Section – I

1. The following system of equations has a total of x + y = 2; $xy - z^2 = 1$ (x, y, z are real numbers):
   a. Two solutions
   b. One solution
   c. Three solutions
   d. No solutions

2. For how many positive integers V less than 15, $V! + (V+1)! + (V+2)!$ is an integral multiple of 36?
   a. 12
   b. 10
   c. 9
   d. 8

3. Compute the infinite sum V, where

$$V = \frac{1}{2} - \frac{2}{4} + \frac{3}{8} - \frac{4}{16} + \frac{5}{32} - \frac{6}{64} + \ldots + \frac{n}{2^n}(-1)^{n+1} \ldots.$$

   a. $\frac{2}{3}$
   b. $\frac{2}{9}$
   c. $\frac{1}{6}$
   d. $\frac{2}{27}$

4. If log2 = a and log3 = b then [log (1) + log (1 + 3) + log (1 + 3 + 5) + ... + ... + log (1 + 3 + 5 + 7 + ... + 19)] – 2[log1 + log2 + log3 + ... log 7] = p + qa + rb where p, q, r are constants. What is the value of p + 2q + 3r?
   a. 12
   b. 26
   c. 18
   d. Cannot be determined

Directions for questions 5 to 7: Answer the questions based on the following information.
abcd is a four digit number such that abcd = $(ab + cd)^2$
ab and cd are two digit numbers and abcd, ab and cd are all in decimal system i.e. their values are 1000a + 100b + 10c + d, 10a + b and 10c + d respectively.

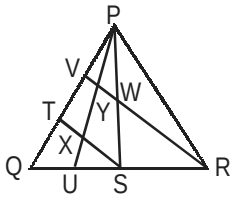
5. If d = 1, what is the value of a + b + c?
   a. 18
   b. 17
   c. 3
   d. This is not possible.

6. If b = 0, then how many different values of a are possible?
   a. 4
   b. 3
   c. 1
   d. 2

7. If c = 2, then how many different values of d are possible?
   a. 1
   b. 2
   c. 4
   d. 3

8. V is a 1001 digit number consisting of 1001 sevens. What is the remainder when V is divided by 1001?
   a. 7
   b. 700
   c. 777
   d. None of these

Directions for questions 9 to 11: In the figure shown here QS = SR, QU = SU, PW = WS and ST || RV



9. The $\frac{\text{area of } \Delta PSX}{\text{area of } \Delta PQR}$ is equal to

a. $\frac{1}{5}$ b. $\frac{1}{7}$ c. $\frac{1}{6}$ d. $\frac{1}{9}$

10. The $\frac{\text{area of } \Delta PVY}{\text{area of } \Delta PQR}$ is equal to

a. $\frac{1}{12}$ b. $\frac{1}{30}$ c. $\frac{1}{15}$ d. $\frac{1}{18}$

11. The $\frac{\text{area of } \Delta SUX}{\text{area of } \Delta PQR}$ is equal to

a. $\frac{1}{20}$ b. $\frac{1}{15}$ c. $\frac{1}{30}$ d. Cannot be determined

12. Consider the quadratic equation $(p + r – q)\, x^2 + 2rx + (q + r – p) = 0$, where p, q and r are distinct real numbers and $p + r – q \neq 0$. Suppose that both the roots of the equation are rational. Then

a. $\frac{r}{q – p}$ is rational b. p^2, q^2, r^2 are rational c. $\frac{q}{p – r}$ is rational d. $\frac{p}{q – r}$ is rational

13. Two dishonest milkmen mix water to milk before selling it in the market. The first milkman mixes water to milk in the ratio 2 : 1 and second milkman mixes water to milk in the ratio 3 : 1. Now if both of them sell the milk at their cost price of Rs. 60 and Rs. 80 respectively, then from which milkman should the customer purchase the milk to get the maximum quantity of pure milk per rupee paid?
a. 1st milkman b. 2nd milkman c. Any one d. Data Inadequate

14. V is a positive integer. When V is divided by any of the numbers 2, 11, 13, 71 and 89 the remainder is 1. What is the remainder when V^{256} is divided by 16?
a. 1 b. 15 c. 3 d. Cannot be determined

15. Let V_1 and V_2 be the roots of the equation $x^2 – 3x + a = 0$, and V_3 and V_4 be the roots of the equation $x^2 – 12x + b = 0$. If $V_1 < V_2 < V_3 < V_4$ and, V_1, V_2, V_3 and V_4 are in G.P., then product of a and b equals
a. 64 b. 128 c. 16 d. 56

16. If $\log_y x = 10$, then $\log_{x^3}(y^6)$ is equal to

a. 5 b. $\frac{1}{5}$ c. 6 d. $\frac{1}{6}$

17. An urn A contains 6 red and 4 black balls and an another urn B contains 4 red and 6 black balls respectively. One ball is drawn at random from urn A and placed in urn B and in turn one ball is drawn from urn B and placed in urn A. Now if one ball is drawn at random from the urn A, then what is the probability that the ball finally drawn from urn A is found to be a red coloured ball?

a. $\frac{32}{65}$ b. $\frac{32}{55}$ c. $\frac{32}{57}$ d. $\frac{64}{89}$

18. The three sides of a right angled triangle are in G.P. The tangents of the two acute angles are

a. $\sqrt{\frac{\sqrt{5}+1}{2}}$ and $\sqrt{\frac{2}{\sqrt{5}+1}}$ b. $\frac{\sqrt{5}+1}{2}$ and $\frac{2}{\sqrt{5}+1}$

c. $\sqrt{\frac{\sqrt{2}+1}{2}}$ and $\sqrt{\frac{2}{\sqrt{2}+1}}$ d. None of these

19. Let k quantities be in A.P., d being the common difference. Let the square of the arithmetic mean exceeds arithmetic mean of squares of these quantities by a quantity V. Then V is

a. always positive b. $\frac{1-k^2}{12}d^2$ c. $\frac{d^2(k^2-1)}{12}$ d. $\frac{1+k^2}{12}d^2$

20. The number of solution/s of the simultaneous equations

$V = 3\log_e K$ and $V = \log_e(3K)$ is / are

a. Three b. One c. Two d. Zero

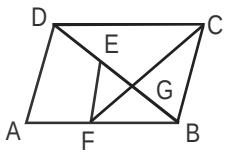
21. Let a, b and c be positive real numbers and

$V = \frac{a^2+b^2+c^2}{ab+bc+ca}$, where $V \le 1$, then

a. a, b and c can take any value b. $a^2+b^2+c^2 \le 1$

c. $ab+bc+ca \ge 1$ d. a = b = c

22. In the adjoining figure, ABCD is a parallelogram. AD is parallel to FE and $\frac{AF}{FB}$ is $\frac{2}{3}$ then $\frac{BG}{GD}$ equals to



a. $\frac{3}{5}$ b. $\frac{3}{8}$ c. $\frac{9}{40}$ d. $\frac{2}{5}$

Directions for questions 23 to 25: Answer the questions based on the following information.

p, q, r and s are real numbers such that $p^2 + q^2 + r^2 + s^2 - pq - qr - rs - s + \frac{2}{5} = 0$

23. What is the value of p + q + r + s?
a. 2 b. 1 c. 0 d. cannot be determined

24. What is the value of $(p + q)^2 + (r + s)^2$?
a. $\frac{9}{25}$ b. $\frac{49}{25}$ c. $\frac{58}{25}$ d. cannot be determined

25. What is the value of $(p - q)^2 + (r - s)^2$?
a. $\frac{1}{25}$ b. $\frac{2}{25}$ c. $\frac{3}{25}$ d. cannot be determined

26. A box contains 90 balls of different colours: 13 yellow, 19 green, 27 red, 10 black, 7 brown and 14 white. The smallest number V such that any V ball drawn from the box will contain at least 14 balls of the same colour is
a. 79 b. 69 c. 70 d. 72

27. The quadratic equation $x^2 + px + q = 0$ (p, q are real numbers) has both roots positive. A necessary and sufficient condition for this is
a. pq < 0 b. p < 0, q > 0
c. q > 0 and $p \le -2\sqrt{q}$ d. q < 0, p > 0

28. The set of all values of V for which $Vy^2 - 6Vy + 5V + 1 > 0$ for all real y is
a. $0 < V < \frac{1}{4}$ b. $V \ge 0$ c. $0 \le V \le \frac{1}{4}$ d. $0 \le V < \frac{1}{4}$

29. If V is a real number such that $|1 - V| - V \ge 0$, then
a. $-\frac{1}{2} \le V \le \frac{1}{2}$ b. $V \le \frac{1}{2}$ c. $0 \le V \le 1$ d. $V \le \frac{1}{4}$

30. Seven numbers are in A.P. such that their sum is 7. The first number is 7 times the fourth number. The sixth number is equal to:
a. –3 b. –5 c. –1 d. Data insufficient

31. The cubic expression in v, which takes the value zero when v = 1 and v = –2, and takes values 100 and 28 when v = 3 and v = 2 respectively is
a. $3v^3 + 4v^2 - 5v - 2$ b. $3v^3 - 4v^2 + 5v - 2$
c. $3v^3 + 4v^2 - 5v + 2$ d. $3v^3 - 4v^2 + 5v + 2$

Directions for questions 32 to 33: Let V_1, V_2, V_3 ..... V_{100} be hundred positive integers such that $V_i + V_{i+1} + V_{i+2} + V_{i+3} = K$, where K is a constant and i = 1, 2, 3, ... 97.

32. If V_3 = 9 then what is the value of V_{99}?

a. 9 b. K – 9 c. $\frac{K}{2} - 9$ d. Cannot be determined

33. If V_5 = 7 then what is the vale of V_{90}?

a. 7 b. K – 7 c. $\frac{K-7}{2}$ d. Cannot be determined

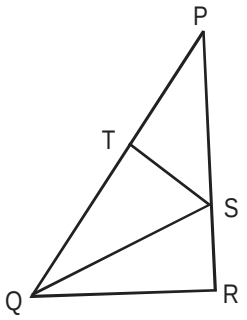
34. When a certain two-digit number is added to another two digit number having the same digits in reverse order, the sum is a perfect square. How many such two-digit numbers are there?

a. 4 b. 6 c. 8 d. 10

35. If V be a root of $x^5 - 1 = 0$, with $V \neq 1$ then the value of $V^{10} + V^{11} + V^{12} + ... + V^{50}$ is

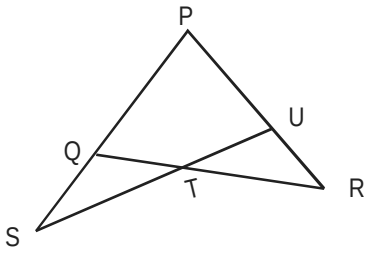
a. V b. V^5 c. V^4 d. V^3

36. In the adjoining figure, in ΔPQR PQ = PR, S and T are points on PR and PQ respectively such that RQ = QS = ST = TP. $\angle PTS$ equals



a. $\frac{\pi}{7}$ b. $\frac{3\pi}{7}$ c. $\frac{5\pi}{7}$ d. $\frac{6\pi}{7}$

37. Through T, the mid-point of the side QR of a ΔPQR, a straight line is drawn to meet PQ produced to S and PR at U, so that PU = PS. If length of UR = 2 units then the length of QS is



a. $2\sqrt{2}$ units b. $\sqrt{2}$ units c. 2 units d. cannot be determined

Directions for questions 38 to 40: Answer the questions based on the following information.
In a particular country all the numbers are expressed with the help of three alphabets a, b and c.

15 is written as abc.
6 is written as bc.
103 is written as acbaa and
60 is written as bcbc.

38. How would one write 17 in that country?
a. abb b. bab c. baa d. aba

39. Which of the following is the value of abcabc?
a. 340 b. 403 c. 240 d. 420

40. Which of the following is the value of acbacb?
a. 308 b. 348 c. 402 d. 304

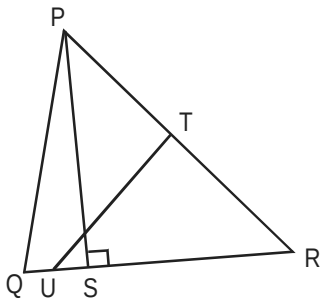
41. A, B and C are contesting in the election for the post of the secretary of a club. The club doesn't allow ladies to become members. The probabilities of winning the election of A, B and C are $\frac{1}{3}$, $\frac{2}{9}$ and $\frac{4}{9}$ respectively. The probabilities that they would include lady members in the club are 0.6, 0.7, and 0.5 for A, B and C respectively. The probability that ladies will become members of the club after the election is

a. $\frac{26}{45}$ b. $\frac{5}{9}$ c. $\frac{19}{45}$ d. None of these

42. Anuj likes to jog in a park. While jogging he also calculates his speed and the number of steps taken by him. He observes that the steps taken by him per minute are 5 times his speed in km/hr. What is the distance he covers per step if he jogs at a uniform speed?

a. $\frac{5}{3}$m b. $\frac{10}{3}$m c. $\frac{100}{3}$m d. Data insufficient

43. Let PQR be an acute angled triangle and PS be the altitude through P. If QR = 16 units and PS = 12 units then the distance between the mid-points of QS and PR is



a. 14 units b. 12 units c. 10 units d. 8 units

44. If $(\log_4 a)\,(\log_a 2a)\,(\log_{2a} 3a) = \log_{2a} 8a^3$, thus a equals

a. $\frac{16}{3}$ b. $\frac{4}{3}$ c. $\frac{128}{3}$ d. $\frac{64}{3}$

45. 5 men start painting a wall. On day 2 three more men join them and on day 3 two of them leave the group. Again on the next day 3 more men join them and 2 leave on the very next day and this process continues. If the wall is completely painted in exactly 15 days, find the number of days required to paint the wall by 29 men working together?

a. 7 b. 5 c. $7\frac{1}{3}$ d. $7\frac{9}{29}$

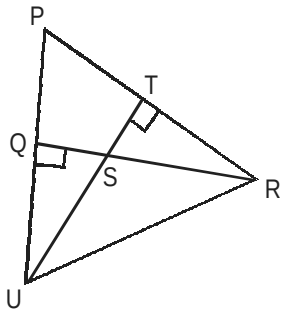
46. A swimmer started swimming from Baghbazar Ghat towards Ahiritola Ghat against the stream. At Ahiritola Ghat he saw a floating log. He continued swimming till Howrah Ghat and after reaching there, he immediately turned and started swimming back. Coincidentally the log and the swimmer both reached Baghbazar Ghat at the same time. If Howrah Ghat is 20 km away from the Ahiritola Ghat and the swimmer take triple the time for one trip than the other, then what is the distance between Baghbazar Ghat and Ahiritola Ghat?

a. 60 km b. 20 km c. 10 km d. 40 km

47. If $V_0 = 3, V_1 = 3$ and $V_n = V_{n-1}.V_{n-2} + 1$ for n > 1, then

a. V_{110} is even and V_{111} is even.
b. V_{110} is even and V_{111} is odd.
c. V_{110} is odd and V_{111} is even.
d. V_{110} is odd and V_{111} is odd.

Directions for questions 48 and 49: Answer the questions based on the following information.
T is the mid-point of the hypotenuse PR of a right angled triangle ΔPQR. The perpendicular TU to PR meets PQ produced at U and intersects QR at S. If TS = 6 and SU = 18.



48. What is the length of PR?

a. 18 b. 12 c. 24 d. 16

49. What is the length of QU?

a. $\frac{18}{\sqrt{5}}$ b. $\frac{36}{\sqrt{5}}$ c. $\frac{48}{\sqrt{5}}$ d. $\frac{24}{\sqrt{5}}$

50. How many consecutive zeroes would be there at the end of $3125^{4}!$?

a. $\frac{5^{19}-1}{4}$ b. $\frac{5^{20}-1}{4}$ c. $\frac{5^{21}-1}{4}$ d. $\frac{5^{22}-1}{4}$