Subject: INFORMATION THEORY & CODING

Time: 3 Hours **JUNE 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The probability that a student passes Mathematics is 2/3 & that of Biology is 4/9. If the probability of passing at least one is 4/5, what is the probability that he will pass both courses.

(A) 14/45 **(B)** 45/14
(C) 0 **(D)** 1

b. The percentage of alcohol in a compound may be considered as a random variable X, where $0 \le X \le 1$, having the PDF $f(X) = 20X^3(1-X)$. Then the value of $P\left(X \le \frac{2}{3}\right)$ is

(A) 0.9064 **(B)** 0.4609
(C) 1 **(D)** 0

c. The probability distribution of a random variable 'X' is given by $P(X = x) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{3-x}, x = 0,1,2,3,$ then the mean of 'X' is

(A) 1.875 **(B)** 2.875
(C) 0 **(D)** 1

d. The binary symbols 0 & 1 are transmitted with probabilities $\frac{1}{4}$ & $\frac{3}{4}$ respectively. The corresponding self information are

(A) 2 bits & 0.415 bits **(B)** 0 & 1 bits
(C) 1 & 0 bits **(D)** 0 & 0 bits

e. A source S = {S_1, S_2, S_3} emits symbols with $P = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right\}$. The total information of all the messages is

(A) 2 bits **(B)** 3 bits
(C) 4 bits **(D)** 5 bits

f. Equivocation with respect to a channel is a measure of

(A) Certainty | **(B)** Uncertainty
(C) Probability | **(D)** Statistics

g. The maximum rate of transmission occurs when the source & channel are

(A) Matched | **(B)** Mismatched
(C) Inductive | **(D)** Capacitive

h. A Gaussian channel has a 10MHz bandwidth. If $\frac{S}{N} = 100$, the channel capacity is

(A) 66.59×10^6 bits/S | **(B)** 55.48×10^7 bits/S
(C) 77.60×10^6 bits/S | **(D)** 44.37×10^6 Bits/S

i. If the message bits appear together at the beginning or end of a codeword, then the code is

(A) Non-systematic | **(B)** Systematic
(C) Forward | **(D)** Backward

j. For a valid code vector, the product CH^T is

(A) Unity | **(B)** Zero
(C) Infinity | **(D)** Not defined

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Binary data are transmitted over a noisy communication channel in blocks of 16 binary digits. The probability that a received binary digit is in error due to channel noise is 0.1. Assume that the occurrence of an error in a particular digit does not influence the probability of occurrence of an error in any other digit within the block (i.e. errors occur in various digit positions within a block in a statistically independent fashion).
(i) Find the average (or expected) number of errors per block.
(ii) Find the variance of the number of errors per block.
(iii) Find the probability that the number of errors per block is greater than or equal to 5. **(8)**

b. A message is coded in binary code, the probabilities of transmission of the two symbols are 0.45 & 0.55 respectively. In the channel of communication, the symbol 1's are distorted into 0's with a probability of 0.1 & 0's are distorted into 1's with a probability of 0.2. Find the probability that
(i) A received '0' has not been distorted.
(ii) A received '1' has not been distorted. **(8)**

Q.3 a. Define the following:
(i) PDF (ii) CDF (iii) Stationarity (iv) Ergodicity **(8)**

b. Consider the random process $X(t) = A\cos(\omega t + \theta)$, where A and ω are constants while θ is a random variable with a uniform pdf
$$f_\theta(\theta) = \begin{cases} \frac{1}{2\pi}, & -\pi < \theta < \pi \\ 0, & \text{otherwise} \end{cases}.$$
Show that X(t) is WSS. **(8)**

Q.4 a. A discrete memoryless source produces four symbols with probabilities P_1, P_2, P_3 & P_4. Show that the entropy of the source is maximum when all the four symbols occur with equal probability. Compute the value of the maximum entropy. **(8)**

b. In a facsimile transmission of a picture, there are about 2.25×10^6 pixels/frame. For a good reproduction 12 brightness levels are necessary. Assume all these levels are equally likely to occur. Find the rate of information if one picture is to be transmitted every 3 Minutes. What is the source efficiency of this facsimile transmitter? **(8)**

Q.5 a. State & explain Shannon's – Hartley law. Show that $\lim_{B \to \infty} C = 1.44\left(\frac{s}{\eta}\right)$. **(8)**

b. Define average information, self information, mutual information and channel capacity. Show that $I(X, Y) = H(X) + H(Y) - H(X, Y)$. **(8)**

Q.6 a. Explain channel diagram, matrix and channel capacity of the following:
(i) Binary Symmetric Channel
(ii) Binary Erasure Channel
(iii) Deterministic Channel
(iv) Cascaded Channel **(8)**

b. A voice grade channel of the telephone network has a bandwidth of 3.4 KHz. Calculate
(i) the capacity of the telephone channel for a $\frac{S}{N}$ ratio of 30 dB.
(ii) the minimum $\frac{S}{N}$ ratio required to support information transmission through the telephone channel at the rate of 4800 bits/Sec. **(8)**

Q.7 a. How does the error control coding differ from the source coding? Classify the family of error control codes. **(4)**

b. Consider a (6,3) linear block code whose Generator Matrix G is

$$G = \begin{bmatrix} 100101 \\ 010110 \\ 001011 \end{bmatrix}$$

(i) Find all code vectors, their distances & Hamming Weights.
(ii) Find minimum weight Parity check matrix.
(iii) Draw the encoder circuit for the above code. **(12)**

Q.8 a. A code is composed of dots & dashes. Assuming that a dash is 3 times as long as a dot and has one third the probability of occurrence. Calculate
(i) The information in a dot and a dash.
(ii) The entropy of a dot-dash code
(iii) The average rate of information if a dot lasts for 10 ms and this time is allowed between the symbols. **(4)**

b. A source X has the following message and respected probabilities

m_1 1/16
m_2 1/16
m_3 1/8
m_4 1/4
m_5 1/2

Obtain the Huffman codes and determine its coding efficiency. **(8)**

c. Show that $H(X^2) = 2H(X)$. **(4)**

Q.9 a. How do burst errors differ from random errors? Explain with an example, how the burst errors could be corrected. **(4)**

b. Consider the (3, 1, 2) convolution code with $g^{(1)} = (110)$, $g^{(2)} = (101)$ & $g^{(3)} = (111)$. (i) Draw the encoder circuit (ii) Find the codeword corresponding to the information sequence (11101). **(8)**

c. Write short note on BCH codes. **(4)**