B. Tech Degree III Semester Examination, December 2006

IT/CS/EC/CE/ME/SE/EB/EI/EE 301 ENGINEERING MATHEMATICS III

(1999 Admission Onwards)

Time : 3 Hours — Maximum Marks : 100

I. (a) Find the half range sine series of $\frac{l}{2} - x \; 0 < x < l$. (7)

(b) Find the Fourier Transform of

$$f(x) = 1 \text{ when } |x| < 1$$
$$= 0 \text{ when } |x| > 1$$

Hence evaluate $\int_0^a \frac{\sin s}{s} ds$. (7)

(c) Convert $\int_0^2 \left(8 - x^3\right)^{1/3} dx$ into β function. (6)

OR

II. (a) An alternating current after passing through rectifier has the form

$$i = I_0 \sin x \text{ for } 0 \le x \le \pi$$
$$= 0 \quad \text{for } \pi \le x \le 2\pi$$

where I_0 is the maximum current and the period is 2π. Express i as a Fourier Series. (10)

(b) Find the Fourier Sine transform of $\frac{e^{-ax}}{x}$. (10)

III. (a) Prove that

(i) $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ (7)

(ii) $\int x J_0(x) dx = x J_1(x)$ (5)

(b) Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre Polynomials. (8)

OR

IV. (a) Prove that $\int_{-1}^{1} x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2 - 1}$. (10)

(b) Prove that

(i) $J_n^1(x) = \frac{1}{2}\left[J_{n-1}^{(x)} - J_{n+1}^{(x)}\right]$ (5)

(ii) $J_n(x) = \frac{x}{2n}\left[J_{n-1}^{(x)} + J_{n+1}(x)\right]$ (5)

V. (a) Solve

(i) $p^2 + q^2 = 4pq$ (4)

(ii) $p(1 + q) = qz$ (6)

(Turn Over)

(b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{e}\right)$. If it is released from rest from this position, find the displacement $y(x,t)$. (10)

OR

VI. (a) Derive the one-dimensional heat equation. (10)

(b) A rectangular plate with insulated surface is 8cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along one short edge $y = 0$ is given by

$u(x,0) = 100\sin\left(\frac{\pi x}{8}\right), 0 < x < 8$ while the two long edges $x = 0$ and $x = 8$

as well as the other short edge are kept at 0^0C, show that the steady state temperature at any point of the plate is given by $u(x,y) = 100e^{\frac{-\pi y}{8}} \sin\left(\frac{\pi x}{8}\right)$. (10)

VII. (a) The following is the probability density function of a random variable X

X	0	1	3	7	13
$P(x = x)$	$\frac{1}{8}$	α	$\frac{1}{6}$	$\frac{1}{4}$	β

(6)

Find α and β if $P\left[x^2 = 4\times -3\right] = \frac{1}{2}$

(b) Find the mean and variance of Poison distribution. (7)

(c) Probability that a batsman scores a century in a cricket match is $\frac{1}{3}$. Find the probability that out of 5 matches he may score century in

(i) Exactly two matches
(ii) No match
(iii) Atleast in one match (7)

OR

VIII. (a) Fit a straight line to the following data

X	:	3	5	8	4	10
Y	:	2	3	7	5	8

(10)

(b) The weekly wages of 1000 workmen are normally distributed around a mean of 70 and with a standard deviation of Rs.5. Estimate the number of workers whose weekly wages will be

(i) between Rs.70 and Rs.72 (ii) less than Rs.63
(iii) more than Rs.75 (10)

IX. (a) An exam was given to 50 students of college A and 60 students of College B. College B has mean score 79, standard deviation 7. College A has mean score 75, standard deviation 9. Is there significant difference between the mean score of two colleges? (10)

(b) A random sample of size 17 from a normal population is found to have $\bar{x} = 4.7,\ s^2 = 5.76$. Find 90% confidence interval for the mean of the population. (10)

OR

X. (a) A random sample of size 18 is taken from a normal distribution $N\left(H, \sigma^2\right)$. Test the Hypothesis $H_0 : \sigma^2 = 0.36$ against $H_1 : \sigma^2 > 0.36$ at $\alpha = 0.05$ given the sample variance $s^2 = 0.68$. (10)

(b) In two independent sample of sizes 8 and 10, the sum of squares of deviation of sample values from the respective sample means were 84.4 and 102.6. Test if the difference of population of variance is significant or not $\alpha = 5\%$. (10)

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