

BTS 115(A1)

B.Tech. Degree III Semester (Supplementary) Examination in Information Technology/Computer Science and Engineering/Electronics and Communication Engineering/Civil Engineering (Habitat Engineering and Construction Management)/Mechanical Engineering (CAD/CAM), June 2001.

IT/CS/EC/CE/ME MATHEMATICS - III
(1995 admissions)

Time: 3 Hours

Max. Marks: 100

(All questions carry equal marks)

I a) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$

b) Obtain Fourier series for the function $f(x)$ given by

$$f(x) = 1 + \frac{2x}{\pi}, -\pi \leq x \leq 0$$

$$= 1 - \frac{2x}{\pi}, 0 \leq x \leq \pi$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

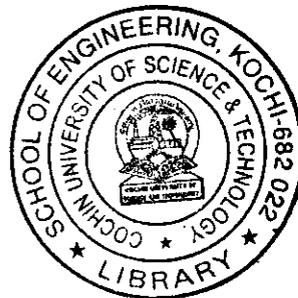
II a) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

b) Evaluate

(i) $\int_0^{\infty} e^{-x^2} dx$

(ii) $\int_0^{\infty} e^{-a^2x^2} dx, a > 0$

III a) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$



(P.T.O)

b) solve

(i) $z^2(p^2x^2 + q^2) = 1$

(ii) $(p^2 - q^2)z = x - y$

OR

IV a) A tightly stretched string with fixed end points $x = 0$ and $x = \ell$ is initially in a position given by $y = y_0 \sin^3(\pi x / \ell)$. If it is released from rest from this position, find the displacement $y(x, t)$.

b) Solve $p(1+q) = qz$

V a) In a certain factory turning out blades there is a small chance $\frac{1}{500}$ for any blade to be defective. The blades are supplied in packets of 10. Use Poisson's distribution to calculate the approximate number of packets containing 2 defective blades in a consignment of 10,000 packets.

b) Let X has the distribution $N(3, 4)$. Find the value of C such that $p(x > c) = 2p(x \leq c)$.

OR

VI a) Eight fair coins are tossed 256 times. In how many tosses do you expect at least one head?

b) If X is a random variable with $E(X) = 3$ and $\text{Var}(X) = 2$. Find h , such that $p\{|X - 3| < h\} \geq .99$.

VII a) Fit a second degree parabola to the following data:
 x : 1.0 1.5 2.0 2.5 3.0 3.5 4.0
 y : 1.1 1.3 1.6 2.0 2.7 3.4 4.1

b) If θ is the angle between the two regression lines, show that

$$\tan \theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

Explain the significance when $r = 0$ and $r = \pm 1$

OR

VIII a) If $4x - 5y + 33 = 0$ and $20x - 9y - 107 = 0$ are the regression lines. Find (i) the correlation coefficient (ii) the mean of x and y (iii) the standard deviation of y when the variance of x is 9.

b) For eight observations on a variable x and y the following details were obtained.

$$\sum x = 544 \quad \sum y = 552 \quad \sum x^2 = 37028$$

$$\sum y^2 = 38132 \quad \sum xy = 37560$$

Find the two regression lines. Estimate the value of y when $x = 68$.

IX a)(i) Find the directional derivative of $f(x, y, z) = 2xy + z^2$ at $(1, -1, 3)$ in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$.

(ii) Prove that $\text{div. grad}(r^n) = n(n+1)r^{n-2}$

b) Evaluate $\int_c \vec{F} \cdot d\vec{r}$, where $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$, curve c is the rectangle in the xy -plane bounded by $y = 0$, $x = a$, $y = b$, $x = 0$

OR

X a) Show that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative force field. Find the scalar potential.

b) Verify Stoke's theorem for a vector field defined by $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ in the rectangular region in the XOY - plane bounded by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$
