DECEMBER 2007

Code: AE07
Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING

Time: 3 Hours

Max. Marks: 100

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. The divided difference $f[x_i, x_{i+1}]$ is equal to (if $x_{i+1} - x_i = h$)

(A) $\Delta f_{i+1}/h$. **(B)** $\nabla f_{i+1}/h$.

(C) $\nabla f_{i+1}/h^2$. **(D)** $\nabla^2 f_{i+1}/h^2$.

b. An approximation to $f'(x_k)$ is written as

$$f'(x_k) = [a\,f(x_{k+1}) + b\,f(x_k) + c\,f(x_{k-1})]/(2h).$$

Then, the values of the coefficients (*a, b, c*) are

(A) (1, 0, 1). **(B)** (1, –2, 1).
(C) (–3, 4, 1). **(D)** (1, 0, –1).

c. Error in composite Simpson's rule for integrating $\int_a^b f(x)dx$ is bounded by $M(b-a)h^4 \max\left|f^{(4)}(x)\right|$. The value of *M* is

(A) $1/180$. **(B)** $1/90$.
(C) $1/2880$. **(D)** $1/160$.

d. Newton-Raphson method for computing $35^{1/5}$ can be written as $x_{n+1} = g(x_n)$, where $g(x_n) =$

(A) $x_n^5 - 35$. **(B)** $(4x_n^5 + 35)/(5x_n^4)$.

(C) $(5x_n^4 + 35)/(5x_n^4)$. **(D)** $(4x_n^5 - 35)/(5x_n^4)$.

e. The Runge-Kutta method $y_{n+1} = y_n + (h/2)[f(x_n, y_n) + f(x_{n+1}, y_n + h f_n)]$, when applied to the initial value problem $y' = a y,\ y(0) = 1,$ gives $y_{n+1} = E y_n$. Then, *E* is equal to

(A) 1 + *ah*. **(B)** $1 + a h + a^2 h^2$.

(C) $1 + a h + (a^2 h^2)/2$. **(D)** $1 + a h + 2a^2 h^2$.

f. The linear least squares polynomial approximation to the following data

x	0	1	2	3
$f(x)$	1	2	5	10

is given by $y(x) = 3x$. Then, the least squares error is given by

(A) 0. **(B)** 1.5.

(C) 4.0. **(D)** 5.2.

g. What will be the output of the following program?

```
void main( )      {
    int arr[ ] = {10, 11, 12, 13, 14};
    int i,  *p;
    for (p=arr, i=0; p+i<=arr+4; p++, i++)
        printf("%d", *(p+i));    }
```

(A) 10 11 12 13 14 **(B)** 10 11 12

(C) 11 13 **(D)** 10 12 14

h. What will be the output of the following program?

```
#include <stdio.h>
main(argc,argv)
int argc;
char *argv[];
{
        int i;
        for (i = 1;i<argc;i++);
                printf("%s",argv[i]);
        prinft("\n");
}
```

if the following command in typed,

$ myecho hello world

(B) myecho hello. **(B)** no output is produced.

(C) myecho world. **(D)** hello world.

i. What will be the output of the following program?

main( )

```
{   static int a [5] = { 1,2,3,4,5 };
    int i ;
    for ( i = 0; i<5; i ++ )
    {     printf("%d",*a);
           a = a + 1;
     }
}
```

(A) 1 2 3 4 5. **(B)** Error
(C) Undefined Output **(D)** 5 4 3 2 1

j. What will be the output of the following programme segment?

```
int m, n=10;
m = n++ * n++;
printf("%d, %d, %d, %d, %d", m, n, m++, m--, --m);
```

(A) 100, 12, 100, 101, 99 **(B)** 100, 12, 100, 111, 109
(C) 110, 12, 110, 111, 109 **(D)** 110, 11, 100, 101, 99

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find an interval of unit length which contains the smallest negative root in magnitude of the equation $2x^3+3x^2+2x+5=0$. Using the end points of this interval as initial approximations, obtain the root correct to three decimals using the Regula-Falsi method. **(8)**

b. It is known that the iterative method $x_{n+1}=\frac{1}{8}x_n\left(6+\frac{3a}{x_n^2}-\frac{x_n^2}{a}\right)$ converges to $\sqrt{a}$. Find the order of the method and the leading term of the error. **(8)**

Q.3 a. Set up the Gauss-Seidel iteration scheme in matrix form to solve the system of equations $\begin{bmatrix}1 & 2 & 4\\ 2 & 1 & 2\\ 4 & 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-1\\ 5\\ 3\end{bmatrix}$ Show that the iteration scheme diverges. **(9)**

b. Solve the system of equations $\begin{array}{ccccccc}4x & + & y & + & z & = & 1.5\\ x & + & 3y & + & 2z & = & 2.8\\ 2x & - & 3y & + & 6z & = & 2.3\end{array}$ by Gauss elimination method. **(7)**

Q.4 a. Find the Choleski decomposition of the matrix $\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 2 \\ 1 & 2 & 4 \end{bmatrix}$. Hence find its inverse. **(10)**

b. The nonlinear system of equations $x^2 + y^2 - xy = 0.1$, $3x^2 - y^2 + 5xy = 0.13$ has a solution near $x = 0.1, y = 0.3$. Perform one iteration of the Newton's method to improve the solution. **(6)**

Q.5 a. Write a *C* program for finding a simple root of $f(x) = 0$ using Regula - Falsi method. Input the end points of the interval (*a*, *b*) in which the root lies, maximum number of iterations *n* and the error bound 'bound'. If the given number of iterations *n*, is not sufficient, the program should display "Iterations are not sufficient". Write the function subprogram using $f(x) = x/(x^3 + 1)$. **(7)**

b. If $\Delta\left(\frac{f_i}{g_i}\right) = a\Delta f_i + b\Delta g_i$, and Δ is the forward difference operator, then find the expressions for *a* and *b*. **(4)**

c. A table of values for the function $f(x) = (1 + x)^4$ in [0, 3] is to be constructed at equispaced points. If we want to use linear interpolation in this table, then find the largest step length *h* that can be used to construct the table, if error of interpolation is to be $\leq 1 \times 10^{-4}$. **(5)**

Q.6 a. Construct the interpolating polynomial that fits the data

x	0	1	3	6	10
f(x)	−3	−1	27	219	1007

(8)

b. Use the method of least squares to fit the curve $y = a + bx + cx^2$, to the table of values

x	1	2	3	4	5
y(x)	2.5	4.5	3.7	5.0	4.2

(8)

Q.7 a. Write a *C* program to evaluate the integral $\int_a^b f(x)dx$ by Simpson's rule with $2N+1$ nodal points. Write a function subprogram using $f(x) = e^{-x}/(1+x^2)$. **(9)**

b. Compute an approximation to $f'(2)$ using the formula $f'(x) = [3f(x) - 4f(x-h) + f(x-2h)]/(2h)$, and possible values of h from the data

x	1.2	1.6	1.8	1.9	2
f(x)	2.5012	2.8019	2.9653	3.0496	3.1353

(7)

Q.8 a. Evaluate the integral $\int_2^3 \frac{\cos 2x}{1+\sin x}dx,$ where *x* is in radians, using Simpson's rule with 3, 5, 9 points. Improve the approximation to the value of the integral using Romberg integration. **(7+3)**

b. Derive the two point Gauss-Hermite formula

$$\int_{-\infty}^{\infty} e^{-x^2} f(x)dx = a f\left(-\frac{1}{\sqrt{2}}\right) + b f\left(\frac{1}{\sqrt{2}}\right). \qquad \left[\text{It is given that } \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}\right].$$

(6)

Q.9 a. Use Euler's method to compute an approximation to y(1.2) for the initial value problem $y' = \frac{x-y}{x+y}, \quad y(1) = 2, \; h = 0.05$ **(5)**

b. A Runge-Kutta method of second order, for solving the initial value problem $y' = f(x, y), y(x_0) = y_0$ is given by

$$y_{n+1} = y_n + k_2,\ k_1 = h\,f(x_n, y_n),\ k_2 = h\,f\left(x_n + \frac{h}{2}, y_n + \frac{1}{2}k_1\right)$$

(i) Find the truncation error of the method.

(ii) Using the above method, obtain an approximation to y(1.1) for the initial value problem $y' = 2x^2 + 3y^2,\ y(1) = 0.5$ with h = 0.1.

(6+5)