DATE : 10/04/2011

Aakash
IIT-JEE

(Division of Aakash Educational Services Ltd.)

Regd. Office : **Aakash Tower, Plot No.-4, Sec-11, MLU, Dwarka, New Delhi-110075**

Ph.: 011-47623456 Fax : 011-47623472

Time : 3 hrs. **Max. Marks: 240**

Solutions

for

IIT-JEE 2011

PAPER - 1 (Code - 4)

INSTRUCTIONS

1. The question paper consists of **3 parts** (Chemistry, Physics and Mathematics). Each part consists of **four sections.**

2. In **Section I** (Total Marks : 21), for each question you will be **awarded 3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.

3. In **Section II** (Total Marks : 16), for each question you will be **awarded 4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. There are **no negative marks** in this section.

4. In **Section III** (Total Marks : 15), for each question you will be **awarded 3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.

5. In **Section IV** (Total Marks : 28), for each question you will be **awarded 4 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** otherwise. There are **no negative marks** in this section.

PART–I : CHEMISTRY

SECTION - I (Total Marks : 21)

(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. Dissolving 120 g of urea (mol. wt. 60) in 1000 g of water gave a solution of density 1.15 g/mL. The molarity of the solution is

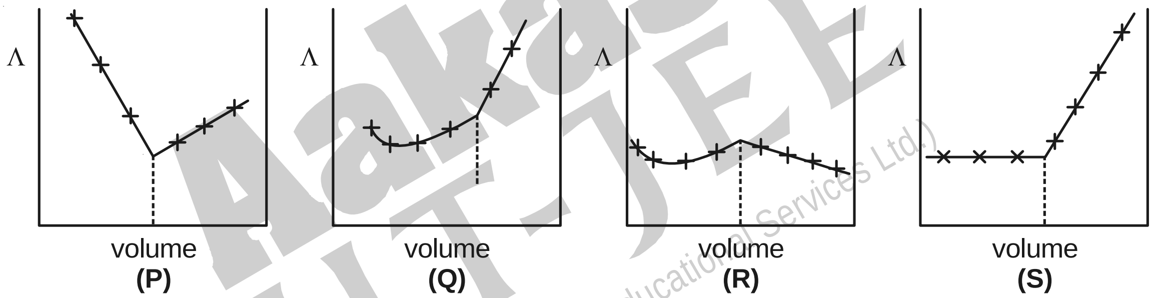
(A) 1.78 M (B) 2.00 M (C) 2.05 M (D) 2.22 M

Answer (C)

Hints : $M = \frac{\text{wt. of solute}}{\text{mol. wt. of solute}} \times \frac{1000}{\text{volume of solution}}$

$$M = \frac{120}{60} \times \frac{1000}{\frac{1120}{1.15}} = 2.05$$

2. $AgNO_3$(aq.) was added to an aqueous KCl solution gradually and the conductivity of the solution was measured. The plot of conductance (Λ) *versus* the volume of $AgNO_3$ is

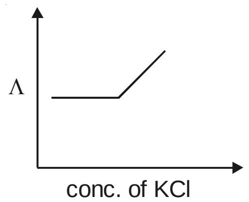


(A) (P) (B) (Q) (C) (R) (D) (S)

Answer (D)

Hints : Ag^+ and K^+ have nearly same ionic mobility

$AgNO_3 + KCl \longrightarrow AgCl(s) + KNO_3$

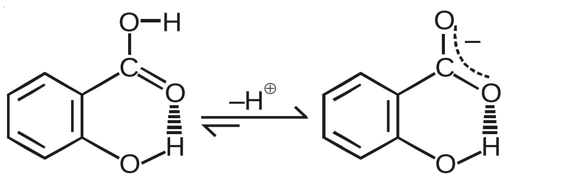


3. Among the following compounds, the most acidic is

(A) p-nitrophenol (B) p-hydroxybenzoic acid (C) o-hydroxybenzoic acid (D) p-toluic acid

Answer (C)

Hints :



Most acidic

Stabilized by strong intramolecular hydrogen bonding

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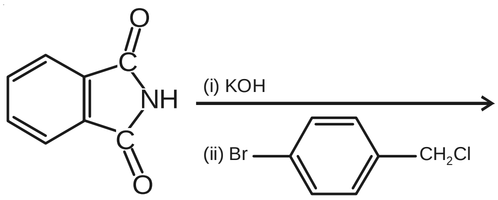
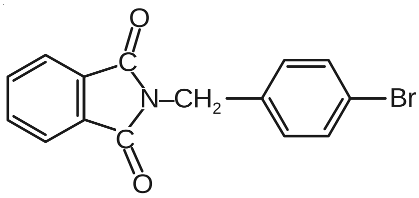
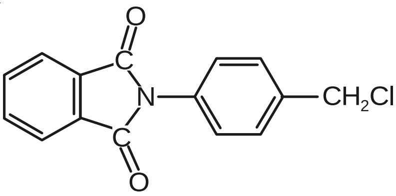
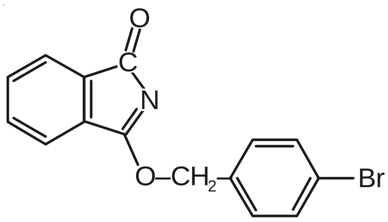
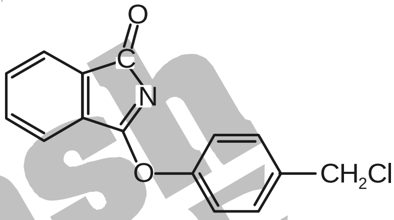
4. The major product of the following reaction is



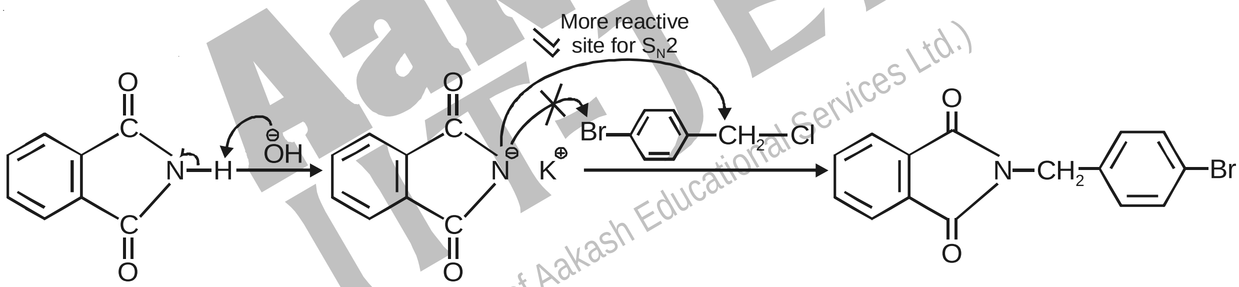
(A) 

(B) 

(C) 

(D) 

Answer (A)

Hints : 

5. Extra pure N_2 can be obtaine by heating

(A) NH_3 with CuO (B) NH_4NO_3 (C) $(NH_4)_2Cr_2O_7$ (D) $Ba(N_3)_2$

Answer (D)

Hints : $Ba(N_3)_2 \longrightarrow Ba(s) + 3N_2 \uparrow$

6. Geometrical shapes of the complexes formed by the reaction of Ni^{2+} with Cl^-, CN^- and H_2O, respectively are

(A) Octahedral, tetrahedral and square planar

(B) Tetrahedral, square planar and octahedral

(C) Square planar, tetrahedral and octahedral

(D) Octahedral, square planar and octahedral

Answer (B)

Hints : $Ni^{+2} + 4Cl^- \longrightarrow [NiCl_4]^{-2}$ (tetrahedral)

$Ni^{+2} + 4CN^- \longrightarrow Ni(CN)_4^{-2}$ (square planar)

$Ni^{+2} + H_2O \longrightarrow Ni(H_2O)_6^{+2}$ (octahedral)

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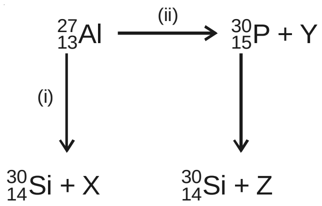
7. Bombardment of aluminium by α-particle leads to its artificial disintegration in two ways, (i) and (ii) as shown. Products X, Y and Z respectively are



(A) Proton, neutron, positron
(B) Neutron, positron, proton
(C) Proton, positron, neutron
(D) Positron, proton, neutron

Answer (A)

Hints : $^{27}_{13}Al \xrightarrow{{}_2He^4} {}^{30}_{14}Si + {}_1H^1$ (X)

$^{27}_{13}Al \xrightarrow{{}_2He^4} {}^{30}_{15}P + {}^{1}_{0}n$ (Y)

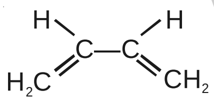
$\downarrow$

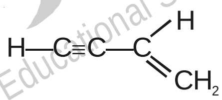
$^{30}_{14}Si + {}^{0}_{+1}e$ (Z)

SECTION - II (Total Marks : 16)
(Multiple Correct Answer Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

8. Amongst the given options the compound(s) in which all the atoms are in one plane in all the possible conformations (if any), is(are)

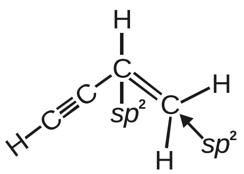
(A) 

(B) 

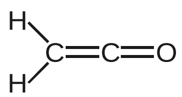
(C) $H_2C = C = O$
(D) $H_2C = C = CH_2$

Answer (B, C)

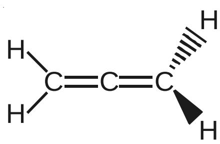
Hints :



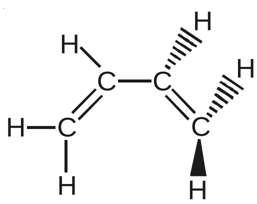
All atoms lie in one plane in all conformation



Planar



is nonplanar



Non-planar in many conformations due to rotation along.
C_2 — C_3 bond.

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9. According to kinetic theory of gases

(A) Collisions are always elastic

(B) Heavier molecules transfer more momentum to the wall of the container

(C) Only a small number of molecules have very high velocity

(D) Between collisions the molecules move in straight lines with constant velocities

Answer (A, B, C)

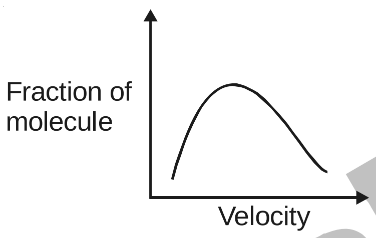
Hints : According to Kinetic theory of gases

(i) Collisions are always elastic

(ii) Momentum transferred on wall

by one collision along x component = 2 mv_x

(iii)



10. The correct statement(s) pertaining to the adsorption of a gas on a solid surface is(are)

(A) Adsorption is always exothermic

(B) Physisorption may transform into chemisorption at high temperature

(C) Physisorption increases with increasing temperature but chemisorption decreases with increasing temperature

(D) Chemisorption is more exothermic that physisorption, however it is very slow due to higher energy of activation

Answer (B, D)

Hints : At high temperature, sufficient activation energy for chemical adsorption is provided.

$\Delta H_{adsorption}$ for chemical lies between 200-400 kJ/mole. Chemisorption of H_2 on glass is an endothermic process.

11. Extraction of metal from the ore **cassiterite** involves

(A) Carbon reduction of an oxide ore

(B) Self reduction of a sulphide ore

(C) Removal of copper impurity

(D) Removal of iron impurity

Answer (A, C, D)

Hints : $SnO_2 \xrightarrow{1200-1300°C} Sn + CO$

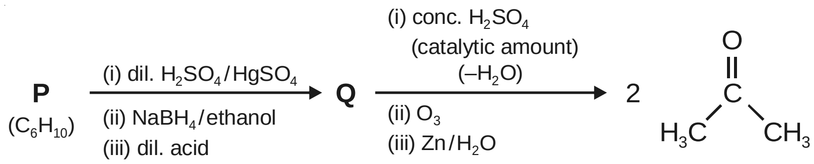
If after contain Fe and Cu as impurity,

SECTION - III (Total Marks : 15)
(Paragraph Type)

This section contains 2 paragraphs. Based upon one of the paragraph **2 multiple choice questions** and based upon the second paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions Nos. 12 and 13

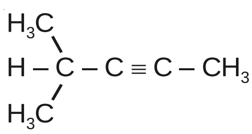
An acyclic hydrocarbon **P**, having molecular formula C_6H_{10}, gave acetone as the only organic product through the following sequence of ractions, in which **Q** is an intermediate organic compound.

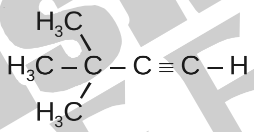


12. The structure of compound **P** is

(A) $CH_3CH_2CH_2CH_2 - C \equiv C - H$

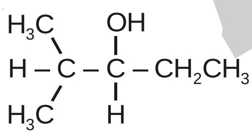
(B) $H_3CH_2C - C \equiv C - CH_2CH_3$

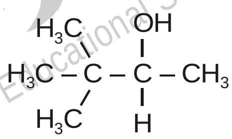
(C) 

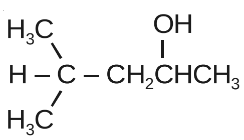
(D) 

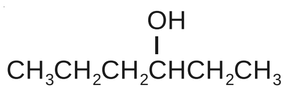
Answer (D)

13. The structure of the compound **Q** is

(A) 

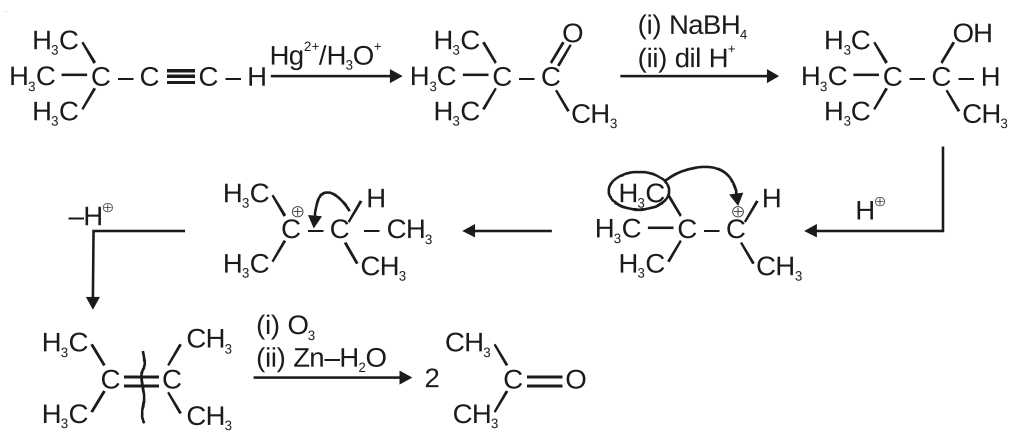
(B) 

(C) 

(D) 

Answer (B)

Hints : (Q. 12 to Q.13)



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Paragraph for Questions Nos. 14 to 16

When a metal rod **M** is dipped into an aqueous colourless concentrated solution of compound **N**, the solution turns light blue. Addition of aqueous NaCl to the blue solution gives a white precipitate **O**. Addition of aqueous NH_3 dissolves **O** and gives an intense blue solution.

14. The metal rod **M** is

(A) Fe (B) Cu (C) Ni (D) Co

Answer (B)

15. The compound **N** is

(A) $AgNO_3$ (B) $Zn(NO_3)_2$ (C) $Al(NO_3)_3$ (D) $Pb(NO_3)_2$

Answer (A)

16. The final solution contains

(A) $[Pb(NH_3)_4]^{2+}$ and $[CoCl_4]^{2-}$
(B) $[Al(NH_3)_4]^{3+}$ and $[Cu(NH_3)_4]^{2+}$
(C) $[Ag(NH_3)_2]^{+}$ and $[Cu(NH_3)_4]^{2+}$
(D) $[Ag(NH_3)_2]^{+}$ and $[Ni(NH_3)_6]^{2+}$

Answer (C)

Hints : (Q. 14 to Q.16)

$$Cu + Ag^{+} \longrightarrow Cu^{+2} + Ag$$

$$Ag^{+} + NaCl \longrightarrow \underset{\text{White ppt}}{AgCl}$$

$$AgCl + NH_3 \longrightarrow [Ag(NH_3)_2]^{+}Cl^{-}$$

$$Cu^{+2} + NH_3 \longrightarrow \underset{\text{(Blue)}}{Cu(NH_3)_4^{2+}}$$

M - Cu

N - Solution of $AgNO_3$

O - AgCl

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SECTION - IV (Total Marks : 28)

(Integer Answer Type)

This section contains **7 questions.** The answer to each of the questions is a **Single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

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17. The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

Answer (4)

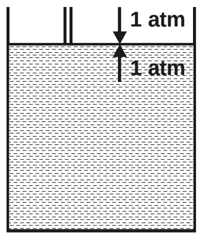
Hints : Energy of incident photon should be higher than work function to slow photoelectric effect

$$\text{Energy of photon} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{300 \times 10^{-9} \times 1.6 \times 10^{-19}} = 4.14 \text{ eV}$$

18. To an evacuated vessel with movable piston under external pressure of 1 atm., 0.1 mol of He and 1.0 mol of an unknown compound (vapour pressure 0.68 atm. at 0°C) are introduced. Considering the ideal gas behaviour, the total volume (in litre) of the gases at 0°C is close to

Answer (7)

Hints :



P_T = 1 atm

vapour pressure of compound = 0.68 atm

$\Rightarrow$ P_{He} = 0.32

$P_{He}V = n_{He}RT$

$$V = \frac{0.1 \times 0.082 \times 273}{0.32} = 7 \text{ litre}$$

19. Reaction of Br_2 with Na_2CO_3 in aqueous solution gives sodium bromide and sodium bromate with evolution of CO_2 gas. The number of sodium bromide molecules involved in the balanced chemical equation is

Answer (5)

Hints : $\begin{bmatrix} Br_2 \rightarrow 2Br^- \\ +2e \end{bmatrix} \times 5 \left| \left[Br_2 \rightarrow 2Br^{+5} + 10e^- \right] \right.$

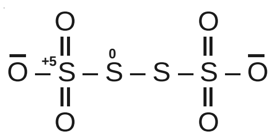
$Br_2 + 5Br_2 \rightarrow 10NaBr + 2NaBrO_3 + 6CO_2 + 6Na_2CO_3$

$3Br_2 + 3Na_2CO_3 \rightarrow 5NaBr + NaBrO_3 + 3CO_2$

20. The difference in the oxidation numbers of the two types of sulphur atoms in $Na_2S_4O_6$ is

Answer (5)

Hints : Structure of $S_4O_6^{2-}$ is



$\therefore$ Difference in oxidation state is +5 – 0 = +5.

21. A decapeptide (Mol. Wt. 796) on complete hydrolysis gives glycine (Mol. Wt. 75), alanine and phenylalanine. Glycine contributes 47.0% to the total weight of the hydrolysed products. The number of glycine units present in the decapeptide is

Answer (6)

Hints : A decapeptide will have (9) peptide linkage

Mass of hydrolyzed product is (796 + 162) gm/mole

$$\text{Number of glycine molecule} = \frac{0.47 \times 958}{75} = 6$$

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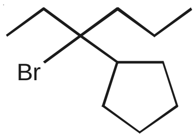
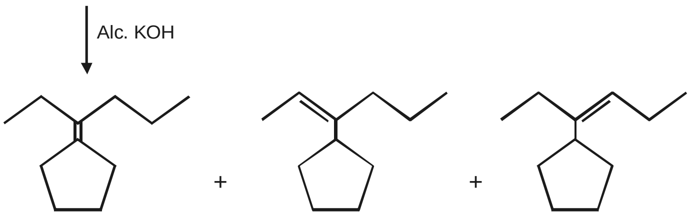
22. The total number of alkenes possible by dehydrobromination of 3-bromo-3-cyclopentylhexane using alcoholic KOH is

Answer (5)

Hints :



3-bromo-3-cyclopentylhexane



1 + 2 + 2

Total = 5

Note : Dehydrobromination of alkyl bromides in the presence of alc. KOH follows E_2 mechanism. Hence no rearrangement in alkylchain is possible

23. The maximum number of electrons that can have principal quantum number, n = 3, and spin quantum number, $m_s = -\frac{1}{2}$, is

Answer (9)

Hints :

3*s* 3*p* 3*d*

Out of 18 electrons present in 3rd shells.

Will have $s = -\frac{1}{2}$

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PART–II : PHYSICS

SECTION - I (Total Marks : 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

24. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is

(A) 1215 Å (B) 1640 Å (C) 2430 Å (D) 4687 Å

Answer (A)

Hints :

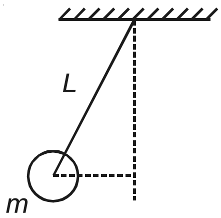
$$\frac{1}{\lambda_1} = R\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\frac{1}{\lambda_2} = 4R\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$\frac{\lambda_2}{\lambda_1} = \frac{5}{27}$$

$\Rightarrow \lambda_2 = 1215$ Å

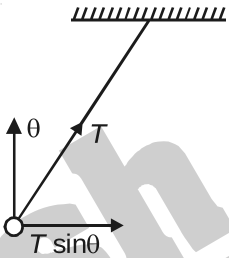
25. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is



(A) 9 (B) 18 (C) 27 (D) 36

Answer (D)

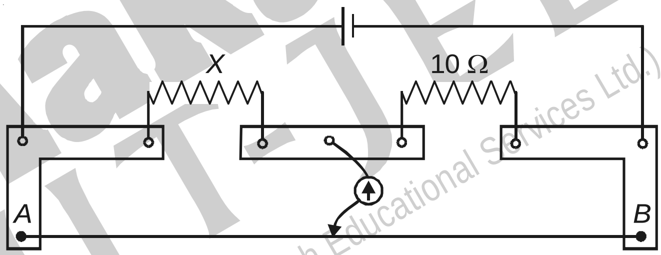
Hints : $T\sin\theta = m\omega^2 l \sin\theta$



$$\Rightarrow \omega_{max} = \sqrt{\frac{T_{max}}{ml}}$$

$= 36$ rad/s

26. A meter bridge is set-up as shown, to determine an unknown resistance X using a standard 10 ohm resistor. The galvanometer shows null point when tapping key is at 52 cm mark. The end corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is



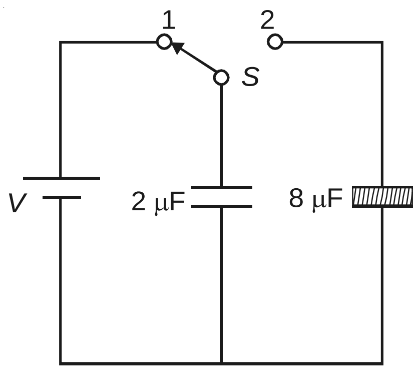
(A) 10.2 ohm (B) 10.6 ohm (C) 10.8 ohm (D) 11.1 ohm

Answer (B)

Hints : $\frac{X}{53} = \frac{10}{50}$

$\Rightarrow X = 10.6\ \Omega$

27. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



(A) 0% (B) 20% (C) 75% (D) 80%

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Answer (D)

Hints : $U_1 = \frac{1}{2}C_1V^2$

$q = C_1V$

$V' = \frac{q}{C_1+C_2} = \frac{C_1V}{C_1+C_2}$

$= \frac{V}{5}$

$U_f = \frac{1}{2}(C_1+C_2)V'^2$

$= \frac{1}{10}C_1V^2$

$\frac{\Delta U}{U_i} \times 100 = 80\%$

28. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is

(A) 8.50 kHz (B) 8.25 kHz (C) 7.75 kHz (D) 7.50 kHz

Answer (A)

Hints : $f = f_0\left[\frac{v}{v-u}\right]\left[\frac{v+u}{v}\right]$

$= 8\times\left[\frac{320+10}{320-10}\right]$

$= 8\times\frac{33}{21}$

$= 8.5$ kHz

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29. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is

(A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$ (C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

Answer (A)

Hints : $T_1V_1^{(\gamma-1)} = T_2V_2^{(\gamma-1)}$

$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$

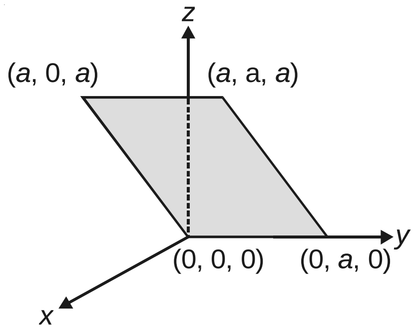
$= 4T_1$ $\{\gamma = 5/3\}$

μ = no. of moles = $\frac{1}{4}$

$\omega_{ext} = \frac{-\mu R(T_2-T_1)}{1-\gamma}$

$= +\frac{9}{8}RT_1$

30. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is



(A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$ (C) E_0a^2 (D) $\dfrac{E_0a^2}{\sqrt{2}}$

Answer (C)

Hints : $\vec{A} = \frac{1}{2}\times\vec{d}_1\times\vec{d}_2$

$= \frac{1}{2}(a\hat{i}+a\hat{j}+a\hat{k})\times(a\hat{i}-a\hat{j}+a\hat{k})$

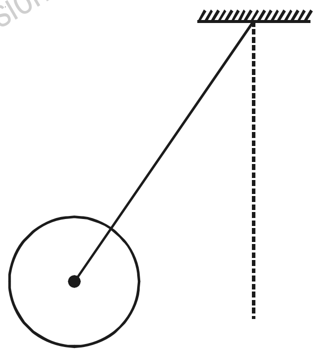
$\phi = \vec{E}\cdot\vec{A}\,\{\vec{E}\text{ is constant}\}$

$= E_0a^2$

SECTION - II (Total Marks : 16)
(Multiple Correct Answer Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

31. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (< L) is attached at its center to the free end of the rod. Consider two ways the disc is attached. ((case A): The disc is not free to rotate about its center and (case B) the disc is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true?



(A) Restoring torque in case A = Restoring torque in case B

(B) Restoring torque in case A < Restoring torque in case B

(C) Angular frequency for case A > Angular frequency for case B

(D) Angular frequency for case A < Angular frequency for case B

Answer (A, D)

Hints : $\omega_B = \sqrt{\dfrac{mgl}{ml^2}}; \omega_A = \sqrt{\dfrac{mgl}{ml^2+\frac{1}{2}mr^2}}$

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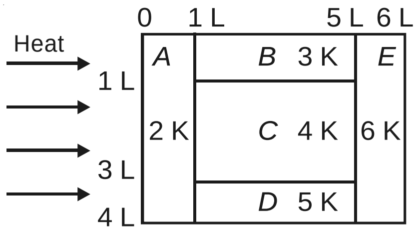
32. A composite block is made of slabs *A, B, C, D* and *E* of different thermal conductivities (given in terms of a constant *K*) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat '*Q*' flows only from left to right through the blocks. Then in steady state



(A) Heat flow through *A* and *E* slabs are same

(B) Heat flow through slab *E* is maximum

(C) Temperature difference across slab *E* is smallest

(D) Heat flow through *C* = heat flow through *B* + heat flow through *D*

Answer (A, C, D)

Hints : $\frac{K\times 4L^2}{L}(T_1-T_0) = \overset{(B)}{\frac{3K\times 4L^2}{4L}(T_2-T_1)} + \overset{(C)}{\frac{4K\times 8L^2}{4L}(T_2-T_1)} + \overset{(D)}{\frac{5K\times 4L^2}{4L}(T_2-T_1)}$

(C) = (B) + (D)

33. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true?

(A) They will never come out of the magnetic field region

(B) They will come out travelling along parallel paths

(C) They will come out at the same time

(D) They will come out at different times

Answer (B, D)

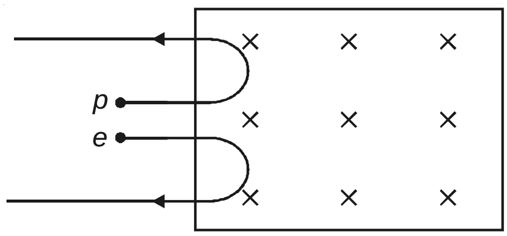
Hints : $\frac{mv^2}{r} = qvB$

$\Rightarrow r = \frac{mv}{qB}$

$m_e < m_p$

$\Rightarrow r_e < r_p$

$\Rightarrow t_e < t_p$



34. A spherical metal shell *A* of radius R_A and a solid metal sphere *B* of radius $R_B(< R_A)$ are kept far apart and each is given charge '+*Q*'. Now they are connected by a thin metal wire. Then

(A) $E_A^{\text{inside}} = 0$ (B) $Q_A > Q_B$ (C) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (D) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

Answer (A, B, C, D)

Hints : After connection,

$V_A = V_B$

$\Rightarrow \frac{q_A}{R_A} = \frac{q_B}{R_B}$

$E_{A(\text{inside})} = 0$

$\Rightarrow q_A > q_B$

$$\frac{\sigma_A}{\sigma_B} = \frac{q_A}{q_B} \times \frac{R_B^{\,2}}{R_A^{\,2}} = \frac{R_B}{R_A}$$

$$\frac{E_A}{E_B} = \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$$

SECTION - III (Total Marks : 15)

(Paragraph Type)

This section contains **2 paragraphs.** Based upon one of the paragraph **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions Nos. 35 and 36

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency 'ω_p', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

35. Taking the electronic charge as 'e' and the permittivity as 'ε_0', use dimensional analysis to determine the correct expression for ω_p

(A) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$ (C) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

Answer (C)

Hints : $\sqrt{\dfrac{m^{-3}\times C^2 \times Nm^{-2}}{kg \times C^2}}$ $\{N = kg\ ms^{-2}\}$

$= \sqrt{s^{-2}}$

36. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}\ m^{-3}$. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units

(A) 800 nm (B) 600 nm (C) 300 nm (D) 200 nm

Answer (B)

Hints : $\lambda = \dfrac{2\pi c}{\omega}$

$$= \frac{2\pi c}{\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}} \approx 600\,\text{nm}$$

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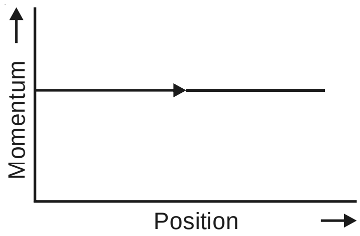
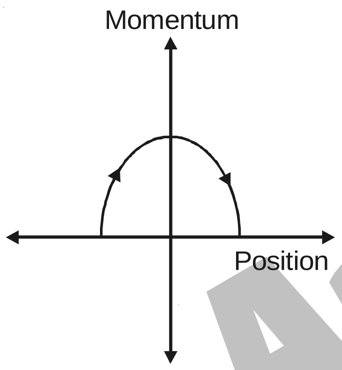
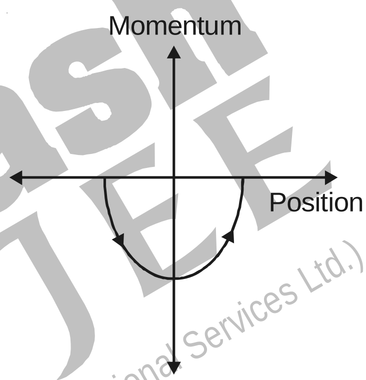
Paragraph for Questions Nos. 37 and 39

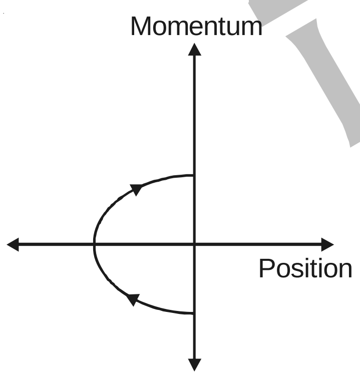
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is $x(t)$ vs. $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.

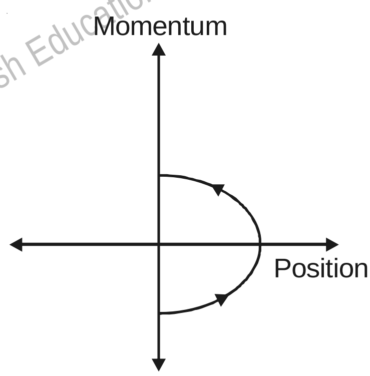


37. The phase space diagram for a ball thrown vertically up from ground is

(A)


(B)


(C)


(D)


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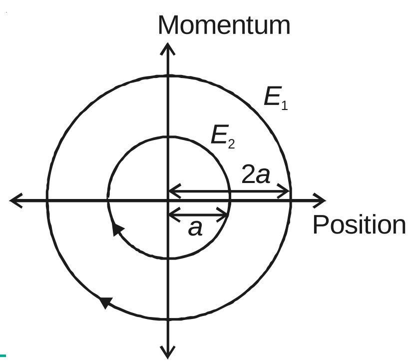
Answer (D)

Hints : $p^2 \propto x$

38. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then



(A) $E_1 = \sqrt{2}E_2$ (B) $E_1 = 2E_2$

(C) $E_1 = 4E_2$ (D) $E_1 = 16E_2$

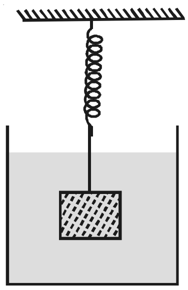
Answer (C)

Hints : $p_1 = 2p_2$

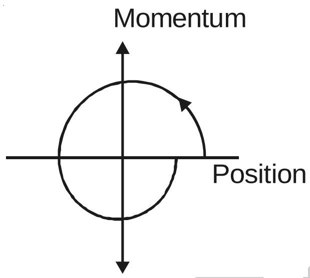
$$\Rightarrow \frac{E_2}{E_1} = \frac{p_2^2}{p_1^2} = \frac{1}{4}$$

$$\Rightarrow E_1 = 4E_2$$

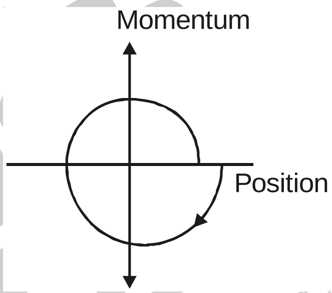
39. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



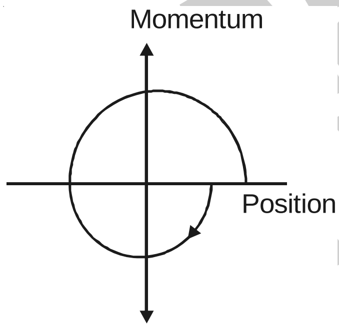
(A)



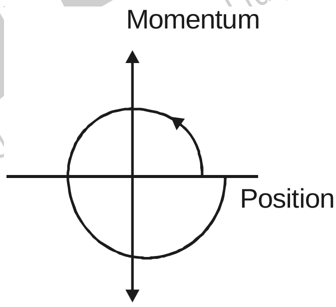
(B)



(C)



(D)



Answer (B)

Hints : Because of viscosity of water, momentum will not be same but less when it returns back.

SECTION - IV (Total Marks : 28)

(Integer Answer Type)

This section contains **7 questions.** The answer to each question is a **Single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

---

40. Steel wire of length 'L' at 40°C is suspended from the ceiling and then a mass '*m*' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length '*L*'. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that $L >>$ diameter of the wire. Then the value of '*m*; in kg is nearly.

Answer (3)

Hints : $$\frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta\ell}{\ell}\right)} = y$$

$$\Rightarrow \frac{mg}{A} = y \times \frac{\Delta\ell}{\ell} \qquad \ldots(i)$$

After cooling

$$\frac{\Delta\ell}{\ell} = \alpha \times 10 \qquad \ldots(ii)$$

So, $\boxed{m = 3\,\text{kg}}$

41. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is

Answer (1)

Hints : $\frac{dN}{dt} = -\lambda N = -10^{10}$

$$\Rightarrow N = 10^{10} \times \left(\frac{1}{\lambda}\right)$$

$m_{total} = N \times m_1$

$= 10^{10} \times 10^9 \times 10^{-25} \times 10^6 mg$

$= 1mg$

42. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\,\mu$, then N is

Answer (5)

Hints : $F_1 = mg(\sin\theta - \mu\cos\theta)$

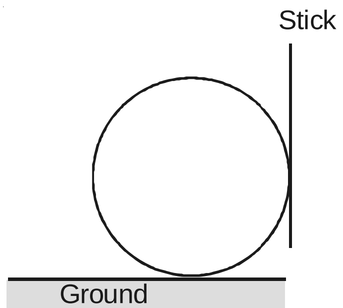
$F_2 = mg(\sin\theta + \mu\cos\theta)$

$= 3F_1$

$$\Rightarrow \mu = \frac{1}{2}\tan\theta$$

$10\mu = N = 5$

43. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is



Answer (4)

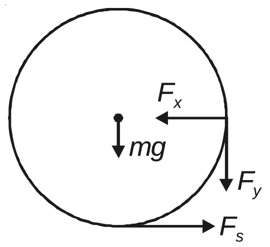
Hints : $F_x^{\,2} + F_y^{\,2} = 2^2 = 4$ …(i)

$(F_x - F_y)r = 2mr^2\alpha = 2mra$

$\Rightarrow F_x - F_y = 2ma = 1.2$ …(ii)

Using $F_y = \dfrac{P}{10} \times F_x$

we get $\boxed{P \simeq 4}$

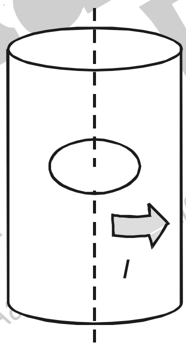


44. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is

Answer (9)

Hints : $I_{AB} = 2\times\left\{\dfrac{2}{5}\dfrac{md}{4}\right\}^2 + 2\times\left\{\dfrac{2}{5}\dfrac{md^2}{4} + \dfrac{ma^2}{2}\right\} = 9 \times 10^{-4}$ kg–m^2

45. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0\cos(300\,t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300\,t)$, then 'N' is



Answer (6)

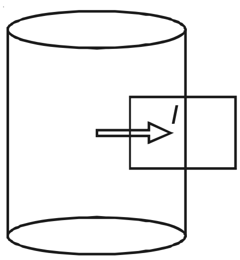
Hints : $B \times \ell = \mu_0 \times \dfrac{I\ell}{L}$

$\Rightarrow B = \dfrac{\mu_0 I}{L}$

$\phi = B \times \pi r^2$

$i = \dfrac{\varepsilon}{R_s} = \dfrac{(d\phi/dt)}{R_s}$

$= \dfrac{\pi r^2 \mu_0}{R_s L}\left(\dfrac{dI}{dt}\right)$



Magnetic moment $= i \times \pi r^2$

$= \dfrac{\pi^2 r^4 \mu_0}{R_s L}\left(\dfrac{dI}{dt}\right)$

$= 6\mu_0 I_0 \sin(300t)$

46. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where '$k$' is a constant. Then N is

Answer (3)

Hints : $(2 \times a \times \gamma) \times c$

$$= 2\times\left\{\frac{kq^2}{a^2}+\frac{Kq^2}{2a^2}\times\frac{1}{\sqrt{2}}\right\}$$

$$\Rightarrow a^3 = k\left\{\frac{q^2}{\gamma}\right\}\left\{1+\frac{1}{2\sqrt{2}}\right\}$$

$$\Rightarrow a^3 = K'\left\{\frac{q^2}{\gamma}\right\}$$

$$\Rightarrow a = K''\left\{\frac{q^2}{\gamma}\right\}^{\frac{1}{3}}$$

PART–III : MATHEMATICS

SECTION - I (Total Marks : 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

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47. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \phi$
(B) $Q \not\subset P$
(C) $P \not\subset Q$
(D) $P = Q$

Answer (D)

Hints : We have

$$\sin\theta - \cos\theta = \sqrt{2}\cos\theta$$

$$\Rightarrow \sin\theta = (\sqrt{2}+1)\cos\theta$$

$$\Rightarrow \frac{1}{\sqrt{2}+1}\sin\theta = \cos\theta$$

$$\Rightarrow (\sqrt{2}-1)\sin\theta = \cos\theta$$

$$\Rightarrow \sqrt{2}\sin\theta - \sin\theta = \cos\theta$$

$$\Rightarrow \sqrt{2}\sin\theta = \sin\theta + \cos\theta$$

and it shows that the given equations

$$\sin\theta - \cos\theta = \sqrt{2}\cos\theta$$

and $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$

are identical. Hence their solution sets are equal

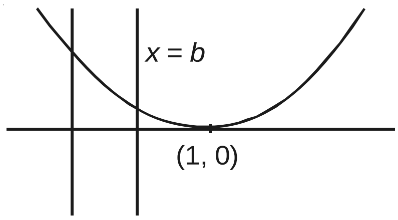
$\Rightarrow$ $P = Q$

48. Le the straight line $x = b$ divide the area enclosed by $y = (1 – x)^2$, $y = 0$ and $x = 0$ into two parts $R_1(0 \le x \le b)$ and $R_2(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Answer (B)

Hints : The given parabola $y = (x – 1)^2$ is as shown below



According to the question,

$$\Rightarrow \int_0^b (x-1)^2 dx - \int_b^1 (x-1)^2 dx = \frac{1}{4}$$

$$\Rightarrow \left[(b-1)^3 + \frac{1}{3}\right]^b - \left[0 - \frac{(b-1)^3}{3}\right] = \frac{1}{4}$$

$$\Rightarrow (b-1)^3 = -\frac{1}{8}$$

$$\Rightarrow b - 1 = -\frac{1}{2}$$

$$\Rightarrow b = 1 - \frac{1}{2} = \frac{1}{2}$$

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49. Let β and α be the roots of $x^2 – 6x – 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n – \beta^n$ for $n \ge 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$

(A) 1 (B) 2 (C) 3 (D) 4

Answer (C)

Hints : We observe that

$(\alpha + \beta)(\alpha^{n-1} – \beta^{n-1}) = \alpha^n – \beta^n + \alpha\beta(\alpha^{n-2} – \beta^{n-2})$

$\Rightarrow 6a_{n-1} = a_n – 2a_{n-2}$

$$\Rightarrow 6 = \frac{a_n - 2a_{n-2}}{a_{n-1}}$$

$$\Rightarrow 3 = \frac{a_n - 2a_{n-2}}{2a_{n-1}}, \forall\, n > 2$$

Putting $n = 10$, we get

$$\frac{a_{10} - 2a_8}{2a_9} = 3$$

50. A straight line L through the point (3, – 2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$

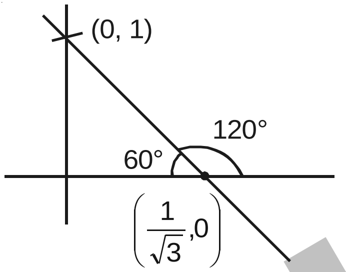
(B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$

(D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

Answer (B)

Hints : The question is too simple from the diagram, the given line



$\sqrt{3}x + y = 1$ makes an angle 120° with x-axis and intersects at $\left(\frac{1}{\sqrt{3}}, 0\right)$. A line making an angle 60° with the given line is either x-axis or different from x-axis. By observation it is clear that the straight line $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$ is the required line.

Second Solution

The equation of the line through (–3, – 2) may be written as

$y + 2 = m(x - 3)$

which will make 60° with $\sqrt{3}x + y = 1$ if

$$\Rightarrow \quad \tan 60° = \left| \frac{m + \sqrt{3}}{1 - \sqrt{3}m} \right|$$

$$\Rightarrow \quad \sqrt{3} = \pm \frac{m + \sqrt{3}}{1 - \sqrt{3}m}$$

$$\Rightarrow \quad m = \sqrt{3} \text{ or } m = 0$$

Since the line intersects x-axis also, hence $m \neq 0$ consequently $m = \sqrt{3}$ and the required line is

$$y \neq 2 = \sqrt{3}(x - 3)$$

$$\Rightarrow \quad y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$$

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51. Let (x_0, y_0) be the solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$
$$3^{\ln x} = 2^{\ln y}$$

Then x_0 is

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) 6

Answer (C)

Hints : We have

$(2x)^{\ln 2} = (3y)^{\ln 3}$

$\Rightarrow$ $(\ln 2)\log_3 2x = (\ln 3)\log_3 3y$

$\Rightarrow$ $\log_{3y} 2x = \log_2 3$

$\Rightarrow$ $(2x) = 3^a$, $(3y) = 2^a$, say

Also $3^{\ln x} = 2^{\ln y}$

$\Rightarrow$ $\ln x = \ln y(\log_3 2)$

$\Rightarrow$ $\log_y x = \log_3 2$

$\Rightarrow$ $x = 2^k$, $y = 3^k$

$\Rightarrow$ $3^a = 2^{k+1}$

$\Rightarrow$ $a = 0$ and $k + 1 = 0$

$\Rightarrow$ $2x = 3^a$

$\Rightarrow$ $x = \frac{1}{2}$

52. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$ is

(A) $\frac{1}{4}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln\frac{3}{2}$ (C) $\ln\frac{3}{2}$ (D) $\frac{1}{6}\ln\frac{3}{2}$

Answer (A)

Hints : We have

$$\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$$

$$= \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin t}{\sin t + \sin(\ln 6 - t)} dt$$

$$\frac{1}{2}\cdot\frac{1}{2}[\ln 3 - \ln 2)] = \frac{1}{4}\ln\frac{3}{2}$$

53. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(A) $\hat{i} - 3\hat{j} + 3\hat{k}$ (B) $-3\hat{i} - 3\hat{j} - \hat{k}$ (C) $3\hat{i} - \hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

Answer (C)

Hints : A vector in the plane of $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ is given by

$\vec{a} + \lambda\vec{b}$

$= (\hat{i} + \hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

$= (1+\lambda)\hat{i} + (1-\lambda)\hat{j} + (1+\lambda)\hat{k}$

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whose projection on $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ is given to be $\frac{1}{\sqrt{3}}$, hence

$$\frac{(1+\lambda)-(1-\lambda)-(1+\lambda)}{\sqrt{3}} = \pm\frac{1}{\sqrt{3}}$$

$\Rightarrow$ $\lambda - 1 = \pm 1$

$\Rightarrow$ $\lambda = 0$ or $\lambda = 2$

But $\lambda = 0$ does not serve our purpose

when $\lambda = 2$, $\vec{v} = 3\hat{i} - \hat{j} + 3\hat{k}$

SECTION - II (Total Marks : 16)

(Multiple Correct Answer Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

54. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y)$, $\forall x, y \in \mathbb{R}$. If $f(x)$ is differentiable at $x = 0$, then

(A) $f(x)$ is differentiable only in a finite interval containing zero

(B) $f(x)$ is continuous $\forall x \in \mathbb{R}$

(C) $f'(x)$ is constant $\forall x \in \mathbb{R}$

(D) $f(x)$ is differentiable except at finitely many points

Answer (B, C)

Hints : We have,

$f(x + y) = f(x) + f(y)$ and $f(x)$ is differentiable at $x = 0$

Clearly $f(x) = kx$ serves our purpose and hence $f(x)$ is continuous for all $x \in \mathbb{R}$ and $f'(x) = k$ = constant.

55. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(A) The equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(B) A focus of the hyperbola is (2, 0)

(C) The eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) The equation of the hyperbola is $x^2 - 3y^2 = 3$

Answer (B, D)

Hints : The equation of the given ellipse is

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

Whose eccentricity $e = \frac{\sqrt{3}}{2}$

Foci of ellipse are $(\sqrt{3}, 0)$ and $(-\sqrt{3}, 0)$

Eccentricity of hyperbola = $\frac{2}{\sqrt{3}}$

Since given hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through $(\sqrt{3}, 0)$, Hence $\frac{3}{a^2}-\frac{0}{b^2}=1$

$\Rightarrow$ $a^2 = 3$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow$ $b^2 = 3\left(\frac{4}{3}-1\right)=1$

Thus, the equation of the hyperbola is

$\frac{x^2}{3}-\frac{y^2}{1}=1$

$\Rightarrow$ $x^2 - 3y^2 = 3$

Foci of the hyperbola are (± 2, 0).

56. Let M and N be two 3 × 3 non-singular skew-symmetric matrices such that $MN = NM$. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

Answer (Statement of the question is incorrect)

Hints : Every skew-symmetric matrix of odd order is always singular and inverse of a singular matrix does not exist. The statement of the given question seems incorrect.

If the matrix is given to be symmetric and non-singular, then

$M^T = M$

$(MN)^T = MN$ (as $MN = NM$ given)

$M^2N^2(M^TN)^{-1}(MN^{-1})^T = M^2N^2(MN)^{-1}((N^T)^{-1}M^T)$

$= M^2N^2(N^{-1}M^{-1})(N^{-1}M)$

$= M^2NM^{-1}N^{-1}M$

$= M^2$

If there may exist a non-singular skew-symmetric matrix (which does not exist), then

$M^2N^2(M^TN)^{-1}(MN^{-1})^T = M^2N^2(-MN)^{-1}((N^{-1})^TM^T)$

$= -M^2NM^{-1}(-N^{-1}(-M)) = -M^2$

57. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are

(A) $\hat{j}-\hat{k}$ (B) $-\hat{j}+\hat{k}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

Answer (A, D)

Hints : Required vector will be parallel or antiparallel to the vectors

$\left(\hat{i}+\hat{j}+\hat{k}\right)\times\left((\hat{i}+\hat{j}+2\hat{k})\times(\hat{i}+2\hat{j}+\hat{k})\right)$

$=4\left(\hat{i}+\hat{j}+2\hat{k}\right)-4\left(\hat{i}+2\hat{j}+\hat{k}\right)$

$=-4\hat{j}+4\hat{k}$

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SECTION - III (Total Marks : 15)
(Paragraph Type)

This section contains 2 paragraphs. Based upon one of the paragraph **2 multiple choice questions** and based upon the second paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions Nos. 58 and 59

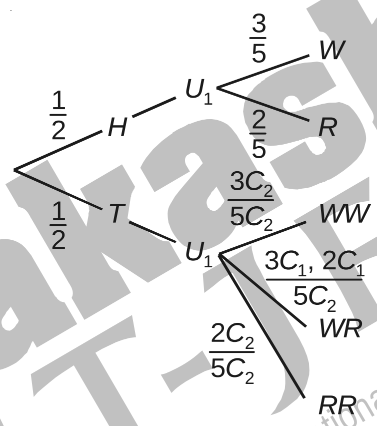
Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

58. The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

Answer (B)

Hints : We have the following tree-diagram under given conditions.



Probability of drawing white ball from U_2

$$= \frac{1}{2}\cdot\frac{3}{5}\cdot 1 + \frac{1}{2}\cdot\frac{2}{5}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{3}{10}\cdot 1 + \frac{1}{2}\cdot\frac{3}{5}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{1}{10}\cdot\frac{1}{3}$$

$$= \frac{3}{10} + \frac{1}{10} + \frac{3}{20} + \frac{1}{5} + \frac{1}{60}$$

$$= \frac{18+6+9+12+1}{60}$$

$$= \frac{23}{30}$$

59. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

Answer (D)

Hints : Using Baye's theorem,

$$\text{Required probability} = \frac{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{2}{5}\times\frac{1}{2}}{\frac{23}{30}}$$

$$= \frac{\frac{4}{10}}{\frac{23}{30}} = \frac{12}{23}$$

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Paragraph for Questions Nos. 60 and 62

Let a, b and c be three real numbers satisfying

$$[a \quad b \quad c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0 \quad 0 \quad 0] \qquad ...(E)$$

60. If the point $P(a, b, c)$, with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

(A) 0 (B) 12 (C) 7 (D) 6

Answer (D)

Hints : From the given condition,

$a + 8b + 7c = 0$

$9a + 2b + 3c = 0$

$7a + 7b + 7c = 0$

$a + b + c = 0$

Also, $2a + b + c = 1$

$\Rightarrow \ a = 1$

$\Rightarrow \ 7a + b + c = 6a + (a + b + c) = 6$

61. Let ω be a solution of $x^3 - 1 = 0$ with $\text{Im}(\omega) > 0$. If $a = 2$ with b and c satisfying (E), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to

(A) –2 (B) 2 (C) 3 (D) –3

Answer (A)

Hints : When $a = 2 \ \Rightarrow \ b + c = -2$

Also, $8b + 7c = -2$

$\Rightarrow \ b = 12$ and $c = -14$

Thus, $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c} = \frac{3}{\omega^2} + \frac{1}{\omega^{12}} + \frac{3}{\omega^{-14}}$

$= 1 + 3\omega + 3\omega^2$

$= 1 - 3$

$= -2$

62. Let $b = 6$, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$

(A) 6 (B) 7 (C) $\frac{6}{7}$ (D) ∞

Answer (B)

Hints : For $b = 6 \ \Rightarrow \ a + c = -6$

$a + 7c = -48$

$a + c = -6$

whence $a = 1$, $c = -7$

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α and β are roots of the given equation $ax^2 + bx + c = 0$, hence $\alpha + \beta = -6$, $\alpha\beta = -7$

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{6}{7}$$

$$\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n=1+\frac{6}{7}+\left(\frac{6}{7}\right)^2+......=\frac{1}{1-\frac{6}{7}}=7$$

SECTION - IV (Total Marks : 28)

(Integer Answer Type)

This section contains **7 questions.** The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

63. Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function such that $f(1) = 2$. If $6\int_1^x f(t)\,dt = 3x\,f(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is

Answer (6)

Hints : From the given condition, we have

$$6\int_1^x f(t)\,dt = 3x\,f(x) - x^3$$

$\Rightarrow$ $6f(x) = 3f(x) + 3xf'(x) - 3x^2$

$\Rightarrow$ $xf'(x) - f(x) = x^2$

$\Rightarrow$ $f'(x) - \frac{1}{x}f(x) = x$

which is linear differential equation, whose I.F. = $e^{-\int \frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

Solution of the given equation is

$$f(x)\cdot\frac{1}{x}=\int x\cdot\frac{1}{x}\,dx + c$$

$\Rightarrow$ $f(x) = x^2 + cx$

Initially, for $x = 1$, $f(1) = 2$ $\Rightarrow$ $c = 1$

$\Rightarrow$ $f(x) = x^2 + x$

$\Rightarrow$ $f(2) = 4 + 2 = 6$

64. If z is any complex number satisfying $|z - 3 - 2i| \leq 2$, then the minimum value of $|2z - 6 + 5i|$ is

Answer (5)

Hints : From the given condition,

$|z - 3 - 2i| \leq 2$

$\Rightarrow$ $|2z - 6 - 4i| \leq 4$

$\Rightarrow$ $4 \geq |(2z - 6 + 5i) - 9i|$

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$\geq || 9i | - | 2z - 6 + 5i ||$

$\Rightarrow \quad 4 \geq 9 - | 2z - 6 + 5i |$

$\Rightarrow \quad | 2z - 6 + 5i | \geq 5$

Minimum value of $| 2z - 6 + 5i |$ is = 5

65. Let $a_1, a_2, a_3, ..., a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \leq p \leq 100$. For any integer n with $1 \leq n \leq 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is

Answer (9)

Hints : We have,

$a_1 = 3, S_p = a_1 + a_2 + a_3 + ... + a_p, 1 \leq p \leq 100$

By the given condition,

$\frac{S_m}{S_n} = \frac{m}{n}\frac{6+(m-1)d}{6+(n-1)d} = k$, which is independent of m and n

$\Rightarrow \quad 5(6 + (m - 1)d) = k(6 + (n - 1)d)$

$\Rightarrow \quad 30 + 25nd - 5d = 6k + nkd - kd$

which is independent of n

Hence, on comparing the coefficients of like terms, we get

$k = 25$

$30 - 5d = 150 - 25d$

$\Rightarrow \quad 20d = 120$

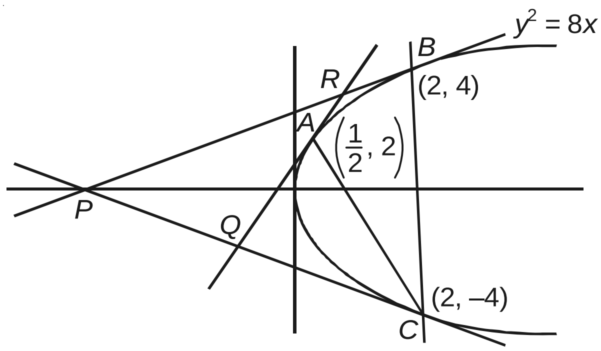
$\Rightarrow \quad d = 6$

$a_2 = 9$

66. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is

Answer (2)

Hints :



Clearly, ar(ΔABC) = 2(arΔPQR)

$\Delta_1 = 2\Delta_2$

$\Rightarrow \quad \frac{\Delta_1}{\Delta_2} = 2$

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67. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

Answer (1)

Hints : We have,

$$f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$$

$$= \sin\left(\tan^{-1}\frac{x}{\sqrt{1-2x^2}}\right); \; x = \sin\theta$$

$$= \frac{x}{\sqrt{1-x^2}}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\frac{df(\theta)}{d\tan\theta} = 1$$

68. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is

Answer (8)

Hints : For $a > 0$,

$$\frac{a^{-5} + a^{-4} + 3a^{-3} + 1 + a^8 + a^{10}}{8} \ge \sqrt[8]{a^{-5} \cdot a^{-4} \cdot a^{-9} \cdot 1 \cdot a^8 \cdot a^{10}}$$

$\Rightarrow$ $a^{-5} + a^{-4} + 3a^{-3} + 1 + a^8 + a^{10} \ge 8$

$\Rightarrow$ $\min(a^{-5} + a^{-4} + 3a^{-3} + 1 + a^8 + a^{10}) = 8$

69. The positive integer value of $n > 3$ satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

Answer (7)

Hints : According to the question,

$$\frac{1}{\sin\frac{\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} + \frac{1}{\sin\frac{3\pi}{n}}$$

$$\Rightarrow \sin\frac{2\pi}{n}\sin\frac{3\pi}{n} = \sin\frac{\pi}{n}\sin\frac{3\pi}{n} + \sin\frac{\pi}{n}\sin\frac{2\pi}{n}$$

$$\Rightarrow \cos\frac{\pi}{n} - \cos\frac{5\pi}{n} = \cos\frac{2\pi}{n} - \frac{4\pi}{n} + \cos\frac{\pi}{n} - \cos\frac{3\pi}{n}$$

$$\Rightarrow \sin\frac{9\pi}{2n} \times \sin\frac{\pi}{2n} = \sin\frac{5\pi}{2n} \cdot \sin\frac{\pi}{n}$$

$$\Rightarrow \sin\frac{9\pi}{2n} = \sin\frac{5\pi}{2n}$$

$$\Rightarrow \frac{9\pi + 5\pi}{2n} = (2k+1)\pi, \text{ say}$$

$$\Rightarrow n = \frac{7}{2k+1}, \; k \text{ is an integer}$$

Let us put $k = 0$

Hence $n = 7$

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