# **SOCIETY OF ACTUARIES Advanced Portfolio Management**

# **Exam APMV**

# MORNING SESSION

**Date:** Friday, May 9, 2008 **Time:** 8:30 a.m. – 11:45 a.m.

#### **INSTRUCTIONS TO CANDIDATES**

#### **General Instructions**

- 1. This examination has a total of 120 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 60 points).
  - a) The morning session consists of 10 questions <u>numbered 1 through 10</u>.
  - b) The afternoon session consists of 12 questions <u>numbered 11 through 22</u>.

The points for each question are indicated at the beginning of the question. Questions 1 through 5 pertain to the Case Study, which is enclosed inside the front cover of this exam booklet.

- 2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
- 3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

#### Written-Answer Instructions

- 1. Write your candidate number at the top of each sheet. Your name must not appear.
- 2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
- 3. The answer should be confined to the question as set.
- 4. When you are asked to calculate, show all your work including any applicable formulas.
- 5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam APMV.
- 6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

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#### CASE STUDY INSTRUCTIONS

The case study will be used as a basis for some examination questions. Be sure to answer the question asked by referring to the case study. For example, when asked for advantages of a particular plan design to a company referenced in the case study, your response should be limited to that company. Other advantages should not be listed, as they are extraneous to the question and will result in no additional credit. Further, if they conflict with the applicable advantages, no credit will be given.

### \*\*BEGINNING OF EXAMINATION\*\* MORNING SESSION

## **Questions 1 – 5 pertain to the Case Study**

- **1.** (4 points) You are a consulting actuary for a rating agency. Particular Motors is considering a purchase of \$500 Million in Separate Account Institutional GICs from LifeCo, and has engaged your company to assign a credit rating to this line of business as a stand alone entity using the internal risk rating system methodology as described in *Crouhy*. The term to maturity that Particular Motors is looking for is 10 years.
  - (a) Describe the purpose and the criteria for the following elements of the internal risk rating system:
    - (i) Financial Assessment
    - (ii) Management and other qualitative factors
  - (b) Evaluate LifeCo's management and other qualitative factors with respect to the proposed GIC purchase.

# **2.** (7 points)

- (a) Describe the concept of a benchmark in performance measurement.
- (b) Describe the properties of a valid benchmark.
- (c) Construct an appropriate benchmark to measure the performance of LifeCo's GIC block of business.

Available Benchmarks are:	Yield	Duration
Money market	4.5%	0.25
Medium-term Treasury bonds	5	3
Real Estate I (50% pass-through / 50% CMOs)	6	3.5
Real Estate II (50% pass-through / 50% CMOs)	5.3	1.3
Corporate Bonds I (70% inv. grade / 30% non-inv. grade)	7.5	4.15
Corporate Bonds II (70% inv. grade / 30% non-inv. grade)	6.5	3.2
Corporate Bonds III (50% inv. grade / 50% non-inv. grade)	6	3.1
Equity I (15% in foreign securities)	8	8
Equity II (15% in foreign securities)	6.8	5.2

(d) Compare the yield of LifeCo's GIC portfolio to that of the benchmark recommended in (c).

- **3.** (6 points) LifeCo has revised downward the effective duration of the accumulation annuity block of business from 4.7 years to 4.1 years. You are responsible for reviewing the portfolio of assets backing the block of business (Appendix A to the Case Study).
  - (a) (*1 point*) Demonstrate that the assets backing the accumulation annuity block of business do not comply with LifeCo's ALM guidelines after the revision of effective liability duration.
  - (b) (4 points) Analyze the characteristics of each asset class in the portfolio that should be considered in rebalancing the portfolio to be in compliance with the ALM guidelines.
  - (c) (1 point) Recommend changes to the portfolio to bring it in compliance with the ALM Guidelines, given that the yield curve is expected to shift upward and the accumulation annuity block of business is coming out of its surrender charge period.

- **4.** (12 points) The management of LifeCo has asked you to select a stochastic equity model to use for consistent modeling of all equity-linked products. It is expected that the model would be used mainly for valuation and capital modeling purposes. The implementation would be similar to the one recommended by the Canadian Institute of Actuaries Research Paper on the "Use of Stochastic Techniques to Value Actuarial Liabilities under Canadian GAAP".
  - (a) (2 points) Identify the LifeCo products with equity exposure and describe that equity exposure.
  - (b) (*3 points*) Describe the following stochastic equity models:
    - (i) lognormal
    - (ii) regime-switching lognormal
    - (iii) AAA stochastic log-volatility model
    - (iv) GARCH(1,1)
  - (c) (*1 point*) Recommend a stochastic equity model appropriate for LifeCo's equity exposure.
  - (d) (*3 points*) Describe the following techniques to estimate parameters:
    - (i) Maximum likelihood estimation
    - (ii) Left-tail calibration method
    - (iii) Markov Chain Monte Carlo estimation
  - (e) (*1 point*) Recommend a parameter estimation technique for the model recommended in (c).
  - (f) (*1 point*) Assess whether or not the recommended model in (c) would be appropriate to price hedging strategies.
  - (g) (*1 point*) Describe how results across products should be aggregated to ensure a consistent CTE measure at the company level.

**5.** (6 points) LifeCo's ALM Committee decided to fully hedge the guarantee risk in the Equity Linked GIC portfolio effective Dec. 31, 2000. You are to prepare a static hedge proposal using only options.

S&P Price Index, Dec. 31, 2000	1378
S&P Total Return Index, Dec. 31, 2000	1590
Interest (annual)	12%
S&P Index dividend yield	3%
Volatility (annual)	
S&P Price Index =	18%
S&P Total Return Index =	18%

Assume that the Equity Linked GIC asset portfolio has tracked the S&P Total Return Index performance very well since issue. Options with the following parameters are available in the over-the-counter market:

Underlying	S&P Total Return S&P Price Index
Option	Put Call
Exercise	American European

- (a) Determine option(s) for LifeCo's Equity Linked GIC portfolio; specify type of underlying, option, and exercise.
- (b) Determine the term, strike rate, and notional amount of the option(s) recommended in (a).
- (c) Calculate the premium of the option(s) recommended in (a) using the information given.
- (d) Demonstrate how the option(s) purchased will hedge the risk if the S&P Price Index and Total Return Index drop to 1170 and 1350, respectively, ignoring premium paid for the option(s) in (c).

- **6.** (8 points) In-House, a large financial institution, is currently paying a rate of 3.00% to its depositors and charging a 7.25% loan rate. It has the following three subsidiaries:
  - A Consumer bank
  - B Mortgage lender
  - C Derivatives trading company

Subsidiary C earned 11.25% on its trades last year.

At the beginning of last year, In-House has the following available data (in millions):

	А	В	С
Loans	1	22	0
Deposits	30	1	0
Investments	0	0	8
Expenses	0.175	0.405	0.550

In order to more accurately measure the financial performance of each subsidiary, In-House decides to revise its net income statement, by adding a transfer pricing charge to account for the loans A made to B and C to fund their transactions.

- (a) (2 points) Calculate the net income for each subsidiary before the transfer pricing adjustment.
- (b) (3 points) Calculate the transfer pricing charge used and the revised net income for each subsidiary, assuming that after the transfer pricing adjustment, subsidiary A's revised net income is \$597,720.
- (c) (1 point) Describe the concept of transfer pricing for a life insurance company.
- (d) (2 points) Describe the advantages and disadvantages of the use of transfer pricing for a life insurance company.

- 7. (4 points) You are an investment actuary assisting your CIO in setting the company's asset allocation.
  - (a) Outline the considerations in setting an equity risk premium for equity mutual funds based on historic averages.
  - (b) Identify and describe factors that may cause future returns for equity mutual funds to differ from their historic averages.

**8.** (6 points) You are a credit derivative trader in an investment company. To mitigate the credit exposure on your company's investment portfolio, your manager asked you to buy a 100 million notional 3-year binary credit default swap (CDS) on company ABC.

ABC's Rating	AA
Coupon Rate	6% per annum
Coupon Payment Frequency	Semi-annual
Estimated Recovery Rate	40%
2-year Bond Price	99.27
3-year Bond Price	98.42

You have the following information about the company's bonds:

Term	<b>Risk-free Discount Factor</b>	<b>Risk-free Bond Price</b>
		With coupon rate 6% per annum
		(compounded semi-annually)
0	1	
0.5	0.975310	100.46
1	0.951229	100.90
1.5	0.927743	101.34
2	0.904837	101.76
2.5	0.882497	102.17
3	0.860708	102.58

Historical AA Bond default rates (%)

Thistoffear Thi T Dona default fates (70)				
Year	$1^{st}$	$2^{nd}$	3 <sup>rd</sup>	
Unconditional default probability	0.02	0.01	0.02	

Assume the yield curve is flat at 5%, continuously compounded, defaults happen halfway through the year (immediately before coupon payments), and CDS payments are annual, at the end of each year.

- (a) Calculate the estimated implied default probabilities from years 1 to 3.
- (b) Explain the difference between the implied default probabilities calculated in (a) and historical bond default rates.
- (c) Calculate the binary CDS spread.

**9.** (*4 points*) You are a consultant for a U.S. taxable corporation with strong earnings that sponsors a large defined benefit pension plan. The pension plan's assets are invested primarily in equities.

Company management is concerned about the plan's impact on the company's cash flow and financial statements due to volatile financial markets. As a result, the company is considering using a liability-matching bond strategy for its pension assets.

Your analysis shows the following:

- the company's publicly-traded shares currently have a Beta above 1.0
- if the pension assets were moved to the new strategy, then
  - $\blacktriangleright$  the Beta of the company's shares are estimated to drop below 1.0
  - > the expected rate of return on plan assets drops significantly
  - pension plan annual required contributions are estimated to increase significantly

You are preparing a report for the company's CFO regarding the liability-matching bond pension investment strategy. In the report:

- (a) Describe the advantages and disadvantages for:
  - (i) plan participants
  - (ii) shareholders
- (b) Assess the impact on the price of the company's shares.
- (c) Compare the effectiveness of the current strategy vs. the new strategy as a hedge for future salary increases of plan participants.

- **10.** (*3 points*) You are a consultant to BL Life Insurance on using behavioral finance in conducting company's business.
  - (a) Explain short-term momentum in stock price return using behavioral finance, and how to profit from it.
  - (b) Explain how Myopic Loss Aversion drives consumer insurance purchase behavior.

BL Life currently has a forced rebalancing strategy. BL Life will rebalance to the target portfolio mix at the end of every month.

- (c) Discuss how the forced rebalancing strategy will affect the company's stochastic pricing for each of the following, given the existence of serial correlation:
  - (i) Average results
  - (ii) Tail results

#### \*\*END OF EXAMINATION\*\* MORNING SESSION

# **SOCIETY OF ACTUARIES Advanced Portfolio Management**

# **Exam APMV**

# AFTERNOON SESSION

**Date:** Friday, May 9, 2008 **Time:** 1:30 p.m. – 4:45 p.m.

#### INSTRUCTIONS TO CANDIDATES

#### **General Instructions**

- 1. This afternoon session consists of 12 questions <u>numbered 11 through 22</u> for a total of 60 points. The points for each question are indicated at the beginning of the question. There are no questions that pertain to the Case Study in the afternoon session.
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#### \*\*BEGINNING OF EXAMINATION\*\* AFTERNOON SESSION

- **11.** (*4 points*) One of the assumptions for the Black-Scholes-Merton formula is that "the stock price changes smoothly".
  - (a) Describe how Merton's mixed jump-diffusion model and Variance-gamma model address violation of the above assumption.
  - (b) Explain how you can make money, assuming that the market prices options using the Black-Scholes-Merton model and that the market dynamics can be best described by:
    - (i) Merton's mixed jump-diffusion model
    - (ii) Variance-gamma model

**12.** (5 points) The stock price of ethanol producer MPG Inc. depends on three variables: the price of gasoline, the price of corn, and the performance of the Russell 2000 index. You are given the following information:

Risk-free Rate $r_f$	6%
Gasoline Forward: Expected Return	9%
Volatility	15%
Corn: Market Price of Risk	0.1
Volatility	25%
Russell 2000 stock index: Market Price of Risk	-0.05
Volatility	30%

- (a) Calculate the market price of risk for gasoline.
- (b) Calculate the expected return on MPG Inc. stock.
- (c) Define a martingale, and show its formula form.

The dollar money market account, g, follows process dg = rg dt, where r is the instantaneous risk-free rate. In a no-arbitrage world with g as the numeraire, the price of any traded derivative, f, depends on the same source of uncertainty as the dollar money market account, g.

- (d) Determine:
  - (i) the market price of risk of this world to make the ratio of f/g a martingale.
  - (ii) whether this is a risk-neutral world and provide the reason for it.

Suppose that *x* follows the Ito process:

$$dx = a(x,t)dt + b(x,t)dZ$$

If G is a function of x and t, then Ito's Lemma shows that G follows the process:

$$dG = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)dt + \frac{\partial G}{\partial x}bdZ$$

(e) Verify that the ratio f/g is a martingale.

- **13.** (4 points) The yield of a stock life insurance company's surplus portfolio has decreased and the company's portfolio management is considering adding hedge funds and/or private equities. The existing asset portfolio is comprised of 20% public equities and 80% bonds, consistent with the investment policy, and is earning 5%. The company requires a target yield of 5.25%.
  - (a) Outline the pros and cons to management of investing in
    - (i) private equities and
    - (ii) hedge funds.

	Current Portfolio	Hedge Fund W	Hedge Fund X	Private Equity Y	Private Equity Z
Implied premium		100bp	205bp	50bp	75bp
to current portfolio					
Volatility	10%	10%	20%	15%	15%

- (b) Analyze the above four investment options with respect to whether the investment will meet or exceed the company's target on a risk adjusted basis.
- **14.** (6 points) A pool of mortgage loans backing a particular mortgage backed security has the following information available at the end of the month, assuming the scheduled balance resets at the beginning of each month.

Scheduled Balance	\$250,000,000
Actual Balance	\$245,000,000
Age (months)	32

- (a) Calculate the Single Monthly Mortality (SMM), Conditional Prepayment Rate (CPR) and Public Securities Association (PSA) model of the pool based on the provided information.
- b) Identify and describe the key factors that impact the prepayment behavior of Agency mortgage backed securities.
- c) Describe how the prepayment profile of subprime mortgage collateral differs from conforming agency mortgages and identify the key underlying reasons that drive the differences.

**15.** (6 points) You are a consultant to a defined benefit pension fund manager. The manager is interested in bidding on a Longevity Bond issued by an AA-rated life insurer. The bond makes three annual payments of  $1,000,000 \times S(t)$ , where S(t) is the value of the survivor index at the end of year t, and S(0) = 1.0.

You have been given the following estimates for the central death rates for the cohort underlying the survivor index:

Т	Central death rate for cohort		
1	0.040		
2	0.045		
3	0.050		

The LIBOR curve is currently flat at 5.0% (annual effective rate).

- (a) Calculate the maximum bid price for the bond if the pension fund is willing to pay 30 basis points annually to hedge part of its longevity risk.
- (b) Describe the benefits and the risks to the pension fund associated with investing in the longevity bond.
- (c) Assess the reasons for interest in mortality-linked securities for each of the following entities:
  - (i) Hedge funds
  - (ii) Government

**16.** (6 points) You own a portfolio of corporate bonds. You are interested in hedging the value of your portfolio against changes in interest rates and credit spreads.

Security	DVBP	Conversion Factor for CTD*	Price
Portfolio	100,000	n/a	10,000,000
Treasury Future	80.5	1.20	95-05

Given the following information:

\*CTD = Cheapest To Deliver

- (a) Determine the number of Treasury futures contracts needed to implement a hedge against a small parallel shift in the yield curve.
- (b) Calculate the impact on the value of the hedged portfolio determined in (a), assuming that there is a 10 basis point upward shift in the yield curve.
- (c) Identify the risk related to the changing credit spreads and describe approaches that could be used to mitigate that risk.
- (d) Identify other hedging instruments that may be used to manage interest rate risk.

**17.** (6 points) As a pricing actuary, you have been asked to recommend one of the following underlying funds for your company's new annuity product, a segregated fund contract. Given the following information:

Underlying	Value of	Management	Initial	Value of term annuity of \$1
Fund	the Option	Expense Ratio	Value	per month for a male aged 40
X	1.7	2.50%	90	52.0
Y	1.3	2.50%	100	48.0
Z	1.0	2.50%	80	51.0

- (a) Describe the benefit features and the key hedging challenges associated with the following guarantees:
  - (i) Guaranteed Minimum Maturity Benefit (GMMB)
  - (ii) Guaranteed Minimum Death Benefit (GMDB)
  - (iii) Guaranteed Minimum Accumulation Benefit (GMAB)

The marketing department would like to offer both a GMMB and a GMDB but is concerned that the market will only support a total cost of 3% or lower. Assume that the GMMB annual margin offset is 20bp.

(b) Recommend the fund from the above list that could best afford to support the guarantees for a male aged 40.

- **18.** (6 points) A 45 year old client has accumulated an investment portfolio of \$1 million over his lifetime. His portfolio returns to date have averaged in excess of 10% per year due to his 100% concentration in equities. He expects to retire at age 65 and expects his employment income to be sufficient to support him and his spouse until then. At retirement, he will sell his business and use the proceeds to provide for all his retirement income. He has no other sources of income.
  - (a) Create an outline of an Investment Policy Statement (IPS) for your client and justify your recommendation. Assume that your client has no other dependents.
  - Revise the IPS given the following additional information: your client has an elderly mother, who has recently fallen ill, and may have large medical care bills. Assume your client will be responsible for paying for his mother's future medical care.
  - (c) Revise the IPS given that the client's mother has been institutionalized and has fixed monthly long-term care costs. Recommend revisions to the IPS, for each of the following risk tolerances:
    - (i) conservative (with minimal investment risk)
    - (ii) moderately aggressive (with some investment risk, but also some protection of capital)

**19.** (6 points) Your company is considering selling Bond A and replacing it with Bond B.

Data for Bond A and Bond B is as follows:

Bond	Α	В
Market Price/100 par value	80	70
Years to Maturity	2	2
Coupon	0	0

Your company estimates the default risks associated with the bonds are as follows:

Bond	А	В
Annual Default rate	0.25	0.05
Loss given default	40	90

The risk-free rate is currently at 10% per annum, but your company estimates that there is a 75% probability that the risk-free rate will move up to 12% at the end of year 1, and a 25% probability that it will decline to 9% at the end of year 1.

Evaluate the effectiveness of this transaction based on your company's estimates using the Reduced-Form Approach for the valuation of bonds.

**20.** (*4 points*) Given that an American-style put option on a dividend-paying stock satisfies the following partial differential equation:

$$\frac{\partial f}{\partial t} + (r - q)S\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2} = rf$$

- (a) Explain the concept of solving this partial differential equation using the implicit finite difference method.
- (b) Identify the additional assumptions needed to apply the explicit finite difference method.
- (c) Explain how to use the representative scenarios approach and why this technique is superior to Monte Carlo simulation for evaluating economic capital required by a financial institution.

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# **21.** (3 points)

- (a) Describe the process of valuation of interest rate derivatives using market benchmark prices.
- (b) Determine the appropriateness of each of the following models for the process described in (a).
  - (i) Black-Derman-Toy
  - (ii)  $dr(t) = a(b-r(t))dt + \sigma dZ$
  - (iii)  $dr(t) = a(b-r(t))dt + \sigma\sqrt{r(t)} dZ$
  - (iv)  $dr(t) = (\alpha(t) \beta r(t))dt + \sigma dW(t)$

# **22.** (4 points)

- (a) Outline and describe the explanatory variables that are critical to the modeling of consumer behavior for estimating:
  - (i) Surrender rates on insurance policies
  - (ii) Mortgage prepayment rates
- (b) Describe the following features of residential mortgage-backed securities and how the performance of the underlying mortgage collateral affects each:
  - (i) "Shifting interest" structure
  - (ii) Deep Mortgage Insurance
  - (iii) Available Funds Cap

## \*\*END OF EXAMINATION\*\* AFTERNOON SESSION