Total No. of Questions—**12**] [Total No. of Printed Pages—**8+2**

[4062]-201

S.E. (First Semester) EXAMINATION, 2011

(Common to Computer Engineering and IT)

DISCRETE STRUCTURES

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

N.B. **:—** (*i*) Answers to the two Sections should be written in separate answer-books.

(*ii*) In Section I attempt : Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

(*iii*) In Section II attempt : Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*iv*) Neat diagrams must be drawn wherever necessary.

(*v*) Figures to the right indicate full marks.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Prove using mathematical induction : where n is non-negative number : [6]

$$3 + 3.5 + 3.5^2 + ... + 3.5^n = 3.\ (5^{n+1} - 1)/4.$$

(*b*) Represent the following statements in logic and find relevant conclusion. Explain rule of inference used : [6]

(*i*) I am either dreaming or hallucinating. If I am hallucinating, I see elephants running down the road. I am not dreaming.

(*ii*) If I play hockey then I am sore the next day. I use medication if I am sore. I did not use medication.

P.T.O.

(*c*) Represent the arguments using quantifiers and finds its correctness : [4]

All students in this class understand logic. Ganesh is a student in this class. Therefore Ganesh understands logic.

Or

2. (*a*) A survey on sample 25 new cars being sold out at a local auto dealer was conducted to see which of three popular options Air Conditioner (A), Radio (R), Power Windows (W) were already installed. The survey found 15 had Air Conditioners, 12 had Radios, and 11 had Power Windows. 5 had Air Conditioner and Power Windows, 9 had Air Conditioner and Radio, 4 had Radio and Power Windows. Three had all three options. Find number of cars which had : [6]

(*i*) only one of the option

(*ii*) at least one of the option

(*iii*) none of the options.

Use principle of inclusion exclusion.

(*b*) Draw Venn diagram and prove the expression. Also write dual of the given expression : [6]

(*i*) $(A \overline{s\text{-}b} B \overline{s\text{-}b} C)^c = (A \overline{s\text{-}b} C)^c \barwedge (A \overline{s\text{-}b} B)^c$

(*ii*) $(U \barwedge A) \overline{s\text{-}b} (B \barwedge A) = A$.

(*c*) For given multi-sets find A $\overline{\text{s-b}}$ B, C $\overline{)(}$ D, A – D, B + C

A = {a, a, b, c, d, d, d, e},

B = {a, b, d, f, g},

C = {b, c, e, e, g, h, h},

D = {a, d, d, e, f, f, g, h}. [4]

3. (*a*) Let Z = {0, 1, 2, ... n – 1}. Let □ be a binary operation such that a □ b = remainder of a.b divided by n. Construct a table for n = 4. Is (Z_4, □) a groupoid, monoid, semi-group, and abelian group. [6]

(*b*) What is homomorphism and automorphism in an algebraic system ? Explain by giving example of each. [4]

(*c*) A central groupoid is an algebraic system (A, *) where * is a binary operation such that : [6]

$$(a * b) * (b * c) = b \qquad \square\ a, b, c \square A$$

show that :

(*i*) a * ((a * b) * c) = a * b

(*ii*) (a * (b * c)) * c = b * c.

Or

4. (*a*) Let Z_8 = {0, 1, 2, ... 7}. Let R is relation under the operations addition modulo 7 and multiplication modulo 7. Does this system form ring ? Is it a commutative ring ? [6]

(*b*) Define : [6]

(*i*) integral domain

(*ii*) field

(*iii*) group codes with example of each.

(*c*) What is hamming distance ? Find hamming distance between code words of : [4]

c = {(0 0 0 0), (0 10 1), (1 0 1 1), (0 1 1 1)}.

Rewrite the message by adding even parity check bit.

5. (*a*) R is a relation on set of ordered pairs of positive integers such that (a, b), (c, d) □ R if and only if (a + d) = (b + c). Define this relation. Find if given relation is an equivalence relation. [6]

(*b*) Let A = {1, 2, 3, 4, 6, 9, 12}. Let a relation on A is R = {(a, b) | a divides b □ a, b □ A}. Give list representation of R. Prove that it is a partial ordering relation. Draw Hasse diagram of the same. Prove or disprove if it a lattice. [8]

(*c*) Define partition. X = {1, 2, 3, 4, 5, 6, 7, 8, 9}. Determine whether or not each of the following is a partition of x : [4]

A = [{2, 4, 5, 8}, {1, 9}, {3, 6, 7}]

B = [{1, 3, 6}, {2, 8}, {5, 7, 9}].

Or

6. (*a*) Determine if each is a function. If yes, is it surjective, byjective or injective ? [6]

(*i*) Each person in the earth is assigned a number which corresponds to his age.

(*ii*) Each student is assigned a teacher.

(*iii*) Each country has assigned its capital.

(*b*) Find homogeneous solution of a recurrence relation : [6]

$a_n = 11a_{n-1} - 39a_{n-2} + 45a_{n-3}$ for $a_0 = 5, a_1 = 11, a_2 = 25$.

(*c*) Write generalized pigeonhole principle. Use any form of pigeonhole principle to solve the given problem : [6]

(*i*) Find minimum number of students in the class to be sure that three of them are born in the same month.

(*ii*) Assume that there are 3 men and 5 women in a party. Show that if these people are lined up in a row at least two women will be next to each other.

SECTION II

7. (*a*) Define graph and multigraph. Give any *two* applications of graph and represent them in graph notation. [6]

(*b*) Use Dijkstra algorithm to find the shortest path from a to f (Refer Fig. 1) : [8]

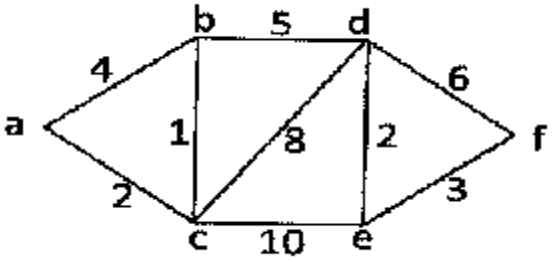

Fig. 1

(*c*) What are self complementary graphs ? Are there any self complementary graphs with 4 and 5 vertices ? If yes, draw them. [4]

Or

8. (*a*) State necessary and sufficient condition for existence of Hamiltonian path and circuit in k_{mn}. [6]

(*b*) Are k_5, k_6 and k_{33} planar graphs ? Which of these non-planar graphs have a property that removal of any vertex and edges incident with the vertex produces a planar graph ? Draw the diagrams and explain. [6]

(*c*) (*i*) What is complement of k_n, and k_{mn} ? [4]

(*ii*) Draw isomorphic graph of a graph shown in Fig. 2 but no crossover of edges : [2]

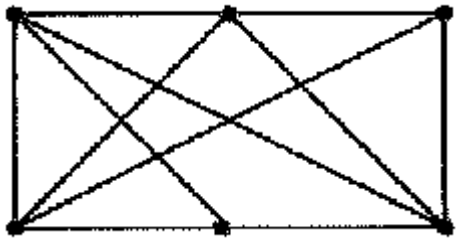

Fig. 2

9. (*a*) Define with example : [6]

(*i*) m-ray tree

(*ii*) search tree

(*iii*) inorder traversal.

(*b*) Use Prim's algorithm to find minimum spanning tree. Take A as starting vertex (Label remaining vertices). (Refer Fig. 3) [6]

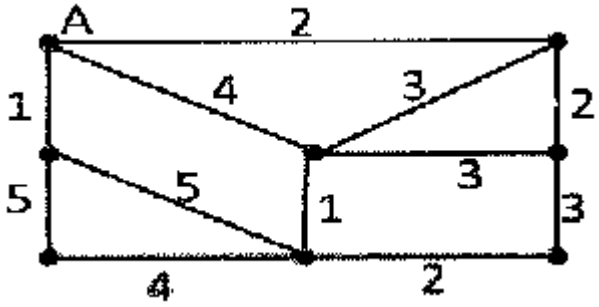

Fig. 3

(*c*) Draw a binary search tree for input data 200, 100, 300, 50, 150, 250, 400, 10, 75, 125, 175. Which is a root, leaf nodes and interior nodes ? [4]

Or

10. (*a*) Use Huffman coding to encode the following symbol with the frequencies listed A : 0.08, B : 0.010, C : 0.12, D : 0.15, E : 0.20, F : 0.35. What is the average number of bits used to encode the character ? [6]

(*b*) Use labeling procedure to find the maximum flow in transport network shown in the Fig. 4. Define corresponding minimal cuts. [8]

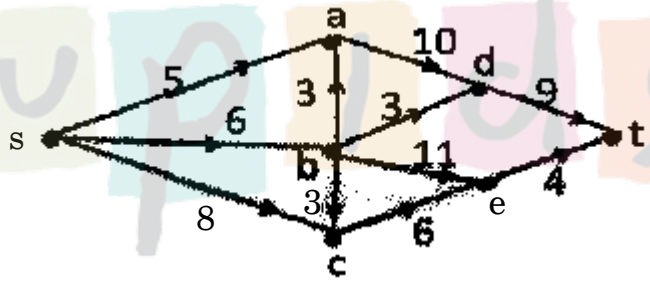

Fig. 4

(*c*) Define pendent vertex and eccentricity of a vertex. [2]

11. (*a*) Three students A, B and C are swimming in the race. A and B have same probability of winning and each is twice as likely to win as C. Find the probability that : [6]

(*i*) B wins

(*ii*) C wins

(*iii*) B or C wins.

(*b*) In a country club 60% of the players play tennis, 40% players play golf, 20% players play both tennis and golf. A member is chosen at random : [6]

(*i*) Find the probability that a member neither plays tennis nor golf.

(*ii*) If a member plays tennis, find the probability that member plays golf.

(*iii*) If a member plays golf, find the probability that member plays tennis.

(*c*) There are 3 bolts and three nuts in a box. Two pairs are chosen at random. Find the probability that one is bolt and one is nut. [4]

Or

12. (*a*) Find number of permutations that can be formed from the letters of the word ELEVEN : [6]

(*i*) How many of them begins and ends with E ?

(*ii*) How many of them have 3 Es together ?

(*iii*) How many of them begins with E and ends with N ?

(*b*) A woman has 11 friends of them six are women : [6]

(*i*) In how many ways can she invite three or more ?

(*ii*) In how many ways can she invite three or more of them if she wants same number of men and women (including herself) ?

(*c*) A student is to answer 10 out of 13 questions in an exam : [4]

(*i*) How many choices has he, if he must answer the first or second questions but not both ?

(*ii*) How many choices has he, if he must answer exactly three out of first five questions ?