Total No. of Questions—**12**] [Total No. of Printed Pages—**8+4**

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SE (I.T. & Comp.) (First Semester) EXAMINATION, 2010

DISCRETE STRUCTURES

(2008 Course)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Attempt from Section I Q. **1** *or* Q. **2**, Q. **3** *or* Q. **4**, Q. **5** *or* Q. **6**. Attempt from Section II Q. **7** *or* Q. **8**, Q. **9** *or* Q. **10**, Q. **11** *or* Q. **12**.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Prove by induction for $n \geq 0$ [6]

$$1 + a + a^2 + ... + a^n = \frac{1 - a^{n+1}}{1 - a}.$$

(*b*) In a survey of 60 people it was found that : [6]

25 read Business India

26 read India Today

26 read Times of India

11 read both Business India and India Today

P.T.O.

9 read both Business India and Times of India

8 read both India Today and Times of India

8 read none of these.

(*i*) How many read all three ?

(*ii*) How many read exactly one ?

(*c*) Prove that $[(p \to q) \wedge (r \to s) \wedge (p \vee r)] \to (q \vee s)$ is a tautology. [4]

Or

2. (*a*) Let P and Q be 2 multisets.

$P = \{a, a, a, c, d, d\}$ and $Q = \{a, a, b, c, c\}$. Find : [4]

(*i*) $P \cup Q$

(*ii*) $P \cap Q$

(*iii*) $P - Q$

(*iv*) $P + Q$

(*b*) $P(x)$: x is even. [6]

$Q(x)$: x is a prime number.

$R(x, y)$: $x + y$ is even.

(1) Using above write an English sentence for each of the symbolic statement given below :

(*i*) $\forall x \, (\sim Q(x))$

(*ii*) $\exists y \, (\sim P(y))$

(*iii*) $\sim (\exists x \, (P(x) \quad Q(x)))$

(2) Using the information given above write the following English sentences in symbolic form :

(*i*) The sum of any two integers is an odd integer

(*ii*) Every integer is even or prime

(*iii*) Every integer is an odd integer.

(*c*) Find the CNF and DNF for the following : [4]

(*i*) $(p \rightarrow q) \wedge (q \rightarrow p)$

(*ii*) $((p \wedge (p \rightarrow q)) \rightarrow q)$

(*d*) Define power set.

List all elements of the set p(A) XA where A = $\{a, b, c\}$. [2]

3. (*a*) Show that (I,) is a commutative ring with identity. Where + and □ are defined as : [6]

A $b = a + b - 1$ and a $b = a + b - ab$.

(*b*) Let Z_n denote the set of Integers as $\{1, ..., n-1\}$. Construct the multiplication table for with $n = 6$. Is $(Z_n, \quad)$ [6]

Where □ is a binary operation on Z_n such that a □ b = remainder of ab divided by n. Is Z_n an abelian group ?

(*c*) Let G be a group of real nos under addition and be the group of +ve real nos under multiplication. Let $f : G \rightarrow$ be defined as $f(x) = e^x$. Show that f is an isomorphism. [4]

Or

4. (*a*) Define : [4]

(*i*) Subgroup

(*ii*) Cyclic Group

(*iii*) Integral domain

(*iv*) Field

(*b*) Prove the following results for the group G : [6]

(*i*) The identity element is unique.

(*ii*) Each a in G has a unique inverse a^{-1}.

(*iii*) $ab = ac$ implies $b = c$.

(*c*) Consider the (3, 6) encoding function e : [6]

$e(001) = 000000$

$e(001) = 000110$

$e(010) = 010010$

$e(011) = 010100$

$e(100) = 100101$

$e(101) = 100011$

$e(110) = 110111$

$e(111) = 110001$

Show that e is a group code.

5. (*a*) Let A = B be the set of real nos. [6]

$f : A \rightarrow B$ given by $f(x) = 2x^3-1$

$g : B \rightarrow A$ given by $g(y) =$

Show that f is a bijection between A and B and g is bijection between B and A.

(*b*) For each of these relations on set $A = \{1, 2, 3, 4\}$ decide whether it is reflexive, symmetric, transitive or antisymmetric. (one relation may satisfy more than one properties). [6]

$R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

$R_2 = \{(1, 1), (1, 2), (2, 2), (2, 1), (3, 3), (4, 4)\}$

$R_3 = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

(*c*) Determine whether the poset represented by each of the Hasse diagram are lattices. Justify your answer. [6]

$\sqrt[3]{\frac{1}{2} y + \frac{1}{2}}$

h

Or

6. (*a*) Find the solution to the recurrence relation

$$a_n = 6\, a_{n-1} - 11\, a_{n-2} + 6a_{n-3}$$

with initial condition $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$. [6]

(*b*) A = {1, 2, 3, 4, 5} and R and S be equivalent relations on A whose matrices are given below. Compute the matrix of smallest relation containing R & S. [6]

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \qquad M_s = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(*c*) Define with examples : [6]

(*i*) Poset

(*ii*) Lattice

(*iii*) Complemented Lattice.

SECTION II

7. (*a*) Which of the following graphs have a Euler circuit or path or Hamiltonian cycle ? Write the path *or* circuit : [6]

d

(*b*) Determine whether graphs G and H are isomorphic or not. Justify your answer. [6]

(*c*) Find the shortest path from a to z in the following graph. [6]

Or

8. (*a*) State and prove Euler's formula for a connected planar graph of order n, size e and with f faces. [6]

(*b*) Define the following with suitable example : [6]

(*i*) Cut set

(*ii*) Factors of graph

(*iii*) Weighted graph.

(*c*) Identify whether the graphs given are planar or not. Draw planar representation if possible : [6]

A

9. (*a*) A binary tree has 10 nodes. The inorder and preorder traversals of the trees are as shown below. Construct the binary tree. [6]

Inorder : ABCEDFJGIH

Preorder : JCBADEFIGH

(*b*) Convert the following tree into binary tree. [4]

(*c*) Using Prim's algorithm construct minimal spanning tree starting at vertex a. [6]

Or

10. (*a*) Find the maximum flow in the transport network given below : [6]

(*b*) Construct the expression tree for the following expression. $(3 - (2(-11 - (9 - 4)))) \div (2 + (3 + (4 + 7)))$. [4]
Also evaluate the expression.

(*c*) Using Kruskal's algorithm construct minimal spanning tree. [6]

11. (*a*) A single card is drawn from an ordinary deck of 52 cards. Find the probability p that : [3]

(*i*) the card is a face card

(*ii*) the card is face card and heart

(*iii*) the card is face card or heart.

(*b*) How many seven letter words can be formed using the letters of the word BENZENE ? [3]

(*c*) Two dice are tossed once. Find the probability of getting an even number on first or a total of 8. [4]

(*d*) If repetitions are not permitted, how many four digit numbers can be formed from digits 1, 2, 3, 7, 8, and 5. [6]

Or

12. (*a*) How many ways can the letters in the word MISSISSIPPI be arranged ? What if P's are to be separated ? [4]

(*b*) Show that :

$C\ (2n,\ 2) = 2C(n,\ 2) + n^2.$ [6]

(*c*) A pair of fair dice is thrown. Find the probability p that the sum is 10 or greater if : [3]

(*i*) 5 appears on first die

(*ii*) 5 appears on at least one die.

(*d*) A coin is tossed 3 times. Find the probability that there will appear : [3]

(*i*) Three heads

(*ii*) Exactly 2 heads

(*iii*) No heads.