STATE ELIGIBILITY TEST
MATHEMATICAL SCIENCES

SUBJECT CODE : 23

SYLLABUS OF PAPER II
SECTION A

1. General information on science and its interface with society to test the candidate's awareness of science, aptitude of scientific and quantitative reasoning.

2. COMMON ELEMENTARY COMPUTER SCIENCE (Applicable to all candidates offering subject areas).

   i) History of development of computers, Mainframe, micro's and Super Computer systems.

   ii) General awareness of computer Hardwar i.e. CPU and other peripheral devices (input/output and auxiliary storage devices)

   iii) Basic knowledge of computer systems software and programming language i.e. Machine language. Assembly language and higher level language.

   iv) General awareness of popular commercial software packages like LOTUS, DBASE, WORDSTAR, other Scientific application packages.

PAPER II
SECTION B

General Information:- Units 1.2.3. and 4 are compulsory for all candidates. Candidates with Mathematics background may omit units 10-14 and units 17,18. Candidates with Statistics background may omit units 6,7,9,15 and 16. Adequate alternatives would be given for candidates with O.R. background.

1. Basic concepts of Real and Complex analysis-sequences and series, continuity, uniform continuity, Differentiability, Mean Value Theorem, sequences and series of functions, uniform convergence, Riemann integral-definition and simple properties. Algebra of Complex numbers, Analytic functions, Cauchy's Theorem and integral formula, Power series, Taylor's and Laurent's series, Residues, Contour integration.

2. Basic Concepts of Linear Algebra-Space of n-vectors. Linear dependence, Basis, Linear transformation, Algebra of matrices, Rank of a matrix, Determinants, Linear equations, Quadratic forms. Characteristic roots and vectors.

3. Basic concepts of probability-Sample space, discrete probability, simple theorems on probability, independence of events, Bayes Theorem. Discrete and continuous random variables, Binomial, Poisson and Normal distributions;

Expectation and moments, independence of random variables, Chebyshev's inequality.

4. Linear Programming Basic Concepts- Convex sets. Linear Programming Problem (LPP) Examples of LPP. Hyperplane, open and closed half-spaces. Feasible, basic feasible and optimal solutions. Extreme point and graphical method.

5. Real Analysis- finite, countable and uncountable sets, Bounded and unbounded sets, Archimedean property, ordered field, completeness of R, Extended real number system, limsup and liminf of a sequence, the epsilondelta definition of continuity and convergence, the algebra of contlinous functions, monotonic functions, types of discontinuities, infinite limits and limits at infinity, functions of bounded variation, elements of metric spaces.

6. Complex Analysis-Riemann Sphere and Stereographic projection, Lines, Circles, crossratio. Mobius transformations, Analytic functions, Cauchy-Riemann equations, line integrals, Cauchy's theorem, Morera's theorem, Liouville's theorem, integral formula, zero-sets of analytic functions, exponential, sine and cosine functions, Power series representation, Classification of singularities, Conformal Mapping.

7. Algebra-Group, subgroups, Normal subgroups, Quotient Groups, Homomorphisms, Cyclic Groups, permutation Groups, Cayley's Theorem, Rings, ideals, Integral Domains, Fields, Polynomial Rings.

8. Linear Algebra-Vector spaces, subspaces, quotient spaces, Linear independence, Bases, Dimension. The algebra of linear Transformations, kemel, range, isomorphism, Matrix Representation of a linear transformation, change of bases, Linear functionals, dual space, projection, determinant function, eigenvalues and eigen vectors, Cayley-Hamilton Theorem, Invariant Sub-spaces, Canonical Forms: diagonal form, Triangular form, Jordan Form, Inner product speces.

9. Differential Equations-First order ODE, singular solutions, initial value Problems of First Order ODE, General theory of homogeneous and non-homogeneous Linear ODC, Variation of Parameters. Lagrange's and Charpit's methods of solving. First order Partial Differential Equations. PDE's of higher order with constant coefficients.

10. Data Analysis Basic Concepts- Graphical representation, measures of central tendency and dispersion. Bivariate data, correlation and regression. Least squares-polynomial regression, Application of normal distribution.

11. Probability- Axiomatic definition of probability. Random variables and distribution functions (univariate and multivariate); expectation and moments; independent events and independent random variables; Bayes' theorem; marginal and conditional distribution in the multivariate case, covariance matrix and correlation coefficients (product moment, partial and multiple), regression. Moment generating functions, characteristic functions; probability inequalities (Tchebyshef, Markov, Jensen). Convergence in probability and in

distribution; weak law of large numbers and central limit theorem for independent identically distributed random variables with finite variance.

12. Probability Distribution-Bernoulli,Binomial,Multinomial, Hypergeomatric, Poisson, Geometric and Negative binomial distributions, Uniform, exponential, Cauchy, Beta, Gamma, and normal (univariate and multivariate) distributions Transformations of random variables; sampling distributions. t F and chi-square distributions as sampling distributions, Standard enors and large sample distributions. Distribution of order statistics and range.

13. Theory of Statistics; Methods of estimation: maximum likelihood method, method of moments, minimum chi-square method, least-squares method. Unbiasedness, efficiency, consistency. Cramer-Rao ineguality. Sufficient Statistics. Rao-Blackwell Theorem.Uniformly minimum variance unbiased estimators. Estimation by confidence intervals. Tests of hypotheses: Simple and composite hypotheses, two types of errors, critical region, randomized test, power function, most pawerful and Unifirmly most powerful tests. Likelihood-ratio tests. Wald's sequential probability ratio test.

14. Statistical methods and Data Analysis-Tests for mean and variance in the normal distribution : one -population and two-population cases; related confidence intervals. Tests for product moment, partial and multiple correlation coefficients; comparison of k linear regressions. Fitting polynomial regression; related test. Analysis of discrete data: chi-square test of goodness of fit, contingency tables. Analysis of variance: one-way and two-way classification (equal number of observations per cell). Large-sample tests through normal approximation. Nonparametric tests, median test, Mann-Whitney test, Wilcoxon test for one and two-samples, rank correlation and test of independence.

15. Operational Research Modelling-Definition and scope of Operational Research. Different types of models. Replacement models and sequencing theory, Inventory problems and their analytical structure. Simple deterministic and stochastic models of inventory control.

    Basic characteristics of queueing system, different performance measures. Steady state solution of Markovian queueing models:

    M/M/1.M/M/1 with limited waiting space M/M/C , M/M/C with limited waiting space.

16. Linear Programming-Linear Programming. Simplex method, Duality in linear programming. Transformation and assignment problems. Two person-zero sum games. Equivalence of rectangular game and linear programming.

17. Finite Population: Sampling Techniques and Estimation: Simple random sampling with and without replacement. Stratified sampling; allocation problem: systematic sampling. Two stage sampling. Related estimation problems in the above cases.

18. Design of Experiments: Basic principles of experimental design. Randomisation structure and analysis of completely randomised, randomised blocks and

Latin-square designs. Factorial experiments. Analysis of 2n factorial experiments in randomised blocks.

PAPER III

1. Real Analysis: Riemann integrable functions; improper integrals, their convergence and uniform convergence. Eulidean space R", Bolzano-Weierstrass theorem, compact Subsets of R", Heine-Borel theorem, Fourier series. Continuity of functions on R", Differentiability of F:R" →Rm Properties of differential, partial and directional derivatives, continuously differentiable functions. Taylor's series. Inverse function theorem, Implicit function theorem. Integral functions, line and surface integrals, Green's theorem, Stoke's theorem.

2. Complex Analysis: Cauchy's theorem for convex regions, Power series representation of Analytic functions. Liouville's theorem, Fundamental theorem of algebra, Riemann's theorem on removable singularities, maximum modulus principle. Schwarz lemma, Open Mapping theorem, Casoratti-Weierstrass- theorem, Weierstrass's theorem on uniform convergence on compact sets, Bilinear transformations, Multivalued Analytic Functions, Riemann Surfaces.

3. Algebra: Symmetric groups, alternating groups, Simple groups, Rings, Maximal Ideals, Prime Ideals, Integral domains, Euclidean domains, principal Ideal domains, Unique Factorisation domains, quotient fields, Finite fields, Algebra of Linear Transformation, Reduction of matrices to Canonical Forms, Inner Product Spaces, Orthogonality, quadratic Forms, Reduction of quadratic forms.

4. Advanced Analysis: Elements of Metric Spaces, Convergence, continuity compactness, Connectedness, Weierstrass's approximation Theorem, Completeness, Bare category theorem, Labesgue measure, Labesgue Integral, Differentiation and Integration.

5. Advanced Algebra: Conjugate elements and class equations of finite groups, Sylow theorems, solvable groups, jordan Holder Theorem, Direct products, Structure Theorem for finite abelian groups, Chain conditions on Rings; Characteristic of Field, Field extensions, Elements of Galois theory, solvability by. Radicals, Ruler and compass construction.

6. Functional Analysis Branach Spaces, Hahn-Banach Theorem, Open mapping and closed Graph Theorems. Principle of Uniform boundedness, Boundedness and continuity of Linear Transformations, Dual Space, Embedding in the second dual, Hilbert Spaces, Projections. Orthonormal Basis, Riesz-representation theorem, Bessel's Inequality, parsaval's identity, self adjoined operators, Normal Operators.

7. Topology; Elements of Topological Spaces, Continuity, Convergence, Homeomorphism, Compactness, Connectedness, Separation Axioms, First and Second Countability, Separability, Subspaces, Product Spaces, quotient spaces. Tychonoffs Theorem, Urysohn's Metrization theorem, Homotopy and Fundamental Group.

8. Discrete Mathematics: Partially ordered Sets, Lattices, Complete Lattices, Distributive lattices, Complements' Boolean Algebra, Boolean Expressions, Application to switching circuits, Elements of Graph Theory, Eulerian and Hamiltonian graphs, planar Graphs, Directed Graphs, Trees. Permutations and Combinations, Pigeonhole principle, principle of Inclusion and Exclusion, Derangements.

9. Ordinary and Partial Differential Equations: Existence and Uniqueness of solution dy/dx=f(x,y) Green's function, sturm □

   Liouville Boundary Value Problems, Cauchy Problems and Characteristics, Classification of Second Order PDE, Separation of Variables for heat equation, wave equation and Laplace equation, Special functions.

10. Number Theory: Divisibility: Linear diophantine equations. Congruences. Quadratic residues; Sums of two squares, Arithmatic functions Mu, Tau, Phi and Sigma (and).

11. Mechanics Generalised coordinates; Lagranges equation; Hamilton's cononical equations; Variational principles- Hamilton's principles and principles of least action; Two dimensional motion of rigid bodies; Euler's dynamical equations for the motion of rigid body; Motion of a rigid body about an axis; Motion about revolving axes.

12. Elasticity: Analysis of strain and stress, strain and stress tensors; Geometrical representation; Compatibility conditions; Strain energy function; Constitutive relations; Elastic solids Hookes law; Saint-Venant's principle, Equations of equilibrium; Plane problems-Airy's stress fuction, vibrations of elastic, cylindrical and spherical media.

13. Fluid Mechanics : Equation of continuity in fluid motion; Euler's equations of motion for perfect fluids; Two dimensional motion complex potential; Motion of sphere in perfect liquid and motion of liquid past a sphere; Vorticity, Navier-Stokes's equations for viscous flows-some exact solutions.

14. Differetial Geometry: Space curves-their curvature and torsion; Serret Frehat Formula; Fundamental theorem of space curves; Curves on surfaces; Fisrt and second fundamental form, Gaussian curvatures; Principal directions and principal curvatures; Goedesics, Fundamental equations of surface theory.

15. Calculus of Variations: Linear functionals, minimal functional theorem general variation of a functional, Euler-Lagrange equation; Variational methods of boundary value problems in ordinary and partial differential equations.

16. Linear Integral Equations: Linear intergral Equations of the first and second kind of Fredholom and Volterra type; solution by successive substitution and succesive approximations; Solution of equations with separable kernels; The Fredholm Alternative; Holbert-Schmidt theory for symmetric kernels.

17. Numerical analysis: Finite differences, interpolation; Numerical solution of algebric equation; Iteration; Newton-Raphson method; algebric equation Meration Nowton Raphion method selection on linear system, Director Method Gauss elmination method, Matrix- Inversion, eigenvalue problems;Numerical differentiation and integration.

    Numerical solution of ordinary differential equation; iteration method, Picard's method, Euler's method and improved Euler's method.

18. Integral Transform: Laplace transform; Transform of elementary functions, Transform of Derivatives, Inverse Transform, Convolution Theorem, Applications, Ordinary and Partial differential, equations; Fourier transforms. sine and cosine transform, Inverse Fourier Transform, Application to ordinary and partial differential equations.

19. Mathematical Programming: Revised simplex method, Dual simplex method, Sensitivity analysis and parametric linear programming. Kuhu-Tucker conditions of optimality. Quadratic programming; methods due to Beale, Wofle and Vandepanne, Duality in quadratic programming, self duality. Integer programming.

20. Measure Theory: Measurable and measure spaces; Extension of measures, signed measures, Jordan-Hahn decomposition theorems. Integration, monotone convergence theorem. Fatom's lemma, dominated convergence theorem Absolute continuity, Radon Nikodym theorem, Product measures, Fubini's theorem.

21. Probability: Sequences of events and candom variables; Zero- one laws of Borel and Kolmogorov. Almost sure convergence, convergence in mean square, khintchine's weak law of large numbers; Kolmogorov's inequality, strong law of large numbers.

    Convergence of series of random variables, three-series criterion. Central limit theorems of Liapounov and Lindeberg-Feller. Conditional expectation, martingales.

22. Distrinbution Theory: Properties of distribution functions and characteristic functions: continuity therem, inversion formula, Representation of distribution function as a mixture of discrete and continuous distribution functions; Convolutions, marginal and conditional distributions of bivariate discrete and continuous distributions. Relations between characteristic functions and moments; inequalities of Holder and Minkowski.

23. Statistical Inference and Decision Theory: Statistical decision problem: non-randomized, mixed and randomized decision rules; risk function, admissibility, Bayes' rules, minimax rules, least favourable distributions, complete class, and minimal complete class. Decision problem for finite parameter space. Convex loss function. Role of sufficiency.

    Admissible, Bayes and minimax estimators; illustrations. Unbiasedness. UMVU estimators. Families of distributons with monotone likelihood property,

exponential family of distributions. Test of a simple hypothesis against a simple alternative from decision-theoretic viewpoint. Tests with Neyman structure. Uniformly most powerful nubiased tests. Locally most powerful tests. Inference on location and scale parameters; estimation and tests. Equivariant estimators. Invariance in hypothesis testing.

24. Large sample statistical methods: Various modes of convergence. Op and Op' CLT. Sheffe's theorem, Polya's theorem and Slutsky's theorem. Transformation and variance stabilizing formula. Asymptotic distribution of function of sample moments. Sample quantiles. Order statistics and their functions. Tests on correlations, coefficients of variation. skewness and kurtosis. Pearson Chi-square, contingency Chisquare and likelihood ratio statistics. U-statistics. Consistency of Tests. Asymptotic relative efficiency.

25. MultivariageStatistical Analysis: Singular andnon-singular multivariate distributions. Characteristics functions. Multivariate normal distribution Marginal and conditional distribution, distribution of of linear forms, and quadratic forms, Cochran's theorem.

    Inference on parameters of multivariate normal distributions: One-population and two-population cases. Wishart distribution. Hotellings T2, Mahalanobis D2. Discrimination analysis, Principal components, Cononical correlations, Cluster analysis.

26. Linear Models and Regression: Standard Gauss-Markov models; Estimability of parameters; best linear unbiased estimates (BLUE): Method of least squares and Gauss-Markov theorem; Variance-covariance matrix of BLUES. Tests of linear hypothesis; One-way and two-wayclassifications. Fixed, random and mixed effects models (two-way classifications only); variance components, Bivariate and multiple linear regression; Polynomial regression; use of orthogonal polynomials. Analysis of covariance. Linear and nonlinear regression. Outliers.

27. Sample Surveys: Sampling with varying probability of selection, Hurwitz-Thompson estimator; PPS sampling; Double sampling. Cluster sampling. Non-sampling errors: Interpenetrating samples. Multiphase sampling. Ratio and regression methods of estimation.

28. Design of Experiments: Factorial experiments, confou-nding and fractional replication. Split and strip plot designs; Quasi-Latin square designs; Youden square. Design for study of response surfaces; first and second order designs. Incomplete block designs; Balanced, connects-dness and orthogonality, BIBD with recovery of inter-block information; PBIBD with 2 associate classes. Analysis of series of experiments. esimation of residual effects. Construction of orthogonal-Latin squares, BIB designs. and confounded factorial designs. Optimality criteria for experimental designs.

29. Time-Series Analysis: Discrete-parameter stochastic processes; strong and weak stationarity; autocovariance and autocorrelation. Moving average, autoregressive, autoregressive moving average and autoregressive integrated

moving average processes. Box-jenkins models. Estimation of the parameters in ARIMA models; forecasting. Periodogram and correlogram analysis.

30. Stochastic Processes: Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution; branching processes: Random walk; Gambler's ruin. Markov processes in continuous time; Poisson procsses, birth and death processes, Wiener process.

31. Demography and vital Statistics: Measures of fertility and mortality, period and Cohort measures. Life tables and its applications; Methods of construction of abridged life tables. Appliaction of stable population theory to estimate vital rates. Population projections. Stochastic models of fertility and reproduction.

32. Industrial Statistics: Control charts for variables and attributes; Acceptance sampling by attributes; single, double and sequential sampling plans; OC and ASN functions, AOQL and ATI; Acceptance sampling by varieties. Tolerance limts. Reliablity analysis: Hazard function, distribution with DFR and IFR; Series and parallel systems. Life festing experiments.

33. Inventory and Queueing theory: Inventory (S,s) policy, periodic review models with stochastic demand. Dynamic inventory models. Probabilistic re-order point, lot size inventory system with and without lead time. Distribution free analysis. Solution of inventory problem with unknown density function. Warehousing problem. Queues: Imbedded morkov chain method toobtain steady state solution of M/G/1, G/M/1 and M/D/C, Network models. Machine maintenance models. Design and control of queueing systems.

34. Dynamic Programming and Marketing: Nature of dynamic programming, Deterministic processes, Non-sequential discrete optimisation-allocation problems, assortment problems. Sequential discrete optimisation long-term planning problems, multi stage production processes. Functional approximations. Marketing systems, application of dynamic programming to marketing problems. Introduction of new product, objective in setting market price and its policies, purchasing under fluctuating prices, Advertising and promotional decisions, Brands switching analysis, Distribution, decisions.

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