SOLUTIONS

1. **Answer A.** $\dfrac{302.476 \times 0.040328}{5.96247}$ is approximately $\dfrac{300 \times 0.04}{6} = \dfrac{3 \times 4}{6} = \dfrac{12}{6} = 2$.

2. **Answer B.** Let $BC = x$, so $AB = 3x$, and the area is $3x^2$. By Pythagoras, $AC^2 = AB^2 + BC^2$, so $25 = 9x^2 + x^2 = 10x^2$. Thus $x^2 = \frac{5}{2}$ so the area $= 3x^2 = \frac{15}{2}$.

3. **Answer C.** $x^3 = x(x^2) = x(x+3) = x^2 + 3x = (x+3) + 3x = 4x + 3$.

4. **Answer C.** Let $T = 2006$. Then the expression is equal to

$$T^2 - (T+1)(T-1) + (T+2)(T-2) - (T+3)(T-3)$$

which equals $T^2 - (T^2 - 1) + (T^2 - 4) - (T^2 - 9) = 0 + 1 - 4 + 9 = 6$.

5. **Answer B.** Draw up a table of the numbers and their differences:

$$\begin{array}{ccccccccccc} 2 & & 3 & & 6 & & 15 & & 42 & & X \\ & 1 & & 3 & & 9 & & 27 & & Y. & \end{array}$$

The numbers in the second row are powers of 3, so $Y = 81$. Thus $X - 42 = 81$, so $X = 123$. Alternatively, notice that the sequence is generated by the formula, $x_{n+1} = 3x_n - 3$ with $x_0 = 2$. Therefore $X = 3 \times 42 - 3 = 123$.

6. **Answer A.** We need only consider the last digit, so look at the last digits of the powers of 3: 3^1 ends in 3, 3^2 ends in 9, 3^3 ends in 7, 3^4 ends in 1, then 3^5 ends in 3 again. Thus the sequence of last digits $\{3, 9, 7, 1\}$ repeats itself in blocks of length 4. Since 444 is at the end of a block, it follows that 333^{444} ends in 1.

7. **Answer E.** $\dfrac{2x - 3y}{x + 2y} = 3$, so $2x - 3y = 3x + 6y$, giving $-x = 9y$, so $x = -9y$.
Thus $\dfrac{2x + y}{3x + 10y} = \dfrac{-18y + y}{-27x + 10y} = \dfrac{-17y}{-17y} = 1$.

8. **Answer C.** $4^{n+1} + 4^{n+2} = 4^{n+1}(1 + 4) = 5 \times 4^{n+1} = 5 \times (2^2)^{n+1} = 5 \times 2^{2n+2}$.

9. **Answer A.** Put $n = 2$: $f(2) = f(1) + 2 \times 2 - 1 = 0 + 4 - 1 = 3$. Similarly, $f(3) = f(2) + 2 \times 3 - 1 = 3 + 6 - 1 = 8$.
[In fact, it can be shown that $f(n) = n^2 - 1$ for all $n \geq 0$.]

10. **Answer B.** The interior angles in a regular n-sided polygon are equal to $180(1 - \frac{2}{n})^\circ$, so $A\widehat{B}C = 180(1 - \frac{2}{5})^\circ = 108^\circ$. Since triangle ABC is isosceles, it follows that $C\widehat{A}B = \frac{1}{2}(180 - 108)^\circ = 36^\circ$, and this is the same as $O\widehat{A}B$. Similarly, $O\widehat{B}A = 36^\circ$. Finally, $E\widehat{O}C = A\widehat{O}B = (180 - 2 \times 36)^\circ = 108^\circ$.

11. **Answer A.** The point A is the intersection of the lines $y = 4$ and $y = 2x + 2$, so $4 = 2x + 2$, giving $x = 1$ and $y = 4$, so A has co-ordinates $(1, 4)$. Similarly, B is the intersection of the lines $y = 4$ and $y = 10 - 2x$, so B has co-ordinates $(3, 4)$. Finally, C is the intersection of the lines $y = 2x + 2$ and $y = 10 - 2x$, so C has co-ordinates $(2, 6)$. Thus the triangle has base AB of length $3 - 1 = 2$ (difference in x-values) and height $6 - 4 = 2$ (difference in y-values), so its area is $\frac{1}{2} \times 2 \times 2 = 2$.

12. **Answer E.** Draw up a table:

	Aneesa	Bongi	Carol
Dress			
Shoes	Yellow		

Bongi does not have the black dress or shoes, so Carol must have the black shoes. Carol's dress is the same colour as her shoes, so Carol has the black dress. Aneesa's dress is not the same colour as her shoes, so she has the green dress. Thus Bongi must be wearing the yellow dress and the green shoes.

13. **Answer A.** As a rough estimate, find one-eighth of the circumference of a circle with area 500. If the radius is r, then $\pi r^2 = 500$, so $r^2 = \frac{500}{\pi} \approx \frac{500\times 7}{22} \approx 160$. Therefore $r \approx \sqrt{160} \approx 12.6$, and one-eighth of the circumference $= \frac{1}{4}\pi r \approx \frac{11}{14} \times 12.6 \approx 10$.

    [If each side has length x, then the exact area is $2x^2(\sqrt{2}+1)$, but you do not need this fact.]

14. **Answer B.** $a_1 + a_2 + \cdots + a_{99} + a_{100} = (\frac{1}{1} - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + \cdots + (\frac{1}{99} - \frac{1}{100}) + (\frac{1}{100} - \frac{1}{101})$. If you remove the brackets, then you see that everything cancels out except for the first and last fractions. Thus the sum is equal to $\frac{1}{1} - \frac{1}{101} = \frac{101-1}{101} = \frac{100}{101}$.

15. **Answer C.** Suppose he spends x minutes travelling through urban areas, and $120-x$ minutes through rural areas. Then the distance will be $\frac{x}{60} \times 40 + (2 - \frac{x}{60}) \times 105 = 210 + \frac{40-105}{60}x = 210 - \frac{13}{12}x$, which is equal to 177.5. Therefore $x = \frac{12}{13}(210 - 177.5) = 30$.

16. **Answer B.** Since DF is parallel to BE, it follows that triangle BEC is similar to triangle DFC, so $B\widehat{E}C$ is a right angle. Thus triangle ABC is isosceles, and $AB = BC = 2\times DC = 10$.

17. **Answer C.** We start by finding the *greatest* number of balls he can take *without* having three of the same colour. Obviously, the answer is six (two balls of each colour). If he takes one more ball, for a total of seven balls, then he must have three balls of the same colour (either blue or yellow).

18. **Answer C.** $y = 1 + 2^{-p} = 1 + \dfrac{1}{2^p} = \dfrac{2^p+1}{2^p} = \dfrac{x}{x-1}$.

19. **Answer A.** Rewrite the equation as $(x^2 + 6x + 9) + (y^2 - 4y + 4) = 0$. By completing the square we see that $(x+3)^2 + (y-2)^2 = 0$. Now a perfect square cannot be negative, so if the sum of two perfect squares is zero, then both of them must be zero. Therefore $x + 3 = 0$ and $y - 2 = 0$, so $x + y = -3 + 2 = -1$.

20. **Answer E.** Let P and Q be the centres of the circles closest to the vertices A and B respectively, and let T and U be the points where these circles touch the side AB. Firstly, $TU = PQ = 2$. Next, APT is a $(30°, 60°, 90°)$-triangle with $PT = 1$, so $AP = 2$ and $AT = \sqrt{3}$. Finally, $BU = AT = \sqrt{3}$, so $AB = 2 + 2\sqrt{3}$.