DECEMBER 2006

Code: A-07
Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING
Time: 3 Hours
Max. Marks: 100

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. Euler's method when applied to the initial value problem $y' = \lambda y, y(0) = 1$, gives $y_{n+1} = E y_n$. Then E is equal to

(A) $(1+\lambda h)^2$. **(B)** $1+\lambda h$.
(C) λh. **(D)** $1-\lambda h$.

b. In the integral $\int_{-1}^{1} f(x)dx$, the values of the integrand are $f(-1/\sqrt{3}) = 0.3842$, and $f(1/\sqrt{3}) = 0.1592$. Then, the Gauss-Legendre two point formula gives

(A) 0.5434. **(B)** 0.5406.
(C) 0.2250. **(D)** 0.6536.

c. The numerical differentiation formula $f'(x_0) = [f(x_0+h) - f(x_0)]/h$ is given. Then, the error is defined by

$$f'(x_0) - [\{f(x_0+h) - f(x_0)\}/h] = Mf''(\xi), x_0 < \xi < x_1$$

The value of M is

(A) $-h^2/2$. **(B)** $h^2/2$.
(C) $h/2$. **(D)** $-h/2$.

d. The linear least squares polynomial approximation to the following data is

x	-1	1	2	3
f(x)	4	6	13	32

(A) $3+2x$ **(B)** $3.214-4.5x$.
(C) $5.857+6.314x$. **(D)** $6.314+5.857x$.

e. If Δ is the forward difference operator, then $\Delta(1/f_i)$ is equal to

(A) $\Delta f_i / f_i^2$. (B) $\Delta f_i / \Delta^2 f_i$.

(C) $\Delta f_i / (f_i f_{i+1})$. (D) $-\Delta f_i / (f_i f_{i+1})$.

f. The system of equations $\begin{bmatrix} 2 & -p \\ -p & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$ is being solved by Gauss-Jacobi method. The iteration converges if $|p|$ is less than

(A) 2. (B) 2.5

(C) 3.0 (D) 4.0

g. The iteration method $x_{k+1} = \frac{1}{9}\left[5x_k + \frac{5N}{x_k^2} - \frac{N^2}{x_k^5}\right]$ converges and is determining a particular quantity. This quantity is

(A) $N^{1/2}$. (B) $N^{1/3}$.

(C) $N^{1/4}$. (D) $N^{1/5}$.

h. What is the output of the following C program

```
main()
{
  int a[ ] = { 3.8, 4.4, 5, 7.6, 6 };
  int i, *s = a,*t = a;
       for ( i=0; i<5; i++ )
       {
        printf ( "% d",*a );
          ++t ;
       }
       for ( i=0; i<5; i++ )
       {
        printf ( "% d",*s );
          ++s ;
       }
 }
```

(A) 3333334567. **(B)** 3333344586.

(C) 3333335576. **(D)** 3334444586.

i. What is the output of the following C program

```
main()
{
   int i = 2;
   switch ( i )
   {
     default : printf ( "hundred" );
      case 1: printf  ( "two hundred" );
      break ;
```

```
            case 2: printf  ( “five hundred” );
            break ;
            case 3: printf  ( “six hundred” );
            break ;
            }
    }
```

(A) hundred. **(B)** two hundred.

(C) five hundred. **(D)** six hundred.

j. What is the output of the following C program

```
# define int char
main()
{
  int i = 87;
  printf ("sizeof ( i ) = %d", sizeof ( i ));
}
```

(A) 87 **(B)** 1

(C) 8 **(D)** 2

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. The smallest positive root of the equation $x^2 - 4x + \ln x = 0.4$ is to be determined. Find an interval of unit length in which the root lies. Taking the end points of this interval as initial approximations, obtain the root correct to three decimal places using secant method. **(8)**

b. A method for finding $\sqrt{N}, N > 0$ is written as

$$x_{k+1} = \frac{1}{2}x_k\left(3 - \frac{x_k^2}{N}\right) + \frac{3}{8}x_k\left(1 - \frac{x_k^2}{N}\right)^2$$

Find the order of the method and the error constant. **(8)**

Q.3 a. Solve the system of equations

$$\begin{aligned} 3x + y + 4z &= 10 \\ x + 3y + z &= 0 \\ 2x + 3y + 5z &= 9 \end{aligned}$$

using the Gauss-elimination method. **(8)**

b. The system of equations $3x^2 - y^2 + xy = 2.5, x^2 + 3y^2 - xy = 7$ has a solution near $x = 1.3$, $y = -1.1$. Perform two iterations of the Newton’s method to improve the solution. **(8)**

Q.4 a. The following system of equations is given

$$\begin{aligned} 4x + y + 2z &= 4 \\ 3x + 5y + z &= 10 \\ x + y + 3z &= 10 \end{aligned}.$$

The system is to be solved by the Gauss-Seidel iteration method. Obtain the iteration matrix of the method. Hence, find the rate of convergence of the method. **(10)**

b. Find the Choleski decomposition of the symmetric matrix

$$\begin{bmatrix} 6.25 & 0 & 0 & 5 \\ 0 & 6.25 & 0 & -3.75 \\ 0 & 0 & 2.25 & 0 \\ 5 & -3.75 & 0 & 42.25 \end{bmatrix}$$

(6)

Q.5 a. An equi-spaced data with step length *h* is to be constructed for a function *f* (*x*) and approximations to function values are to be calculated by linear interpolation from this table. Find the maximum step size that can be used. The function values are to be correct to 10^{-6} for the function $f(x) = 2^x$ over the interval [0, 1]. **(8)**

b. Write a C program for finding a simple root of *f* (*x*) = 0 using Newton-Raphson method. Input initial approximation to the root as *xold* , maximum number of iterations as *n*, and error tolerance as *tol*. Output the value of the root , number of iterations taken and the value of *f* (*root*). If iterations *n* is not sufficient, the program should indicate the same. Assume that $f(x) = x^5 + 6x^3 - 8.$ **(8)**

Q.6 a. A physicist wants to fit an approximation of the form $f(x) = a + (b/x)$ to a data. Derive the normal equations using least squares approximation. Fit the above approximation to the following data.

x	1.0	1.2	1.4	1.5	1.6
f(x)	5.7	5.3	5.0	4.8	4.7

(8)

b. Construct the backward difference table for the data

x	0.0	0.2	0.4	0.6	0.8
f(x)	0.500	1.108	1.764	2.516	3.412

Hence, compute an approximation to $f(0.7)$. **(8)**

Q.7 a. A differentiation formula is defined as follows

$$D(h) = [-3y(x) + 4y(x+h) - y(x+2h)]/(2h).$$

Using the Taylor series expansions, show that $y'(x) - D(h) = c_1 h^2 + c_2 h^3 + \ldots$ and write the expression for c_1. Use the above formula to compute approximations to $y'(1.0)$ with step lengths *h* = 0.2 and *h* = 0.1 from the following table of values

x	1.0	1.1	1.2	1.4
f(x)	2.000	2.331	2.728	3.744

(8)

b. The error in composite Simpson's rule with step length *h* for computing $\int_a^b f(x)dx$ is bounded by $|\text{Error}| \le [(b-a)h^4 M_4]/180,\ M_4 = \max |f^{(4)}(x)|,\ x \in [a,b]$.

Assume that the composite Simpson's rule is being used to compute $\int_0^1 \frac{dx}{3+x}$.

Using the above error estimate find *h* such that $|\text{Error}| \le 10^{-8}$.

(8)

Q.8 a. Derive the two point Gauss-Laguerre formula of integration $\int_0^\infty e^{-x} f(x)dx = af(2-\sqrt{2}) + b\,f(2+\sqrt{2})$. Hence, evaluate the integral $\int_0^\infty e^{-x}\log(1+x)dx$. **(6+3)**

b. Write a C program for solving an initial value problem $y' = f(x,y), y(x_0) = y_0$ using Euler's method. Initial values x_0, y_0, step length *h* and final value *xf* up to which the computations are to be carried out are to be read. Write $f(x,y) = x^2 + y^2$ as a function sub program. Output all the given data and all computed values. **(7)**

Q.9 a. Use Taylor series method of second order to compute *y*(1.4) with *h* = 0.2, for the initial value problem $y' = \sqrt{x+y},\ y(1) = 1$. **(8)**

b. Error in Euler's method for solving the initial value problem $y' = f(x,y), y(x_o) = y_0$ is given by $(h^2/2)y''(\xi)$. Euler's method is being used to solve the initial value problem $y' = \cos y, y(0) = 1$. Find the largest value of the step length *h* that can be used such that the magnitude of the error is ≤ 0.04. **(8)**