ME

BTS 115(A1)

**B.Tech. Degree III Semester (Supplementary) Examination in Information Technology/Computer Science and Engineering/Electronics and Communication Engineering/Civil Engineering (Habitat Engineering and Construction Management)/Mechanical Engineering (CAD/CAM), June 2001.**

**IT/CS/EC/CE/ME MATHEMATICS - III**

***(1995 admissions)***

*Time: 3 Hours* *Max. Marks: 100*

*(All questions carry equal marks)*

I a) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$

b) Obtain Fourier series for the function $f(x)$ given by

$$f(x) = 1 + \frac{2x}{\pi}, -\pi \le x \le 0$$

$$= 1 - \frac{2x}{\pi}, 0 \le x \le \pi$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$

**OR**

II a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

b) Evaluate

(i) $\int_o^\infty e^{-x^2} dx$

(ii) $\int_o^\infty e^{-a^2 x^2} dx, \ a > 0$

III a) Solve $\left(x^2 - y^2 - z^2\right)p + 2xyq = 2xz$

(P.T.O)

b) solve

(i) $z^2\left(p^2x^2+q^2\right)=1$

(ii) $\left(p^2-q^2\right)z=x-y$

**OR**

IV a) A tightly stretched string with fixed end points $x = 0$ and $x = \ell$ is initially in a position given by $y=y_o\sin^3(\pi x/\ell)$. If it is released from rest from this position, find the displacement $y(x, t)$.

b) Solve $p(1+q)=qz$

V a) In a certain factory turning out blades there is a small chance $\frac{1}{500}$ for any blade to be defective. The blades are supplied in packets of 10. Use Poisson's distribution to calculate the approximate number of packets containing 2 defective blades in a consignment of 10,000 packets.

b) Let X has the distribution N(3, 4). Find the value of C such that $p(x>c)=2p(x\le c)$.

**OR**

VI a) Eight fair coins are tossed 256 times. In how many tosses do you expect at least one head?

b) If X is a random variable with E(X) = 3 and Var(X) = 2. Find h, such that $p\{|X-3|<h\}\ge .99$.

VII a) Fit a second degree parabola to the following data:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$: | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| y: | 1.1 | 1.3 | 1.6 | 2.0 | 2.7 | 3.4 | 4.1 |

b) If $\theta$ is the angle between the two regression lines, show that

$$\tan\theta=\frac{1-r^2}{r}\cdot\frac{\sigma x\ \sigma y}{\sigma x^2+\sigma y^2}$$

Explain the significance when $r = 0$ and $r = \pm 1$

Contd...3



**OR**

VIII a) If $4x-5y+33=0$ and $20x-9y-107=0$ are the regression lines. Find (i) the correlation coefficient (ii) the mean of $x$ and y (iii) the standard deviation of y when the variance of $x$ is 9.

b) For eight observations on a variable $x$ and y the following details were obtained.

$\Sigma x=544$ $\Sigma y=552$ $\Sigma x^2=37028$

$\Sigma y^2=38132$ $\Sigma xy=37560$

Find the two regression lines. Estimate the value of y when $x=68$.

IX a)(i) Find the directional derivative of $f(x,y,z)=2xy+z^2$ at $(1,-1,3)$ in the direction of the vector $\bar{i}+2\bar{j}+2\bar{k}$.

(ii) Prove that div. grad $\left(r^n\right)=n(n+1)r^{n-2}$

b) Evaluate $\int_c \bar{F}.d\,\bar{r}$, where $\bar{F}=\left(x^2+y^2\right)\bar{i}-2xy\bar{j}$, curve c is the rectangle in the $x$y-plane bounded by $y=0,\ x=a,\ y=b,\ x=0$

**OR**

X a) Show that $\bar{F}=\left(2xy+z^3\right)\bar{i}+x^2\bar{j}+3xz^2\bar{k}$ is a conservative force field. Find the scalar potential.

b) Verify Stoke's theorem for a vector field defined by $\bar{F}=\left(x^2-y^2\right)\bar{i}+2xy\bar{j}$ in the rectangular region in the XOY - plane bounded by the lines $x=0,\ x=a,\ y=0$ and $y=b$

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