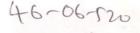
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FACULTY OF SCIENCE

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Paper 1.1

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Time : 3 Hours]

[Max. Marks : 100

Answer all questions.

- **Section A** (Marks : 8 × 5 = 40)
- 1. Prove that $P \leftrightarrow Q$ and $(P \rightarrow Q) \land (Q \rightarrow P)$ are equivalent.
- ~ 2 . Prove that there are 2^{2^n} boolean functions on *n*-variables.
- 3. Prove that a tree always has one fewer edge than vertices.
- 4. Define Euler and Hamiltonian paths.
- 5. How many 9 letter words can be formed that contain 3, 4 or 5 vowels allowing repetition of letters?
- 6. From a group of 10 professors how many ways a committee of 5 members be formed so that at least one of professors *A* and *B* be included.
- 7. Find the coefficient of X^{14} in $(1 + X + X^2 + X^3)^{10}$.
- 8. Solve the recurrence relation $a_n = a_{n-1} + n$, $a_0 = 2$ by substitution method.

Section B – (Marks : 4 × 15 = 60)

(a) (i) Analyze the following argument and then determine whether it is a valid argument.

"If I buy a new car then I will not be able to go to Delhi in December. Since I am going to Delhi in December, I will not buy a new car".

(b) (i) Find a particular solution to $a_{\rm c}$ **rO** $(+10a_{\rm c}) = 7.3^{9} +$

- (ii) Show that $p \lor (q \land r)$ is equivalent to $(p \lor \neg q) \lor \neg r$.
- (b) (i) In a boolean algebra with < + ordering, prove that a + b is the least upper bound of *a* and *b*, and *ab* is the greatest lower bound.

(ii) Construct a minimal switching circuit for the boolean expression.

xyzw + xyz'w + xyzw' + xyz'w + x'yz'w' + x'yzw' + xy'z'w' + x'y'z'w'

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- 10. (a) (i) Let G = (V, E) be a graph, where $V = \{a, b, c, d, e\}$,
 - $E = \{(a, b), (b, a), (a, c), (a, d), (b, c), (d, e)\}$. Draw representation of G. Find the adjacency matrix for G and determine the in-degree and out-degree of each vertex.
 - (ii) Explain why Dijkstra's algorithm is of no use in solving the travelling salesman problem.

Or

- (b) (i) Prove that every connected graph has at least one spanning tree.
 - (ii) Define a planar graph and show that $K_{3,3}$ is non-planar.
- - (ii) Find the number of integral solutions to $x_1 + x_2 + x_3 + x_4 = 50$ where $x_1 \ge -4$, $x_2 \ge 7$, $x_3 \ge -14$ and $x_4 \ge 10$.

Or

(b) (i) If A_i are finite subsets of a universal set U, then prove that

$$|A_1 \cup A_2 \cup \dots \cup A_n| = \sum_{i=1}^n |A_i| - \sum_{i,j} |A_i \cap A_j| + \sum_{i,j,k} |A_i \cap A_j \cap A_k| + \dots$$

$$(-1)^{n-1}|A_1 \cap A_2 \cap \dots \cap A_n|.$$

- (ii) In usual notation prove that $D_5 = 44$.
- 12. (a) Explain Fibonacci sequence of numbers. If F_n ($n \ge 2$) satisfies the Fibonacci relation then prove that there are constants C_1 and C_2 such that

$$F_{\rm n} = C_{\rm l} \left(\frac{1+\sqrt{5}}{2}\right)^{\rm n} + C_{\rm 2} \left(\frac{1-\sqrt{5}}{2}\right)^{\rm n}.$$

(b) (i) Find a particular solution to $a_n - 7a_{n-1} + 10a_{n-2} = 7.3^n + 4^n$.

(ii) Let $F = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, for $n \ge 0$, solve for the entries of F^n using recurrence relations.

 $|\mathbf{x}| \sim Show that p \vee [q \wedge r]$ is equivale $\mathbf{r} \mathbf{O}$ of p $\vee \neg q$ $| \vee \neg r$

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