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Entrance Examination, 2004

M.Sc. (Statistics-OR)

Hall Ticket No.							

Time: **2** hours

Max. Marks: **100**
Part A: **25**
Part B: **75**

Instructions

1. Calculators are not allowed.
2. Part A carries 25 marks. Each correct answer carries **1 mark** and each wrong answer carries $-\frac{1}{4}$ **mark**. So do not gamble. If you want to change any answer, cross out the old one and circle the new one. Over written answers will be ignored.
3. Part B carries 75 marks. Instructions for answering Part B are given at the beginning of Part B.
4. Do not detach any page from this answer book. It contains **18** pages in addition to this top page. Pages **15** to **18** are for rough work.

Answer Part A by **circling** the correct letter in the array below:

1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e
15	a	b	c	d	e
16	a	b	c	d	e
17	a	b	c	d	e
18	a	b	c	d	e
19	a	b	c	d	e
20	a	b	c	d	e
21	a	b	c	d	e
22	a	b	c	d	e
23	a	b	c	d	e
24	a	b	c	d	e
25	a	b	c	d	e

PART-A

Find the correct answer and mark it on the answer sheet on the **top page**. A correct answer gets **1 mark** and a wrong answer gets a **-(1/4) mark**.

1. The standard deviation of marks of 100 students is 13. Every student is later awarded 5 more marks. The standard deviation of the new set of marks is
   (a) 13
   (b) 18
   (c) $13 + \sqrt{5}$
   (d) $13 - \sqrt{5}$
   (e) none of the above.

2. The event A occurs whenever the event B occurs, then for any event $C \neq \phi$
   (a) $P(C|A) > P(C|B)$
   (b) $P(C|A) = P(C|B)$
   (c) $P(C|A) < P(C|B)$
   (d) $P(C|A^c) < P(C|B^c)$
   (e) none of the above.

3. $P(A \cup B) = 0.9$; $P(A) = 0.5$; $P(B) = 0.7$, then $P(A|B^c)$
   (a) is 1/2
   (b) is 1/3
   (c) is 3/4
   (d) is 2/3
   (e) can not be determined using the information given.

4. A box contains four balls numbered 1,2,3,4. Three balls are drawn at random, without replacement and arranged in increasing order of their numbers. The probability that ball number 3 is in the second place is
   (a) 1
   (b) 1/2
   (c) 0
   (d) 1/3
   (e) none of the above.

5. Two numbers are selected at random from {1,2,...,100} say N_1 and N_2. The probability of the event $A = \{N_1N_2 > \frac{1}{4}(N_1+N_2)^2\}$ is

(a) 1

(b) 1/2

(c) 1/3

(d) 0

(e) 2/3.

6. Three points are marked on a circle. The probability that the triangle got by these points is obtuse is

(a) 1/3

(b) 1/4

(c) 1/2

(d) 1/6

(e) 1/8.

7. Three people A,B,C are playing a game. To make a move the player needs to toss a fair coin and it should show heads, other wise it goes to the next player. If A is first to toss, then it is the turn of B and then of C. The probability that C will be the first one to make a move is

(a) 1/2

(b) 4/7

(c) 3/7

(d) 2/7

(e) 1/7.

8. A machine is in a working state if a least one of the components C_1, C_2, C_3 is working. At any given time the probabilities of C_1, C_2 and C_3 working are 1/3, 1/4 and 1/6 respectively. The probability that the machine is in a working state at any given time is

(a) not more than 3/4

(b) not less than 3/4

(c) equal to 1/2

(d) equal to 71/72

(e) equal to 1/72

9. $X \sim B(16,\ 1/2)$ $E\left[X(X-1)(X-2)(X-3)\right]$ is
   (a) 16x15x14
   (b) 15x14x12
   (c) 16x15x14x13
   (d) 16x15
   (e) 15x14x13

10. $P(X = a) = \frac{1}{7}$, $a \in \left\{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, 4\right\}$, $E\left(\frac{X^2-1}{X}\right)$ is
    (a) 0
    (b) 1
    (c) ∞
    (d) between 2 and 3
    (e) more than 1 and less than 2.

11. X and Y are two random variables and $E(X|Y=y) < 0$ for all values of y. Then
    (a) $E(X) > 0$
    (b) $E(X) < 0$
    (c) $E(X) = 0$
    (d) X and Y are uncorrelated
    (e) none of the above.

12. $X \sim N(1,1)$; $Y \sim N(2,2)$ and are independent. Then X+Y
    (a) is not a symmetric random variable and has expected value 3
    (b) is not a symmetric random variable and its expected value is not 3
    (c) has mean 3 and is symmetric about it
    (d) is not symmetric and has variance 3
    (e) none of the above.

13. A box contains N balls numbered 1,...,N, N is not known. n balls are drawn without replacement, suppose their numbers are $x_1, x_2, \ldots, x_n$. The Maximum Likelihood Estimator (MLE) of N is
    (a) $\frac{x_1 + \ldots + x_n}{n}$
    (b) $x_{(n)} = \max\{x_1, x_2, \ldots, x_n\}$
    (c) $x_{(1)} = \min\{x_1, \ldots, x_n\}$
    (d) $\frac{x_{(1)} + x_{(n)}}{2}$
    (e) none of the above.

14. X_1, X_2, X_3 is a random sample from the Bernoulli distribution B(p), i.e., $P(X_i = 1) = p$; $P(X_i = 0) = 1-p$, $0<p<1$ $i = 1,2,3$ and they are independent. Which of the following is sufficient for p?

(a) $X_1^2+X_2^2+X_3^2$

(b) $X_1+2X_2+X_3$

(c) $2X_1-X_2-X_3$

(d) X_1+X_2

(e) $3X_1+2X_2-4X_3$

15. The four places in a 2x2 matrix are 0 or 1 according as the outcome of a toss fo a fair coin is tails or heads respectively. The probability that A is singualr is

(a) 1/8

(b) 3/8

(c) 5/8

(d) 7/8

(e) none of the above.

16. The set $A = \{x;\ 2 < |x-2| < 4,\ x \in \mathbb{R}\}$ is

(a) an open interval

(b) a closed interval

(c) empty

(d) an open set

(e) a closed set.

17. The set $C = \{(x,y);\ 2x^2 - 4x + 3y^2 - 6y + 8 = 0,\ x \in \mathbb{R},\ y \in \mathbb{R}\}$ is

(a) the interior of a circle

(b) the interior of an ellipse

(c) an ellipse

(d) a circle

(e) none of the above.

18. $$\left[2\binom{2n}{2} + 4\binom{2n}{4} + \ldots + 2n\binom{2n}{2n}\right] - \left[\binom{2n}{1} + 3\binom{2n}{3} + \ldots + (2n-1)\binom{2n}{2n-1}\right]$$

$\left(\text{where } \binom{n}{r} = {}^nC_r\right)$ is equal to

(a) 0

(b) 1

(c) 2^n

(d) 3^n

(e) none of the above.

19. The rank of the matrix $\begin{pmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & 3 \end{pmatrix}$ is

    (a) 1

    (b) 2

    (c) 3

    (d) 4

    (e) 5.

20. A and B are two nxn matrices, which of the following is not always true?

    (a) A+B = B+A

    (b) Trace(A+B) = Trace(B+A)

    (c) Trace(AB) = Trace(BA)

    (d) Rank(-AB) = Rank(AB)

    (e) Rank(AB) = Rank(A).

21. $\lim\limits_{x \to 0} \frac{\sin x}{x}$

    (a) is 0

    (b) is 1

    (c) does not exist

    (d) is -1

    (e) is none of the above.

22. $\lim\limits_{n \to \infty} \frac{2^n}{n!}$

    (a) is 1/2

    (b) is 1

    (c) is 2

    (d) does not exist

    (e) none of the above.

23. The function $|x| - x$, $x \in \mathbb{R}$ is

    (a) increasing and not continuous at 0.

    (b) decreasing and not continuous at 0.

    (c) decreasing and continuous everywhere.

    (d) increasing and continuous everywhere.

    (e) none of the above.

24. The value of $\int_0^\infty e^{-x^2} dx$ is

(a) $\sqrt{2\pi}$

(b) $\sqrt{\pi}$

(c) $\frac{\sqrt{\pi}}{2}$

(d) $\sqrt{\frac{\pi}{2}}$

(e) $2\sqrt{\pi}$.

25. The negation of the statement, "Ramesh and Suresh missed the bus" is

(a) neither Ramesh nor Suresh missed the bus

(b) at least one of them missed the bus

(c) at most one of them did not miss the bus

(d) at least one of them did not miss the bus

(e) at most one of the missed the bus.

PART-B

There are 15 questions in this part. Each question carries 5 marks. Answer as many as you can. The maximum you can score is 75 marks. Brief proofs are needed for each question in the place provided below the question.

1. A boy has ten 1 rupee coins, five 2 rupee coins and two 5 rupee coins in his pocket (all the coins are similar in size). He has to buy a pen which costs Rs. 4/-. He takes out two coins from his pocket. Compute

   (i) the probability of taking out enough to buy the pen.

   (ii) the probability of taking out enough to buy the pen with only one of the coins.

   (iii) the probability of taking out exactly the amount needed to buy the pen.

   (iv) the probability of not taking out enough to buy the pen.

2. A child dropped a key in one of the 3 shelves S_1, S_2 or S_3 but now does not remember in which he dropped it. The shelves are cluttered and finding a small key in them is very difficult. The probability of finding the key in S_1 if it is there is 1/10, that of finding it in S_2 if it is dropped there is 1/20 and finding it in S_3 if it is dropped there is 1/30. S_2 was searched and the key was not found, evaluate the probabilities that the key is in (i) S_1 (ii) S_2 (iii) S_3.

3. X is a non-negative random variable for which $P(X>x) = e^{-5x}$ $x > 0$. (a) Determine the distribution of $Y = e^{-5X}$ (b) Evaluate its mean and variance.

4. There are 75 multiple choice questions with 5 alternatives in an examination. A candidate knows the correct answers to only 15 of them and decides to just randomly mark one of the alternatives. A correct answer in this examination gets +1 marks and a wrong answer get -1/4 marks for the candidate.
   (a) What are the expected marks of this candidate.
   (b) What can you say about the probability of his marks being less than 10 or greater than 20.
   (c) Evaluate the probability that this candidate will get more than 30 marks (you can use the suitable approximation).

5. A bag contains 100 balls numbered 1,2,...,100. A ball is drawn at random, replaced and then again a ball is drawn. Denote the numbers on them by X_1 and X_2 respectively. Evaluate
   (a) $P(X_1 = X_2)$
   (b) $P(X_1 > X_2)$
   (c) $P(X_1+X_2 = 33)$.

6. (a) Are uncorrelated random variables always independent?

   (b) $X \sim U(-2,2)$, compute the corelation coefficient between X and $Y = |X|$.

7. Let X be a random variable with probability density function (pdf) f(.) and cumulative distribution function (cdf) F(.). $X_1, X_2, \ldots, X_n$ is a random sample from this population. Let N_x = number of observations in the sample that are less than or equal to x. (a) Derive the distribution of N_x (b) Compute its mean and variance.

8. X_1 and X_2 are independent Poisson (λ) random variables. Let $T(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}$. Find $T^*(t) = E(T(X_1)|X_1+X_2 = t)$ and then evaluate $E(T^*)$.

9. Consider the following hypotheses:

H_0: X is Poisson with parameter $\lambda = 1/2$

H_1: $P(X = x) = \frac{1}{2^{x+1}}$ $x = 0,1,2,\ldots$.

(a) Classify these hypotheses as simple or composite.

(b) How will you test H_0 against H_1 at α level of significance based on a sample of size n. Will your cricitcal region be the "best critical region"? Give reasons.

10. $X_1, X_2, \ldots, X_n$ be iid random variables with common pdf given by

$$f(x,\theta) = \frac{1}{\theta} x^{\frac{\theta-1}{\theta}} \quad 0 < x < 1 \qquad \theta>0.$$

Find the MLE for θ and verify whether it is unbiased for it.

11. (a) Find the value of $\displaystyle\sum_{k=0}^{n} \sum_{j=0}^{n-k} \frac{(-1)^j \, n!}{j!k!(n-j-k)!}$.

(b) Prove that $\displaystyle\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$.

12. Find conditions on a, b and c such that

$$f(x) = ax^2 + bx + c \geq 0 \quad \forall\, x \in \mathbb{R}.$$

13. Consider the set $A = \left\{\frac{(-1)^n}{n},\ n=1,2,\ldots\right\} \cup \left\{\frac{(-1)^{n-1}}{n-1},\ n=1,2,\ldots\right\}$.

(a) Find its greatest lower bound and least upper bound.

(b) Do they lie in A?

(c) Does A have limit points?

14. (a) Display the set A Δ B - the symmetric difference of A and B by a Ven diagram.

(b) Prove that (A Δ B) Δ C = A Δ (B Δ C).

(c) U = {1,2,...,100}; A = all even numbers in U, B = all multiples of 3 in U. Identify A Δ B.

15. Shade the region $R_1 \cap R_2$ in $\mathbb{R}^2$ where

$$R_1 = \left\{(x,y);\ |x-2| \le 1,\ |y-1| \le 2\right\},\ R_2 = \left\{(x,y);\ y = x^2-1\right\}.$$