Total No. of Questions—12] [Total No. of Printed Pages—8+4

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S.E. (Comp/IT) (I Sem.) EXAMINATION, 2010

DISCRETE STRUCTURES

(2008 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Attempt from Section I Q. **1** *or* Q. **2**, Q. **3** *or* Q. **4**, Q. **5** *or* Q. **6** from Section II Q. **7** *or* Q. **8**, Q. **9** *or* Q. **10**, Q. **11** *or* Q. **12**.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Prove by induction that for all $n >= 1$

$$1.2 + 2.3 + \ldots\ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$ [6]

(*b*) (*i*) Given that the value of $p \to q$ is false. Determine the value of $(\sim p \vee \sim q) \to q$.

(*ii*) Given that the value of $p \to q$ is true. Can you determine the value of $\sim p \vee (p \leftrightarrow q)$? [4]

P.T.O.

(c) In a survey of 260 college students, the following data were obtained :

64 had taken a mathematics course, 94 had taken a computer science course, 58 had taken a business course, 28 had taken both mathematics and business course, 26 had taken both mathematics and computer science course, 22 had taken both computer science and business course and 14 had taken all 3 types of courses.

(1) How many students were surveyed who had taken none of the three types of courses ?

(2) Of the students surveyed, how many had taken only a computer science course ? [6]

Or

2. (a) Prove the following using Venn diagram :

(i) $A - (A - B) \subseteq B$

(ii) $A - B = A \cap \bar{B}$. [6]

(b) Use p : Today is Monday.

q : The grass is wet.

r : The dish ran away with the spoon.

Write an English sentence that corresponds to each of the following :

(1) $\sim r \wedge q$

(2) $\sim q \vee r$

(3) $\sim(p \vee q)$

(4) $p \vee \sim r$. [6]

(c) Determine whether the argument given is valid or not.

If I try hard and I have talent, then I will become a musician.

If I become a musician, then I will be happy.

Therefore if I will not be happy, then I did not try hard or I do not have talent. [4]

3\. (a) Consider the binary relation * defined on the set A = {a, b, c, d} by the following table :

*	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Is * commutative ? Associative ? [4]

(b) Let $(s_1, *_1)$, $(s_2, *_2)$ and $(s_3, *_3)$ be semigroups and $f : s_1 \to s_2$ and $g : s_2 \to s_3$ be homomorphisms. Prove that $g \circ f$ is homomorphism from s_1 to s_3. [6]

(c) Define the following terms :

(1) Field

(2) Abelian group

(3) Subgroup

(4) Homomorphism

(5) Monoid

(6) Associative operation. [6]

Or

4. (a) Let A = {a, b}. Which of the following tables define a semigroup on A ? Which define a Monoid on A ?

(i)

*	a	b
a	a	b
b	a	a

(ii)

*	a	b
a	a	b
b	b	b

[4]

(*b*) Let G be a group and let a be a fixed element of G. Show that the function $f_a : G \to G$ defined by $f_a(x) = a \times a^{-1}$, for $x \in G$ is an isomorphism. [6]

(*c*) Let R be a commutative ring with additive identity O and multiplicative identity 1, then prove that :

(*i*) For any x in R, $O * x = O$

(*ii*) For any x in R, $-x = (-1) * x$. [6]

5. (*a*) Let R be a binary relation on the set of all positive integers such that $R = \{(a, b) \mid a - b \text{ is an odd positive integer}\}$. Is R reflexive ? Symmetric ? Antisymmetric ? Transitive ? An Equivalence relation ? A Partial ordering relation ? [6]

(*b*) Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3\}$. Given the matrices M_r and M_s of the relations R and S from A to B, compute :

(1) $M_{R \cap S}$

(2) $M_{R \cup S}$

(3) $M_{R^{-1}}$

(4) $M_{\bar{s}}$. [4]

(c) Define the following with suitable example :

(1) One-to-one function

(2) Onto function

(3) Identity function

(4) Invertible function. [4]

(d) Let A = {1, 2, 3} and consider two reflexive relations R = {(1, 1), (1, 2), (1, 3), (2, 2), (3, 3)} and S = {(1, 1), (1, 2), (2, 2), (3, 2), (3, 3)}. Determine the following relations are reflexive or irreflexive :

(1) R^{-1}

(2) $\bar{R}$

(3) $R \cap S$

(4) $R \cup S$. [4]

Or

6. (a) Let A = B = C = R and let $f : A \to B$, $g : B \to C$ be defined by $f(a) = a - 1$ and $g(b) = b^2$. Find :

(1) $(f \circ g)\ (2)$

(2) $(g \circ f)\ (2)$

(3) $(g \circ f)\ (x)$

(4) $(f \circ g)\ (x)$

(5) $(f \circ f)\ (y)$

(6) $(g \circ g)\ (y)$. [6]

(b) For the relation R whose matrix is given, find the matrix of transitive closure using Warshall's Algorithm :

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$ [6]

(c) Draw the Hasse diagram for the relation R on A = {1, 2, 3, 4, 5} whose relation matrix is given below :

$$M_R = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$ [6]

SECTION II

7. (a) State and prove Euler's formula for a connected plane graph of order *n*, size *e* and with *f* faces. [6]

(b) Show that every planar graph of order less than 12 has a vertex of degree at most 4. [3]

(*c*) Let G, of order n, be a connected 3-regular plane graph in which every vertex lies on one face of length 4, one face of length 6 and one face of length 8. Determine the number of faces of G. [6]

(*d*) Show that $K_{n,\ n}$ is Hamiltonian if and only if $n > 2$. Hence show that $K_{n,\ n}$ has :

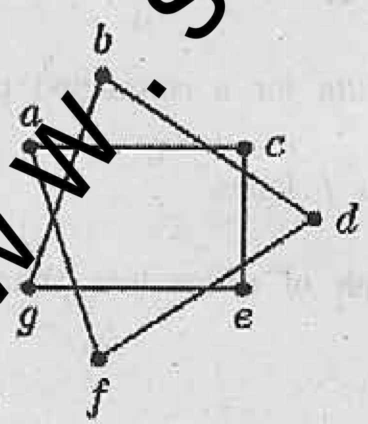
$$\frac{n!(n-1)!}{2}$$

Hamiltonian cycles. [3]

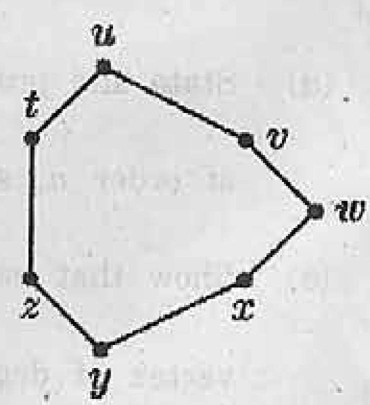
Or

8. (*a*) The graphs G and H, with vertex-sets V(G) and V(H), are drawn below. Determine whether or not G and H drawn below are isomorphic. If they are isomorphic, give a function $g : V(G) \to V(H)$ that defines the isomorphism. If they are not, explain why they are not. [6]

G :



H :

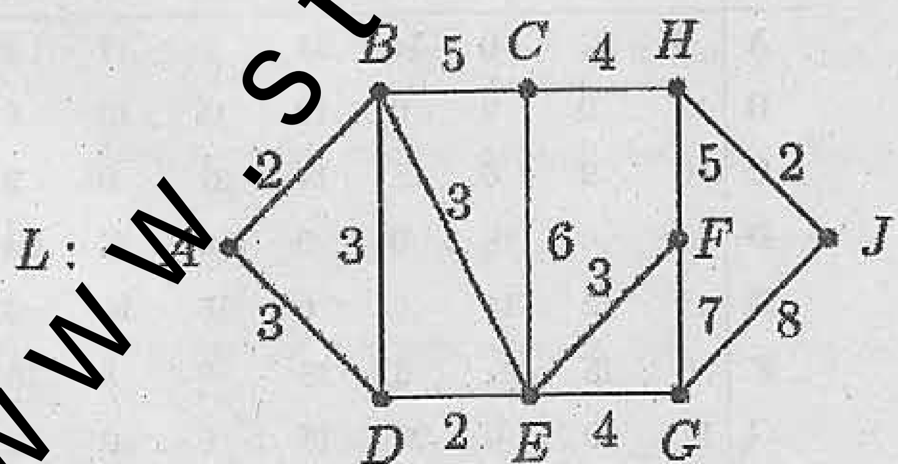


(*b*) What is the maximum number of edges in a simple graph on n vertices ? [3]

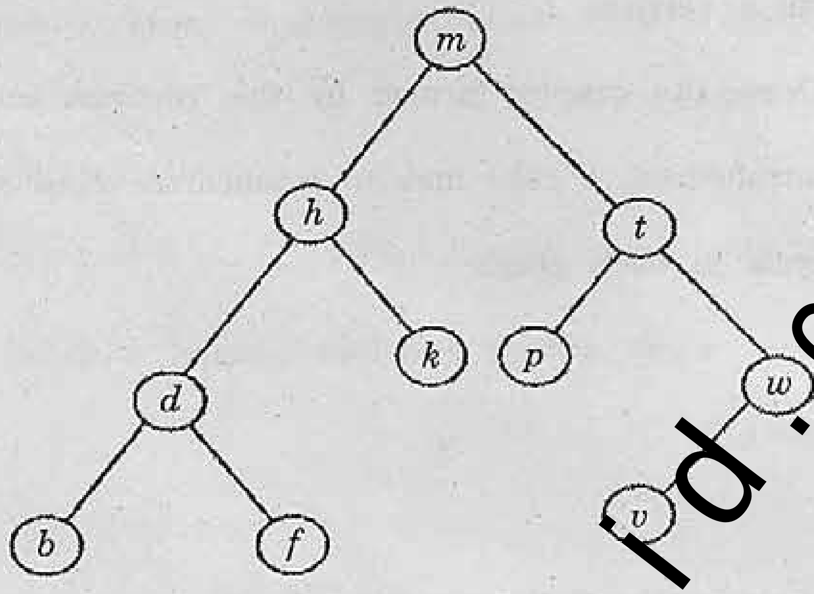
(*c*) Draw the graphs formed by the vertices and edges of a tetrahedron, a cube and an octahedron. Find a Hamiltonian cycle in each graph. [6]

(*d*) How many simple labelled graphs with n vertices are there ? [3]

9. (*a*) Describe Kruskal's algorithm for finding a minimum weight spanning tree for an edge weighted graph. Use the Kruskal's algorithm to find a minimum weight spanning tree for the weighted graph. Write down the weight of the minimum weight spanning tree. [6]



(*b*) Find the preorder, postorder and inorder traversal of the following tree : [6]



(*c*) Construct the labeled tree of the algebraic expression :

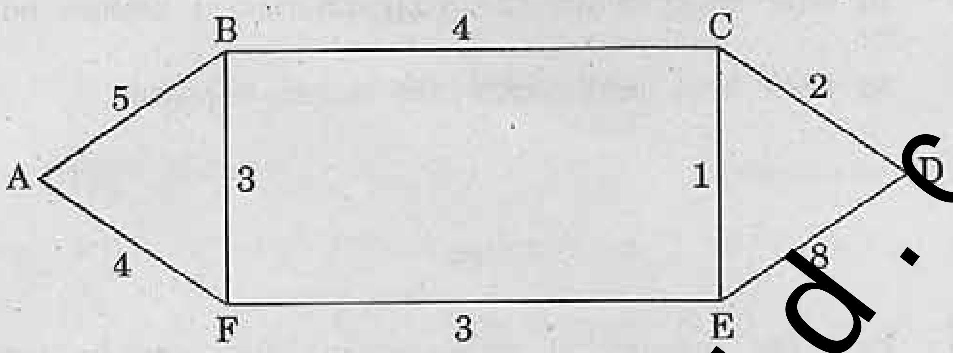
$$(((x + y) * z)/3) + (19 + (x * x)).$$ [4]

Or

10. (*a*) Find a minimum cost spanning tree for the graph with this cost matrix. How many such trees are there ? [6]

	A	B	C	D	E	F	G	H
A	0	12	0	14	11	0	17	8
B	12	0	9	0	12	15	10	9
C	0	9	0	18	14	31	0	9
D	14	0	18	0	0	6	23	14
E	11	12	14	0	0	15	16	0
F	0	15	31	6	15	0	8	16
G	17	10	0	23	16	8	0	22
H	8	9	9	14	0	16	22	0

(*b*) For the network shown below determine the maximum flow between the vertices A and D by identifying the cut set of maximum capacity as shown in the figure below : [6]



(*c*) A binary search tree generated by inserting integer in order 50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24. [4]

11. (*a*) (*i*) In how many ways can 6 men and 5 women be seated in a line so that no two women sit together ? [3]

(*ii*) In how many ways can 6 men and 5 women sit in a line so that women occupy the even places ? [3]

(*b*) (*i*) A family of 4 brothers and 3 sisters is to be arranged in a row for a photograph. In how many ways can they be seated if all sisters are together ? [3]

(*ii*) Given 6 flags of different colours, how many different signals can be generated, if signal requires the use of two flags one below other ? [3]

(*c*) In how many ways can 10 examination papers be arranged so that best and worst are never together ? [4]

Or

12. (*a*) Find the number of arrangements that can be made out of the letters :

(*i*) ASSASSINATION

(*ii*) GANESHPURI. [6]

(*b*) In how many ways can three prizes be distributed among 4 boys when :

(*i*) No one gets more than one prize

(*ii*) A boy can get number of prizes. [6]

(*c*) How many words with or without meanings can be formed using all the letters of the word EQUATION using each letter exactly once ? [4]