ALCCS

Code: CS41 **Subject: NUMERICAL & SCIENTIFIC COMPUTING**
Time: 3 Hours **Max. Marks: 100**

SEPTEMBER 2010

NOTE:

- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**
- **All calculations should be up to three places of decimals.**

Q.1 a. Find a real root of the equation $x\log_{10} x = 1.2$ by Regula-Falsi method correct to 4 decimal places.

b. Solve $x + 4y - z = -5;\ x + y - 6z = 12;\ 3x - y - z = 4$ using Gauss elimination method.

c. Find all the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{pmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$.

d. Find the missing terms in the following table

x	1	2	3	4	5	6	7
f(x)	103.4	97.6	122.9	?	179.0	?	195.8

e. Obtain the least square polynomial approximation of degree two for $f(x) = x^{1/2}$ on [0, 1].

f. Evaluate $\Delta^2\left(\dfrac{5x+12}{x^2+5x+16}\right)$.

g. Evaluate $\int_0^1 \dfrac{x}{1+x^2}\,dx$ by using the Simpson's $\left(3/8\right)^{th}$ rule, dividing the interval into 3 parts. Hence find an approximate value of log $\sqrt{2}$. **(7 × 4)**

Q.2 a. Solve the matrix equation

$$\begin{pmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

using the Cholesky method.

b. Solve the equations by relaxation method $9x - 2y + z = 50;\ \ x + 5y - 3z = 18;\ \ -2x + 2y + 7z = 19$. **(9+9)**

Q.3 a. Transform the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ to tridiagonal form by Given's method. Use exact arithmetic.

b. Find all the eigenvalues and the eigenvectors of the matrix A using Jacobi method where $A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$ **(8+10)**

Q.4 a. Find the cubic polynomial which takes the following values using Newton's interpolation and further evaluate f(4).

x	0	1	2	3
f(x)	1	2	1	10

b. Using Hermite interpolation determine the values of f(-0.5) and f(0.5) for the following given values of f(x) and $f'(x)$ **(9+9)**

x	f(x)	$f'(x)$
-1	1	-5
0	1	1
1	3	7

Q.5 a. A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given below for various values of the time t seconds. Find the velocity of the slider and its acceleration when t = 0.3 second.

t	0	0.1	0.2	0.3	0.4	0.5	0.6
x	30.13	31.62	32.87	33.64	33.95	33.81	33.24

b. The following table gives the temperature θ (in degree Celsius) of a cooling body at different instants of time t (in secs)

t	1	3	5	7	9
θ	85.3	74.5	67.0	60.5	54.3

Find approximately the rate of cooling at t = 8 secs **(9+9)**

Q.6 a. Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ by subdividing the interval [0, 1] into 2 equal parts and then applying the Gauss-Legendre three point formula.

b. Evaluate the integral $I = \int_{-\infty}^{\infty} \frac{e^{-x^2}}{x^2 + x + 1} dx$ using the Gauss-Hermite two point and three point formulas. **(9+9)**

Q.7 a. Solve the initial value problem $y' = (t/y), y(0) = 1$ using the Euler method with stepsize h=0.1 to find y(0.2).

b. Solve the initial value problem $y' = -2ty^2, y(0) = 1$ using fourth order Runge-Kutta method on the interval [0, 0.4] with stepsize h=0.2. Compare your result with the exact solution. **(8+10)**