{3}{4}}$ b. $12^{\frac{6}{7}}$ c. $8^{\frac{3}{4}}$ d. 8

5. If $\log_v x = a$ and $\log_k x = b$, then the value of $\log_{(v/k)} x$ is equal to
   a. $\frac{ab}{a-b}$ b. $\frac{ab}{a+b}$ c. $\frac{a+b}{ab}$ d. $\frac{ab}{b-a}$

6. A table clock loses 5 min per hour. A wall clock loses 5 min per hour of the table clock. A wrist watch gains 5 min per hour of the wall clock. All the watches or clockes are set right at 12:00. What will be the time shown by the wrist watch when it is 2:00 for the first time after 12:00?
   a. 1 : 49 : 14 b. 1 : 49 : 57 c. 1 : 40 : 11 d. 1 : 41 : 57

7. A train of 100 m long is approaching an unmanned railway crossing. The train is travelling at a uniform speed of 90 km/hr and is 1 km away from the crossing. At the same time, a bus, also approaching the crossing, is 700 m away from it. Assuming the bus is also travelling at a uniform speed of B, at what range of B will the bus collide with the train?
   a. 59 km/hr < B < 61 km/hr b. 57 km/hr < B < 63 km/hr
   c. 55 km/hr < B < 65 km/hr d. 53 km/hr < B < 67 km/hr

Directions for questions 8 and 9: Answer the questions based on the following information.

$p^3 + q^3 + r^3 = s^3$, $p^2 + q^2 + r^2 = s^2$ and $p + q + r = s$, where s is a constant $(s \neq 0)$.

8. How many solutions are possible for the given set of equations?
   a. None b. One c. Three d. Six

9. How many different values of p are possible?
   a. None b. One c. Three d. Two

10. There are N boxes, each containing at the most, k balls. If the number of boxes containing at least j balls is N_j for j = 1, 2, 3, ... k, then the total number of balls contained in these N boxes
a. is exactly equal to $N_1 + N_2 + N_3 + ... + N_k$
b. is strictly larger than $N_1 + N_2 + N_3 + ... + N_k$
c. is strictly smaller than $N_1 + N_2 + N_3 + ... + N_k$
d. Cannot be determined

11. If $M = 18! \times 10^5$ and $K = 729 \times 10^8$, then what is the remainder when M is divided by K?
a. 1×10^8 b. 3×10^8 c. 5×10^8 d. 0

12. The external length, breadth and height of a closed box are 10 cm, 9 cm and 7 cm respectively. The total inner surface area of the box is 262 sq. cm. If the walls of the box are of uniform thickness t cm, then t equals
a. 1 cm b. $\frac{23}{3}$ cm c. 1 cm or $\frac{23}{3}$ cm d. None of these

13. Neha wants to visit Nitin's place and hence calls up Nitin at his residence to ask him to come and pick her up. Nitin asks her to leave immediately and start walking towards his house and at the same time he also leaves immediately by car, meets Neha on the way, picks her up and drives back to his place. Had Nitin driven all the way to Neha to pick her up, he would have taken 20 min to reach Neha. But since he asked Neha to walk towards his house, he ended driving only for a total of 30 min. What is the ratio of speed at which Neha walks to the speed at which Nitin drives the car?
a. 1 : 4 b. 1 : 5 c. 1 : 3 d. Cannot be determined

14. For how many values of P will 21P + 4 and 14P + 3 have a common factor other than 1? (P is a natural number.)
a. One b. Two c. Three d. None of these

15. A ball of diameter 15 cm is floating so that the top of the ball is 5 cm above the smooth surface (water) of the pond. What is the circumference in centimetres of the circle formed by the contact of the water surface with the ball?
a. $10\sqrt{2}\,\pi$ b. $50\,\pi$ c. $10\,\pi$ d. $5\sqrt{2}\pi$

16. Five cops go to a cemetery. They leave their caps in the bag and pick them while return. In how many ways can they pick up the caps, so that exactly one person picks up his own cap?
a. 9 b. 5 c. 45 d. 96

17. the maximum value of sinx + cosx is
a. $\sqrt{2}$ b. $\frac{\sqrt{3}}{2} + 1$ c. $\frac{\sqrt{3}+1}{2}$ d. $\frac{3}{2}$

18. x, y and z are the angles of a triangle. Of the following options, which set of values of x, y and z satisfies log $(x \times y \times z)$ = 3 log x + 4 log 2, given that x, y and z are integers?
a. 30°, 70°, 80° b. 30°, 60°, 90° c. 20°, 80°, 80° d. 40°, 40°, 100°

19. Solution P contains three liquids A, B and C in the ratio 2 : 3 : 5 respectively. Another solution Q contains A, B and C in the ratio 5 : 3 : 2 respectively. Solutions P and Q are mixed in the ratio 7 : 3, which gives another solution R. Now 50% volume of R is replaced by another solution having A and B in the ratio 7 : 3. The resulting solution is Z. Calculate the percentage of B in solution Z.
a. 10% b. 20% c. 30% d. Cannot be determined

20. A parallelogram is inscribed in a circle. If the area of the circle is $42\frac{1}{4}\pi$ cm^2 and one of the sides of the parallelogram is 12 cm, what is the area of the parallelogram?

a. 60 cm^2 b. 40 cm^2 c. 80 cm^2 d. Cannot be determined

21. PQR is a right-angled triangle with $\angle$Q = 90°, S is the mid-point of PR, and QS = $\sqrt{117}$ cm. Sum of the length of sides PQ and QR is 30 cm. Area of ΔPQR is

a. 216 cm^2 b. 108 cm^2 c. 54 cm^2 d. 162 cm^2

Directions for questions 22 and 23: Answer the questions based on the following information.

For a natural number x, if x has 2 or more than 2 digits, then f(x) = Sum of all digits of x and f(x) = x, when x has only one digit. Also given that $f^n(x) = f\left\{f^{n-1}(x)\right\}$, where n $\geq$ 2.

22. If x = 123456^{654321}, then $f^{\infty}(x)$ is equal to

a. 9 b. 3 c. 1 d. 7

23. If x = 7^{183}, then $f^{\infty}(x)$ equals

a. 7 b. 4 c. 1 d. 9

24. Let S be as an arbitrary point on the side PQ of an acute-angled ΔPQR. Let T be the point of intersection of QR with the straight line PT drawn parallel to SR through P. Let U be the point of intersection of PR with the straight line QU drawn parallel to SR through Q. If PT = a and QU = b, then the length of SR is

a. $\frac{a+b}{ab}$ b. $\frac{a-b}{ab}$ c. $\frac{ab}{a+b}$ d. $\frac{ab}{a-b}$

25. In set theory, for two sets A and B the following operations are defined as

A – B is the set of all elements of set A that are not elements of set B,

A $\cup$ B is the set of all elements that appear in A or B or both A and B,

A $\cap$ B is the set of all elements that are common to both sets A and B.

With the above definitions, if P, Q and R are three sets, then P – (Q – R) is equivalent to

a. (P – Q) $\cup$ (P $\cap$ R) b. P – (Q $\cup$ R)

c. P – (Q $\cap$ R) d. (P – Q) $\cup$ (P – R)

26. Seven persons named A, B, C, D, E, F and G play a chess tournament, each player playing against every other player plays exactly one game. Assume that each game results in a win for one of the players (that is there is no draw).

Let $L_1, L_2, L_3, ..., L_7$ are the number of games won by players A, B, C, D, E, F and G respectively. Also let $K_1, K_2, K_3, ..., K_7$ are the number of games lost by players A, B, C, D, E, F and G respectively. Then

a. $L_1^2 + L_2^2 + ... + L_7^2 = 49 - (K_1^2 + K_2^2 + K_3^2 + ... + K_7^2)$

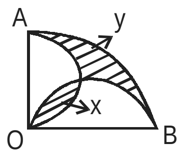
b. $L_1^2 + L_2^2 + ... + L_7^2 = 49 + (K_1^2 + K_2^2 + ... + K_7^2)$

c. $L_1^2 + L_2^2 + ... + L_7^2 = K_1^2 + K_2^2 + ... K_7^2$

d. None of these

27. AOB is a quarter of a circle, and semicircles are drawn on OA and OB. What is the relation between the shaded areas x and y?

a. x = 2y   b. x = y   c. 2x = y   d. 2x = 3y



28. The sum of the first 100 terms of the series $1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2 + 7^2 + 2 \times 8^2 + ...$ is

a. 495000   b. 510050   c. 550010   d. 549000

29. In question 28, what will be the sum of the first n terms if n is odd?

a. $\frac{n(n+1)^2}{2}$ b. $\frac{n^2 (n+1)^2}{4}$ c. $\frac{n^2 (n+1)}{2}$ d. $\frac{n(n+2)}{3}$

30. If $G(x) = (10 - x^{10})^{1/10}$ and $G^n(x) = G\{G^{n-1}(x)\}$ for $n \geq 2$, then what is the value of $G^{50}(2)$?

a. 2^{50} b. 2^{25} c. 2   d. $2^{1/25}$

31. In ΔPQR, the incircle touches the sides QR, RP and PQ at T, U and S respectively. If the radius of the incircle is 4 units and QT, RU and PS are consecutive integers, area of the triangle (in sq. units) is

a. 42   b. 84   c. 21   d. 48

32. Four married couples are to be seated around a circular table with 8 identical seats. In how many ways can they be seated so that males and females sit alternately and no wife sits adjacent to her husband?

a. 12   b. 6   c. 18   d. 24

33. If p, q and r are distinct real numbers such that $p^2 - q = q^2 - r = r^2 - p$, then $(p + q)(q + r)(r + p)$ equals

a. 2   b. 3   c. 1   d. 0

34. Eighty (80) students appeared in a test consisting of three papers QA, DI and English. 24 students passed in QA; 31 passed in DI, and 35 in English. Further, 49 students passed in at least one of QA and DI; 59 in at least one of DI and English; 54 in at least one of QA and English. Only 2 students passed in all the three papers. The number of students who failed in all the three papers is

a. 6   b. 8   c. 10   d. 12

35. In an AP, for any value of p and k, where $p \neq k$, the ratio of the sum of first p terms to the sum of the first k terms is $\frac{p^2}{k^2}$. If the 6th term of the AP is 77, then the 15th term of the AP is

a. 343   b. 302   c. 203   d. None of these

36. If $\log_{pq} p = 2$, then the value of $\log_{pq}\left(\frac{\sqrt[4]{p}}{\sqrt[3]{q}}\right)$ is

a. $2^{\frac{1}{4}}$ b. $3^{\frac{1}{4}} - 2^{\frac{1}{3}}$ c. $\frac{5}{6}$ d. $\frac{6}{5}$

37. If the equation $px^2 + qx + r = 0$ has a root less than –2 and a root greater than 2 (p, q and r are constants and p > 0), then which of the following relations is always true?
a. $4p + 2|q| + r = 0$ b. $4p + 2|q| + r < 0$ c. $4p + 2|q| + r > 0$ d. $4p + 2|q| + r \le 0$

38. A dishonest shopkeeper has two varieties of wheat. One variety costs Rs. 9 per kilogram and the other variety costs Rs. 13 per kilogram. He mixes the two varieties in the ratio 5 : 7. Further, he adds 18 kg of adulterant to the mixture to gain a higher profit percentage. If the shopkeeper makes a profit of 30% by selling the adulterated mixture at the average of the cost price the mixture of the two varieties of wheat, what is the total amount realised by selling the adulterated mixture?
a. Rs. 680 b. Rs. 784 c. Rs. 884 d. None of these

39. In a relay race between two groups, A and B (5 people in each group and the speeds of group members being different), group A defeats group B by 50 m. If each member of group B increases his speed by x%, group B defeats group A by 125 m. If the speeds of the last runners of groups A and B are 25 km/hr and 20 km/hr respectively and the time taken by group A to complete the race is 4 min, what is the value of x? (Assume that each runner has to run more than 125 m.)
a. 11.75% b. 12.15% c. 10% d. 15%

40. A cube is painted and then divided into 64 identical small cubes. Then all those cubes with three painted surfaces are removed. The remaining part of the cube is painted again without dismantling or rearranging it. Again those cubes with three painted surfaces are removed and the cube is once again painted. How many cubes have two sides coloured now?
a. 4 b. 8 c. 16 d. 0

Directions for questions 41 to 46: Each question consists of a question and two statements, I and II. Choose
a. if one of the two statements (I or II) alone is sufficient to answer the question, but cannot be answered by using the other statement alone.
b. if each statement alone is sufficient to answer the question asked.
c. if I and II together are sufficient to answer the question but neither statement alone is sufficient.
d. if even I and II together are not sufficient to answer the question.

41. What is the volume of an open water tank in the form of a right circular cylinder?
I. The total inner surface area = $5\ \pi m^2$
II. Height = Diameter of base

42. What are the integral sides of a right-angled ΔPQR?
I. Sum of squares of sides = 50
II. Perimeter of the triangle = 12

43. A wants to go from city P to city Q via X and Y cities. X is 10 km to the south of P. In which direction is P with respect to Q?
I. Q is 5 km south of Y.
II. X is 10 km to the west of Y.

44. A kite is gaining height due to a good breeze. Find the rate of gain of the height.

I. String pull causes the string roller to make $\frac{3}{2}$ rotations per second.

II. String is being released at 5 m/s at an angle of 60° with the horizontal.

45. There are two drains at the bottom of a water tank. If drain 1 is opened and drain 2 is closed, a full tank will be empty in 15 min. How long will it take to empty a full tank if drain 1 and drain 2 are both opened?

I. If drain 1 is closed and drain 2 is opened it takes 20 min to empty a full tank.

II. In 3 min the same water flows through drain 1 as through drain 2 in 4 min.

46. If a > 1, is M > N? (M and N are positive numbers.)

I. $M = (a - 1)(a^2 + a + 1)$ and $N = (a + 1)(a^2 - a + 1)$

II. $M = a \times N$

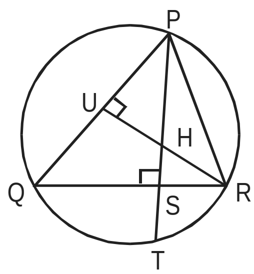
47. A rod of a length of 30 units is to be divided into P parts so that the product of the lengths of the parts is greater than unity. The maximum possible value of P is

a. 30 b. 31 c. 29 d. Cannot be determined

48. If p is strictly negative and is not equal to –2, and the equation $x^2 + p\,|x| + 1 = 0$ has n real roots, then n equals

a. 4 b. 4 or 2 c. 4 or 0 d. 2 or 0

49. If altitude PS meets the circumcircle of ΔPQR at T, and H is the orthocentre, then what is the length of the line segment HT? (Given ST = 3 cm)



a. 4.5 cm b. 5 cm c. 6 cm d. 4 cm

50. $N = a^4 + 4^a$, where 'a' is a natural number. For how many values of 'a' is N prime?

a. One b. Two c. Three d. Infinitely many