IITJEE MATHEMATICS SAMPLE PAPER - I

SECTION – I

Straight Objective Type

This section contains 8 multiple choice questions numbered 1 to 8. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** choice is correct.

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1. If every pair of equation among the equations $x^2 + px + qr = 0$, $x^2 + qx + rp = 0$ and $x^2 + rx + pq = 0$ has a common root then the sum of the three common roots is

(a) $-\frac{1}{2}$ (b) 0

(c) –1 (d) 1

2. The number of integral solutions of $x + y + z = 0$, where $x \geq -5$, $y \geq -5$, $z \geq -5$ is

(a) 135 (b) 136

(c) 455 (d) 105

3. A triangle with vertices represented by complex numbers z_0, z_1, z_2 has opposite side lengths in ratio $2 : \sqrt{6} : \sqrt{3} - 1$ respectively. Then

(a) $(z_2 - z_0)^4 = -9(7 + 4\sqrt{3})(z_1 - z_0)^4$ (b) $(z_2 - z_0)^4 = 9(7 + 4\sqrt{3})(z_1 - z_0)^4$

(c) $(z_2 - z_0)^4 = (7 + 4\sqrt{3})(z_1 - z_0)^4$ (d) none of these

4. Let the function $f(x)$ be defined as follows:

$$f(x) = \begin{cases} x^3 + x^2 - 10x & , \quad -1 \leq x < 0 \\ \cos x & , \quad 0 \leq x < \frac{\pi}{2} \\ 1 + \sin x & , \quad \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

. Then $f(x)$ has

(a) a local minimum at $x = \frac{\pi}{2}$ (b) a local maximum at $x = \frac{\pi}{2}$

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(c) absolute minimum at $x = -1$ (d) absolute maximum at $x = \pi$

5. If $f(x) = \begin{cases} [x], & \text{if } -3 \le x < 0 \\ 2x+1, & \text{if } 0 \le x \le 2 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$, then in [–3, 2]

(a) $\lim_{x \to 0^+} g(x) = 2$

(b) $\lim_{x \to 0^-} g(x) = 0$

(c) $g(x)$ is discontinuous at three points

(d) none of these

6. Let $f(x)$ be a continuous function for all x, such that $(f(x))^2 = \int_0^x f(t) \cdot \frac{2\sec^2 t}{4 + \tan t}\, dt$ and $f(0) = 0$, then

(a) $f\left(\frac{\pi}{4}\right) = \log \frac{5}{4}$

(b) $f\left(\frac{\pi}{4}\right) = \frac{3}{4}$

(c) $f\left(\frac{\pi}{2}\right) = 2$

(d) none of these

7. The equation of the smallest circle passing through the intersection of line $x + y = 1$ and the circle $x^2 + y^2 = 9$ is

(a) $x^2 + y^2 + x + y - 8 = 0$

(b) $x^2 + y^2 - x - y - 8 = 0$

(c) $x^2 + y^2 - x + y - 8 = 0$

(d) none of these

8. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ and $x^2 - y^2 = c^2$ cut each other at right angles, then

(a) $a^2 + b^2 = 2c^2$

(b) $b^2 - a^2 = 2c^2$

(c) $a^2 - b^2 = 2c^2$

(d) $a^2 b^2 = 2c^2$

Space for rough work

SECTION – II
Reasoning Type

This section contains 4 reasoning type questions numbered 9 to 12. Each question contains Statement-1 and Statement-2. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** is correct.

Directions: Read the following questions and choose

(A) If both the statements are true and statement-2 is the correct explanation of **statement-1**.

(B) If both the statements are true but statement-2 is not the correct explanation of **statement-1**.

(C) If statement-1 is True and **statement-2** is False.

(D) If statement-1 is False **and statement-2** is True.

9. Statement-1 : The function $f(x) = \lim_{n \to \infty} \frac{\log_e(1+x) - x^{2n}\sin(2x)}{1+x^{2n}}$ is discontinuous at $x = 1$.

Statement-2 : L.H.L. = R.H.L. $\neq f(1)$.

(a) A (b) B (c) C (d) D

10. Statement-1 : If a, b, c and d are in harmonic progression then $(a + d) > (b + c)$.

Statement-2 : If a, b, c and d are in arithmetic progression, then $ab + cd > 2(ac + bd - bc)$.

(a) A (b) B (c) C (d) D

11. Statement-1 : The function $f(x) = [|\sin x| + |\cos x|]$ is a periodic function having fundamental period $\frac{\pi}{2}$.

Statement-2 : Periodic functions are always many-one.

(a) A (b) B (c) C (d) D

Space for rough work

12. **Statement-1 :** $\sum_{K=1}^{n} K\cdot({}^{n}C_K)^2 = n\cdot {}^{2n-1}C_{n-1}$.

Statement-2 : If 2^{2003} is divided by 15 the remainder is 1.

(a) A (b) B (c) C (d) D

SECTION–III

Linked Comprehension Type

This section contains 2 paragraphs **M_{13}–M_{18}**. Based upon each paragraph, 6 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** choice is correct.

Passage-I

$A(3, 7)$ and $B(6, 5)$ are two points. $C: x^2 + y^2 - 4x - 6y - 3 = 0$ is a circle.

13. The chords in which the circle C cuts the members of the family S of circle passing through A and B are concurrent at

(a) $(2, 3)$ (b) $\left(2, \frac{23}{3}\right)$

(c) $\left(3, \frac{23}{2}\right)$ (d) $(3, 2)$

14. Equation of the member of the family of circles S that bisects the circumference of C is

(a) $x^2 + y^2 - 5x - 1 = 0$ (b) $x^2 + y^2 - 5x + 6y - 1 = 0$

(c) $x^2 + y^2 - 5x - 6y - 1 = 0$ (d) $x^2 + y^2 + 5x - 6y - 1 = 0$

15. If O is the origin and P is the center of C, then difference of the squares of the lengths of the tangents from A and B to the circle C is equal to

(a) $(AB)^2$ (b) $(OP)^2$

(c) $\left|(AP)^2 - (BP)^2\right|$ (d) none of these

Space for rough work

Passage-II

In a parallelogram *OABC*, vectors $\vec{a}, \vec{b}, \vec{c}$ are respectively the position vectors of vertices *A*, *B*, *C* with reference to *O* as origin. A point *E* is taken on the side *BC* which divides it in the ratio of 2 : 1. Also, the line segment *AE* intersects the line bisecting the angle *O* internally in point *P*. If *CP*, when extended meets *AB* in point *F*. Then

16. The position vector of point *P* is

(a) $\frac{3|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$

(b) $\frac{|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$

(c) $\frac{2|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$

(d) none of these

17. The position vector of point *F* is

(a) $\vec{a}+\frac{1}{3}\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

(b) $\vec{a}+\frac{2|\vec{a}|}{|\vec{c}|}\vec{c}$

(c) $\vec{a}+\frac{1}{2}\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

(d) $\vec{a}+\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

18. The vector $\overrightarrow{AF}$ is given by

(a) $\frac{1}{3}\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

(b) $\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

(c) $\frac{2|\vec{a}|}{|\vec{c}|}\vec{c}$

(d) $-\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

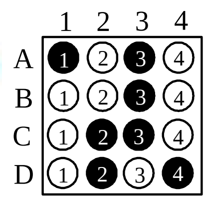
Space for rough work

SECTION–IV

Matrix–Match Type

This section contains 2 questions. Each question contains statements given in two columns which have to be matched. Statements (A), (B), (C), (D) in **Column I** have to be matched with statements (1, 2, 3, 4) in **Column II**. One statement in first column has one or more than one match with the statements in second column. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A–1,3, B–3, C–2,3 and D–2,4, then the correctly bubbled 4×4 matrix should be as follows:



1.

Column I	Column II
(A) $\lim\limits_{x \to -\infty} \frac{\sqrt{x^2}}{x}$ is equal to	**1.** 2
(B) $\lim\limits_{x \to \infty} \int_0^x \frac{1}{\ln 2}\left[\frac{1}{\sqrt{1+t^2}} - \frac{1}{1+t}\right] dt$ is equal to	**2.** 0
(C) $\lim\limits_{x \to 0^+} \frac{\cos^{-1}(\operatorname{sgn} x)}{x}$ is equal to	**3.** 1
(D) For the function $f(x) = \sqrt{1-\sqrt{1-x^2}}$, the critical points is/are at x is equal to	**4.** –1

Space for rough work

2. Match the following column–I with column–II

Column I (Equations)	**Column II (Number of solutions)**
(A) $\tan\left(\frac{\pi}{4}+\left(\frac{1}{2}\right)\cos^{-1}x\right)+\tan\left(\frac{\pi}{4}-\left(\frac{1}{2}\right)\cos^{-1}x\right)=1$	**1.** 0
(B) $\tan^{-1}\frac{1}{2x+1}+\tan^{-1}\frac{1}{4x+1}=\tan^{-1}\frac{2}{x^2}$	**2.** 2
(C) $\tan^{-1}\left(x+\frac{2}{x}\right)-\tan^{-1}\frac{4}{x}-\tan^{-1}\left(x-\frac{2}{x}\right)=0$	**3.** 3
(D) $\tan(\tan^{-1}x)-\cos(\cos^{-1}x)=0$	**4.** Infinitely many

Space for rough work

SECTION–V
Subjective or Numerical Type

The answer to these questions would lie between 0 to 9999. For any answer all four bubbles must be filled, for example if you plan to answer as 16 then fill 0016 and if you plan to answer 0 then fill 0000 in the grid provided in answer sheet. Any incomplete filling will be considered as incorrect filling.

Illustration: If you want to fill 2379 as your answer then it will be filled as

0	0	0	0
1	1	1	1
′	2	2	2
3	′	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	′	7
8	8	8	8
9	9	9	′

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1. Two integers x and y are chosen (with replacement) from the set {0, 1, 2, ..., 10}. If 'P' be the probability that $|x-y|$ is less than or equal to 5, then find the value of 121 P.

2. If A be the area bounded by the curves $y=|x-1|$ and $y+\dfrac{3}{|x+1|}=2$, then find the value of $(2A+3\ln 3)$.

Space for rough work