JUNE 2008

Code: DE01 / DC01 **Subject: MATHEMATICS - I**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. Which term of the series 37+32+27+22+..... is –103?

(A) 24^{th} **(B)** 30^{th}
(C) 15^{th} **(D)** 29^{th}

b. How many terms are there in the expansion of $\left[(x-5y)^5\right]^3$

(A) 4 **(B)** 6
(C) 16 **(D)** 10

c. If $\sin\theta = \frac{-3}{5}$ and $\sec\theta = \frac{5}{4}$, find the value of cot θ

(A) $\frac{2}{3}$ **(B)** $\frac{-4}{3}$
(C) $\frac{5}{3}$ **(D)** $\frac{4}{5}$

d. Expansion of $5\sin\theta\sin 8\theta$ is equal to

(A) $\frac{5}{2}(\cos 7\theta - \cos 9\theta)$ **(B)** $5(\sin 7\theta + \sin 9\theta)$
(C) $10(\cos 7\theta + \cos 9\theta)$ **(D)** $\frac{2}{5}(\sin 7\theta - \cos 9\theta)$

e. For what value of k do the points $(-1,4), (-3,8)$ & $(-k+1, 3k)$ lie on a straight line.

(A) 3 **(B)** 4
(C) 0 **(D)** 1

f. Mid point of the line joining (3, 5) and $(-7,-3)$ is given by

(A) $(-2,1)$ **(B)** (1, 2)
(C) (2, 3) **(D)** (2, 1)

g. $\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}}$ is equal to

(A) $\sqrt{2}$ **(B)** $2\sqrt{2}$
(C) $3\sqrt{2}$ **(D)** $5\sqrt{2}$

h. If y = x sin x, then $\frac{dy}{dx}$ is equal to

(A) cos x + sin x **(B)** cos x + x sin x
(C) x cos x + sin x **(D)** x cos x – sin x

i. $\int \tan^2 x dx$ is equal to

(A) tan x + c **(B)** $\sec^2 x + c$
(C) $x + \tan x + c$ **(D)** $\tan x - x + c$

j. The solution of the differential equation $(1+y^2)dx + (1+x^2)dy = 0$ is

(A) (x + y) = k (1 – xy) **(B)** y – x = kxy
(C) $x^2 + y = kxy$ **(D)** $y + x = k$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. How many terms are there in a finite AP whose first and fifth terms are respectively –14 & 2 and the sum of terms is 40. **(8)**

b. The sum of three numbers in G.P. is $13/12$ and their product is –1. Find the numbers. **(8)**

Q.3 a. If $A + B + C = 180^\circ$, prove that

$$\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$ **(8)**

b. In any triangle ABC, prove that **(8)**

$$\frac{b^2 + c^2 - a^2}{a^2 + b^2 - c^2} = \frac{\tan C}{\tan A}$$

Q.4 a. The acute angle between two lines is $\frac{\pi}{4}$ and slope of one of them is $\frac{1}{2}$. Find the slope of the other line. **(8)**

b. Find the vertex, axis, focus, latus rectum and directrix of the parabola $x^2 + 2y - 3x + 5 = 0$. **(8)**

Q.5 a. Find the equation of the circle which passes through the points (1, 1) & (2, 2) & whose radius is 1. **(8)**

b. Find the equation of the straight line perpendicular to $7x + 9y - 3 = 0$ and passing through (3, 8) **(8)**

Q.6 a. Differentiate from the first principle the function $y = \sin 3x$. **(8)**

b. Evaluate $\lim_{x\to 0} \frac{\tan x - \sin x}{\sin^3 x}$. **(8)**

Q.7 a. Find the points of maxima or minima values of the function $y = x^3 - 18x^2 + 96x$. **(8)**

b. Evaluate $\int \frac{\sin 2x}{a\cos^2 x + b\sin^2 x}\,dx$. **(8)**

Q.8 a. Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$. **(8)**

b. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(8)**

Q.9 a. Solve $x^2 dy + y(x+y)dx = 0$. **(8)**

b. Solve $\frac{dy}{dx} + \frac{1}{x}y = x^3 - 3$. **(8)**