Sample Questions – Theoretical Computer Science

Unlike in Physics, there is only one paper. The test will focus on the following areas:
Analytical Reasoning and Deduction, Combinatorics, Data Structures and Algorithms, Discrete Mathematics, Graph Theory, Principles of Programming

In each of these areas, familiarity with the basics (including the necessary simple mathematics) is assumed and will be tested. There will be questions of both varieties – some requiring short answers as well as some involving detailed problem solving. Some text books which may help the candidate prepare for the test are listed below. The candidate is not expected to read all the books. There is no specified "portion" for the test; rather, the test is designed to check the applicant's understanding of foundational aspects of computing.

Suggested texts:
1. Elements of Discrete Mathematics - C.L. Liu
2. Discrete Mathematical Structures with Applications to Computer Science - Jean-Paul Tremblay and Ram P. Manohar
3. Compilers: Principles, Techniques and Tools - Alfred V. Aho, Ravi Sethi and Jefferey D. Ullman
4. Fundamentals of Data Structures - Ellis Horowitz and Sartaj Sahni
5. An Introduction to Data Structures with Applications - Jean-Paul Tremblay and P.G. Sorenson
6. Fundamentals of Computer Algorithms - Ellis Horowitz, Sartaj Sahni and S. Rajasekaran
7. The Design and Analysis of Computer Algorithms - Alfred V. Aho, John E. Hopcroft and Jefferey D. Ullman
8. Introduction to Algorithms - Thomas H. Cormen, Charles E. Leiserson and Ronald L. Rivest
9. How to Solve it by Computer - R.G. Dromey
10. Programming Languages - Concepts and Constructs, Ravi Sethi

Sample Questions for JEST inTheoretical Computer Science:

1. Select the correct alternative in each of the following:
    (a) Let a and b be positive integers such that a > b and $a^2 - b^2$ is a prime number. Then $a^2 - b^2$ is equal to
        (A) a - b   (B) a + b   (C) a $\times$ b   (D) none of the above
    (b) When is the following statement true? (A [B) \ C = A \ C
        (A) If $^-$ A \ B \ C = _   (B) If A \ B \ C = _   (C) always   (D) never
    (c) If a fair die (with 6 faces) is cast twice, what is the probability that the two numbers obtained di_er by 2?
        (A) 1/12   (B) 1/6   (C) 2/9   (D) 1/2
    (d) T(n) = T(n/2) + 2; T(1) = 1. When n is a power of 2, the correct expression for T(n) is:
        (A) 2(log n + 1)   (B) 2 log n   (C) log n + 1   (D) 2 log n + 1

2. Consider the following function, defined by a recursive program:

```
function AP(x,y: integer) returns integer;
if x = 0 then return y+1
else if y = 0 then return AP(x-1,1)
else return AP(x-1, AP(x,y-1))
```

(a) Show that on all nonnegative arguments x and y, the function AP terminates.
(b) Show that for any x, AP(x, y) > y.

3. How many subsets of even cardinality does an n-element set have ? Justify answer.

4. A tournament is a directed graph in which there is exactly one directed edge between every pair of vertices. Let Tn be a tournament on n vertices.

(a) Use induction to prove the following statement:
Tn has a directed hamiltonian path (a directed path that visits all vertices).

(b) Describe an algorithm that finds a directed hamiltonian path in a given tournament. Do not write whole programs; pseudocode, or a simple description of the steps in the algorithm, will suffice. What is the worst case time complexity of your algorithm?

5. Describe two different data structures to represent a graph. For each such representation, specify a simple property about the graph that can be more efficiently checked in that representation than in the other representation. Indicate the worst case time required for verifying both of your properties in either representation.

6. Two gamblers have an argument. The first one claims that if a fair coin is tossed repeatedly, getting two consecutive heads is very unlikely. The second, naturally, is denying this. They decide to settle this by an actual trial; if, within n coin tosses, no two consecutive heads turn up, the first gambler wins.

(a) What value of n should the second gambler insist on to have more than a 50% chance of winning?

(b) In general, let P(n) denote the probability that two consecutive heads show up within n trials. Write a recurrence relation for P(n).

(c) Implicit in the second gambler's stand is the claim that for all sufficiently large n, there is a good chance of getting two consecutive heads in n trials; i.e. P(n) > 1/2. In the first part of this question, one such n has been demonstrated. What happens for larger values of n? Is it true that P(n) only increases with n? Justify your answer.

7. Consider the following program:

```
function mu(a,b:integer) returns integer;
var i,y: integer;
begin
---------P----------
i = 0; y = 0;
while (i < a) do
```

```
    begin --------Q------------
    y := y + b ;
    i = i + 1
    end
    return y
    end
```
Write a condition P such that the program terminates, and a condition Q which is
 true whenever program execution reaches the place marked Q above.