Your Roll No .

B.Sc. (Hons.)/I   **J**

BIOCHEMISTRY—Paper IV

(Mathematics and Statistics)

(Admissions of 2000 and onwards)

Time   3 *Hours*   *Maximum Marks*   60

Attempt *Five* questions in all, selecting at least

two questions from each Section

Section A

1   (*a*)   If   4

$$f(x) = \log \frac{\sqrt{a + bx} - \sqrt{a - bx}}{\sqrt{a + bx} + \sqrt{a - bx}},$$

find for what value of

$$x, \frac{1}{f'(x)} = 0$$

(b) Let   4

$$f(x) = \frac{1}{x}, \quad x \in (0, 1]$$

Examine the validity of the hypothesis and conclusion

of Lagrange's Mean Value Theorem for the above

function

(c) If   4

$$x = \tan(\log y),$$

prove that

$$(1 + x^2)\frac{d^{n+1}y}{dx^{n+1}} + (2nx - 1)\frac{d^n y}{dx^n} + n(n - 1)\frac{d^{n-1}y}{dx^{n-1}} = 0$$

2   (a) Find the equation of the normal to the curve   4

$$y(x - 2)(x - 3) - x + 7 = 0$$

at the point where it cuts the axis of X

(b) If   4

$$u = \tan^{-1}\frac{(x^3 + y^3)}{(x - y)},$$

then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

(c)   Prove that for the ellipse   4

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

the radius of curvature $\rho = \dfrac{a^2 b^2}{p^3}$, where p is the perpendicular distance from its center upon the tangent at (x, y)

3   (a)   Find all the asymptotes of the curve .   4

$$3x^3 + 2x^2 y - 7xy^2 + 2y^3 - 14xy + 7y^2 + 4x + 5y = 0$$

(b)   Find the maxima and the minima of the function f defined by :   4

$$f(x) = 2x^3 - 15x^2 + 36x + 10.$$

for $x \in \mathbf{R}$.

(c)   Evaluate   4

$$\lim_{x \to \pi/2} \left(\sec x - \frac{1}{1 - \sin x} \right).$$

4   (a) Trace the curve   4

$$y(1 - x^2) = x^3$$

(b) Solve the following differential equation   4

$$(D^2 - 4D + 3)y = 5e^{-2x} + 140$$

(c) Find the position and nature of multiple points of the following curve   4

$$x^3 + y^3 + a^3 = 3axy$$

5   (a) Solve the equation   4

$$27x^3 + 42x^2 + - 28x - 8 = 0$$

the roots being in geometric progression

(b) Evaluate   4

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha + 2\beta)+$$

to n terms, provided $\beta \neq 2k\pi$

(c) Find the rank of the matrix   4

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & 0 & 3 \end{pmatrix}$$

6 (a) If   4

$$I_n = \int_0^{\pi/2} \sin^n x \, dx \quad \text{and} \quad n > 1,$$

prove that .

$$I_n = \frac{n-1}{n} I_{n-2}$$

(b) Evaluate   4

$$\int_0^{\pi/2} \log \sin x \, dx$$

(c) Examine the convergence of the following sum   4

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \quad .$$

Section B

7 (a) If $\bar{X}_w$ is the weighted mean of X_i's with weights

w_i then prove that   4

$$\left(\sum_{i=1}^n w_i \right) \left(\sum_{i=1}^n w_i (X_i - \bar{X}_w)^2 \right) = \sum_{i=1}^n \sum_{j>i} w_i w_j (X_i - X_j)^2$$

where $\Sigma_{i=1}^n w_i \neq 0$

(b) From a sample of n observations, the arithmetic mean

and variance are calculated It is then found that

one of the values x_1 is in error and should be replaced

by x_1' Show that the adjustment to the variance to correct this error is

$$\frac{1}{n}(x_1' - x_1)\left\{x_1' + x_1 - \frac{x_1' - x_1 + 2T}{n}\right\}$$

where T is the total of the original observations 4

(c) A and B throw a pair of dice A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6 If A begins show that his chance of winning is $\dfrac{30}{61}$ 4

8. (a) A person draws 2 balls from a bag containing 3 white and 4 red balls If he is to receive Rs 10 for every white ball drawn and Rs 20 for each red ball Find his expectation   4

(b) A car hire farm has two cars which it hires out day by day The number of demands for a car on each day is distributed as a Poisson distribution with mean 1 5 Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused Given $(e^{15} = 0.2231)$ 4

(c) In a certain examination 2000 students appeared

The average marks obtained were 50% with standard

deviation was 5% If the marks are normally

distributed, how many students do you expect to

obtain more than 60% marks ? (Area under the

standard normal curve from $x = 0$ to $x = 2$ is

0 4772)   4

9   (a) The following table gives the joint probability

distribution of (X, Y)

X \ Y	1	2	3
1	$\dfrac{5}{27}$	$\dfrac{4}{27}$	$\dfrac{2}{27}$
2	$\dfrac{1}{27}$	$\dfrac{3}{27}$	$\dfrac{3}{27}$
3	$\dfrac{3}{27}$	$\dfrac{4}{27}$	$\dfrac{2}{27}$

Find

(i) Marginal distribution of X and Y,

(*u*) _ P[1 $\leq$ X $\leq$ 2, Y $\geq$ 2],

(*uı*) V(X), V(Y),

(*ıv*) Cov(X, Y)

Symbols carry usual meaning   1+1+1+1=4

(*b*)   Let X_1, X_2 be two random variables with zero means and variances σ_1^2 and σ_2^2 respectively having correlation coefficient r between them Determine the values of the constants a and b which are independent of r such that the random variables $X_1 + aX_2$ and $X_1 + bX_2$ are uncorrelated   4

(*c*)   The lines of regression obtained in a certain correlation analysis are

$$X + 9Y = 7 \text{ and } Y + 4X = 16\frac{1}{3}$$

Find

(*ı*)   Correlation coefficient of X and Y,

(*ıı*) σ_X, σ_Y 4

10 (a) In a large city A, 20% of a random sample of 900 school boys had a certain physical defect In another large city B, 18 5% of a random sample of 1600 school boys had same defect Is the difference between the proportions significant ?   4

(b) In an experiment on immunization of cattle from tuberculosis the following results were obtained ·

	Affected	Unaffected
Inocculated	17	23
Not inocculated	8	12

Examine the effect of vaccine in controlling the incedence of the disease   4

[Given $\chi^2_{0\ 05}$ = 3 84 for 1 degree of freedom]

(c) For a random sample of 10 pigs fed on a diet A the increase in weight in pounds in a certain period

were, 10, 6, 16, 17, 13, 12, 8, 14, 15, 9 lbs For

another sample of 12 pigs fed on diet B the increase

in the same peirod were 7, 13, 32, 15, 12, 14, 18,

8, 21, 23, 10, 17 lbs Test whether diets A and B

differ significantly as regard the effect on the increase

in weight You may used the fact that 5% value of

t for 20 d f is 2 09   4