Total No. of Questions—12] [Total No. of Printed Pages—8+2

[3762]-601

S.E. (Comp.) (I Sem.) EXAMINATION, 2010

DISCRETE STRUCTURE

(2003 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** from Section I and Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12** from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Assume suitable data, if necessary.

SECTION I

1. (*a*) A survey has been taken on methods of computer travel. Each respondent was asked to check bus, train or automobile as a major method of travelling to work. More than one answer was permitted. The result reported were as follows :

Bus–30 people, train–35 people, automobile–100 people, bus and train–15 people, bus and automobile–15 people, train and automobile–20 people and all three methods–5 people. How many people completed a survey form ? [6]

P.T.O.

(*b*) Obtain conjunctive normal form of each of the following :

(*i*) $p \wedge (p \rightarrow q)$

(*ii*) $\sim (p \vee q) \leftrightarrow (p \wedge q)$

(*iii*) $(p \rightarrow q) \wedge (q \rightarrow p)$. [6]

(*c*) Using Venn diagrams, show that :

(*i*) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(*ii*) $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$. [6]

Or

2. (*a*) Using Mathematical induction, prove that : [6]

$$1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots(-1)^{n-1} n^2 = (-1)^{n-1} \cdot \frac{n(n+1)}{2}$$

(*b*) Translate the following into logical notations :

(*i*) For any value of x, x^2 is non-negative

(*ii*) For every value of x, there is some value of y such that $x - y = 1$

(*iii*) There are positive values of x and y such that $x \cdot y \geq 0$. [3]

(*c*) Negate each of the following statements :

(*i*) $\forall x, |x| = x$

(*ii*) $\exists x, x^2 = x$

(*iii*) If there is not, then someone is killed. [3]

(*d*) For multisets, define in brief :

(*i*) Multisets

(*ii*) Multiplicity of an element in a multiset

(*iii*) Cardinality of multiset

(*iv*) Union of multiset

(*v*) Intersection of multiset

(*vi*) Difference of multiset. [6]

3. (*a*) A menu card in a restaurant displays four soups, five main courses, three deserts and 5 beverages. How many different menus can a customer select if :

(*i*) He selects one item from each group without omission.

(*ii*) He chooses to omit the beverages, but selects one each from the other group.

(*iii*) He chooses to omit the deserts, but decides to take a beverage and one item each from the remaining groups. [6]

(*b*) How many automobile licence plates can be made if each plate consists of different letters followed by three different digits. Solve the problem if first digit cannot be 0. [6]

(c) A pair of fair dice is thrown. If the two numbers appearing are different, find the probability P that

(i) the sum is 6

(ii) an ace appears. [4]

Or

4. (a) Show that :

$$ {}^{n}C_{1} + 6({}^{n}C_{2}) + 6({}^{n}C_{3}) = n^{3}. \quad [6]$$

(b) A fair coin is thrown 10 times. Find the probability of getting exactly 6 heads and at least 6 heads. [4]

(c) Define :

(i) Trial and event

(ii) Exhaustive events and sample space

(iii) Favourable events

(iv) Mutually exclusive events

(v) Equally likely events

(vi) Independent events. [6]

5. (a) Find the transitive closure of R be Warshall's algorithm where

$$A = \{1, 2, 3, 4, 5, 6\} \text{ and}$$

$$R = \{(x - y);\ |x - y| = 2\}. \quad [6]$$

(b) Draw the Hasse diagram of the following sets under the partial ordering relation 'divides' and indicate those which are chains :

(i) $\{2, 4, 12, 24\}$

(ii) $\{1, 3, 5, 15, 30\}$. [4]

(c) Explain the following with example :

(i) Reflexive relation

(ii) Symmetric relation

(iii) Antisymmetric relation

(iv) Tansitive relation. [6]

Or

6. (a) Let functions f and g be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$ respectively. Find :

(i) gof (4) and $fog(4)$

(ii) $gof(a + 2)$

(iii) $fog(a + 2)$. [6]

(b) Let $X = \{1, 2, \ldots\ldots.7\}$ and $R = \{(x, y) |_{x-y}$ is divisible by $3\}$ Show that R is equivalence relation. Draw graph of R. [6]

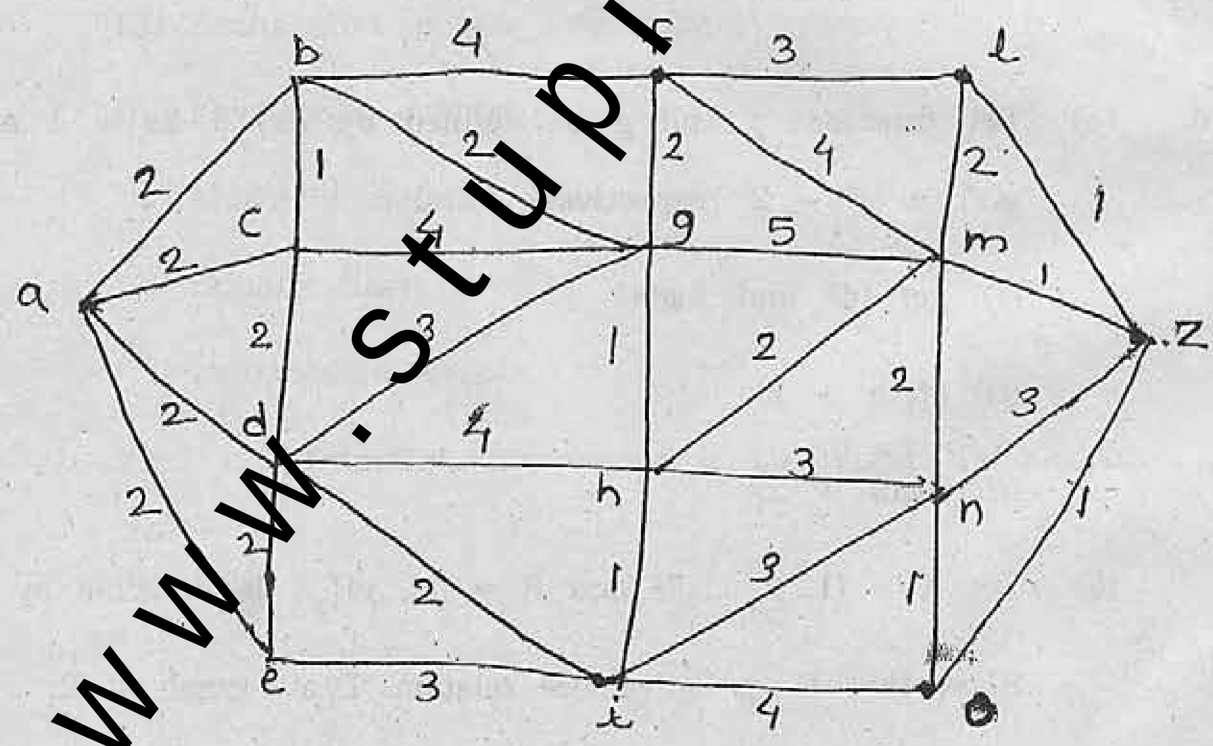
(c) Find the numeric function for :

$$A(z) = \frac{2}{1 - 4z^2}.$$ [4]

SECTION II

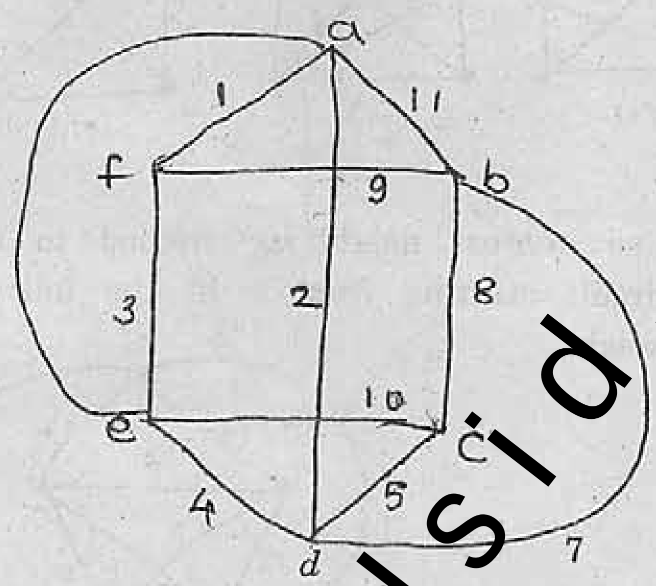
7. (a) How many nodes are necessary to construct a graph with exactly 8 edges in which each node is of degree 2. [6]

(b) State the Dijkstra's algorithm to obtain the shortest path (distance) between two vertices in the given graph and apply the same to obtain the shortest path between a to z in the following graph : [12]

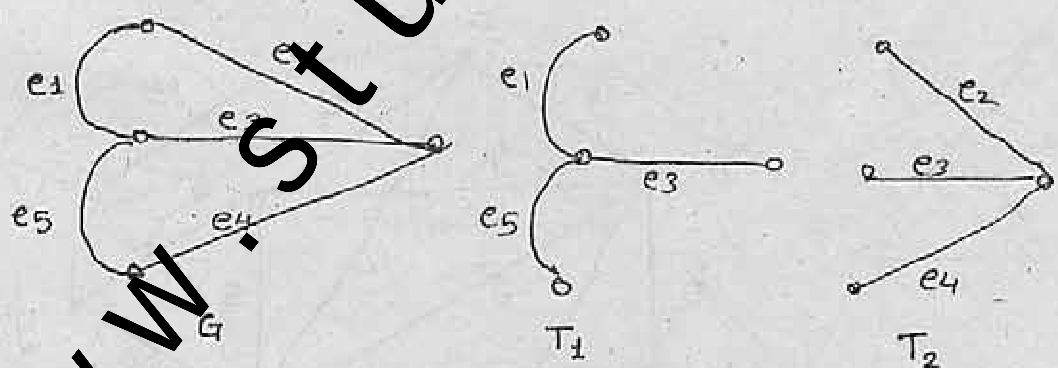


Or

8. (*a*) Determine minimum spanning tree for the given graph using Prim's algorithm. [6]

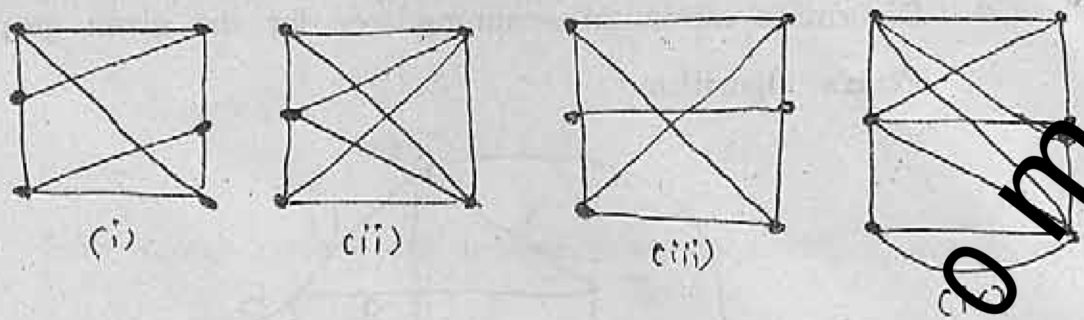


(*b*) Draw fundamental cut-sets and union of edge disjoint fundamental cut-sets of graph G with respect to trees T_1 and T_2 as shown below : [6]

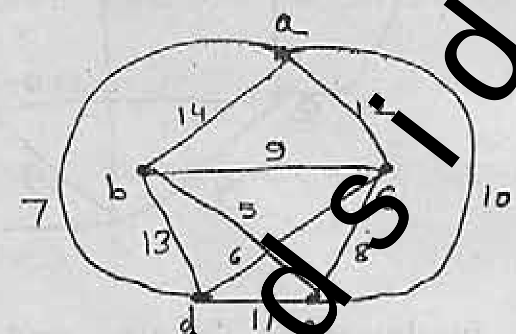


(*c*) For the following set of weights construct an optimal binary prefix code. For each weight in the set give corresponding code word. 5, 7, 8, 15, 35, 40. [6]

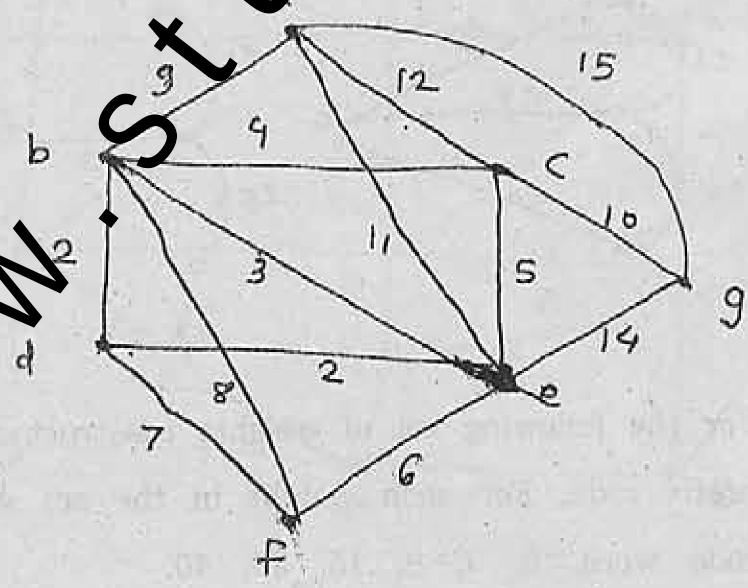
9. (*a*) Identify whether the graph given are planar or not. [4]



(*b*) Use nearest neighbour method to find the Hamiltonian circuit starting from *a* in the following graph. Find its weight. [6]

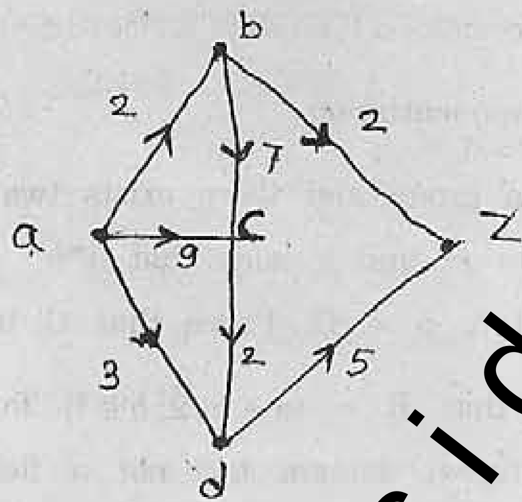


(*c*) Give the stepwise construction of minimum spanning tree for the following graph using Prim's algorithms. [6]

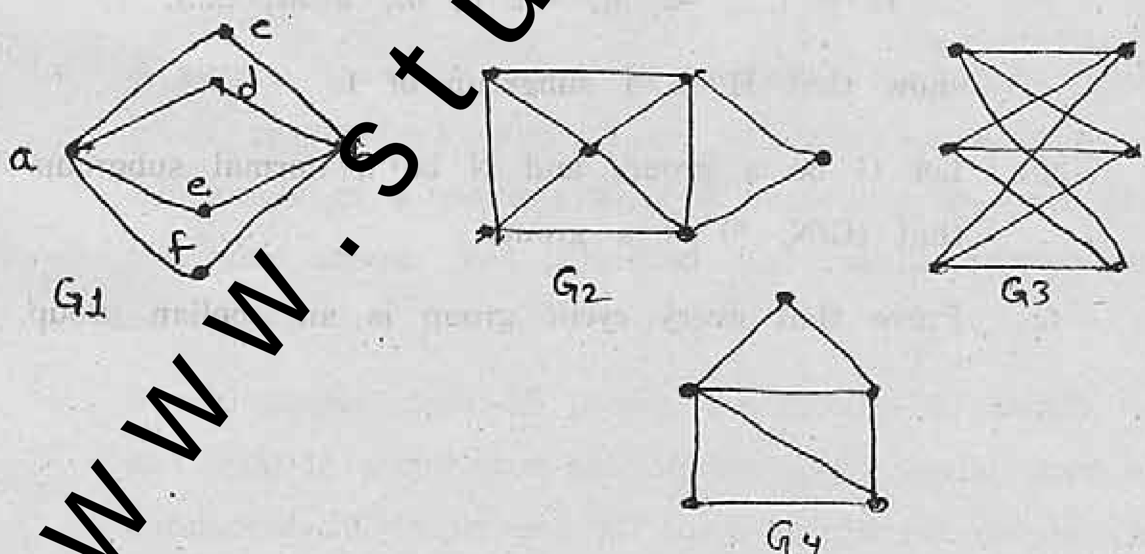


Or

10. (*a*) Determine the maximal flow in the following transport network. [6]



(*b*) Determine which of the graphs of the given figure represent Eulerian circuit, Hamiltonian circuit, Bipartite graphs and planar graph. Justify your answer. [8]



(c) Draw two non-isomorphic trees with six points. [2]

11. (a) Determine whether or not the following operations on the set of integers I are associate :

(i) Division

(ii) Exponentiation. [4]

(b) G is a group and there exists two relatively prime positive integers m and n such that $a^m b^m = b^m a^m$ and $a^n b^n = b^n a^n$ for all $a, b \in$ G. Prove that G is an abelian group. [6]

(c) Show that R = $\{a + b\sqrt{2}; b \in \text{I}\}$ for the operation +, × is an integral domain but not a field. [6]

Or

12. (a) I is a group of integers under addition, H is a subset of I consisting of all multiples of a positive integer m : that is

$$\text{H} = \{...., -2\ m, -m\ 0, m, 2\ m, ...\},$$

show that H is a subgroup of I. [6]

(b) Let G be a group, and N be a normal subgroup. Prove that (G/N, *) is a group. [6]

(c) Prove that every cyclic group is an abelian group. [4]