Download From www.JbigDeaL.com Powered By © JbigDeaL

MATHEMATICAL SCIENCES

Paper – III

SECTION – I

Note : i) Answer both the questions.

ii) Each question carries twenty marks. $2 \times 20 = 40$

1. (a) Discuss the cubic spline interpolation by using Hermite cubic interpolant and apply it to find cos (3·14159) for free boundary conditions by using the following data :

***x* :**	0	1	3	3·5	5
cos *x* :	1	0·54030	– 0·98999	– 0·93646	0·28366

(b) Deduce the minimizing property of cubic splines.

OR

Show that a set $M \subset C[a, b]$ is compact in $C[a, b]$ if and only if the aggregate of functions $x(t) \in M$ are uniformly bounded and equi-continuous.

OR

Derive the steady-state equation of the multiserver Merkovian model (M/M/C) and obtain its solution.

2. A homogeneous solid sphere of radius R has the initial temperature distribution $f(r)$, $0 \le r \le R$, where r is the distance measured from the centre. The surface temperature is maintained at 0°. Show that the temperature $T(r, t)$ in the sphere is the solution of

$$T_t = c^2 \left(T_{rr} + \frac{2}{r} T_r \right)$$

where c^2 is a constant. Show that the temperature in the sphere for $t > 0$ is given by

$$T(r, t) = \frac{1}{r} \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{R} r \right) exp\left(- \lambda_n^2 t \right), \ \lambda_n = \frac{c\, n\pi}{R}.$$

OR

A rigid body is set rotating under no forces (moment of finite forces about the principal axes being zero) about its one point with angular velocity components $\omega_j = n$, $\omega_2 = 0$, $\omega_3 = n\sqrt{2}$ about the principal axes, respectively. If the respective principal moments are $4A$, $3A$ and A, respectively then discuss the ultimate motion.

OR

What is sampling distribution ? Derive non-central t-distribution.

[P.T.O.

Download From www.JbigDeaL.com Powered By © JbigDeaL

SECTION – II

Note : i) Answer all questions.

ii) Each question carries fifteen marks. $3 \times 15 = 45$

3. (a) Suppose the function $f(z)$ is analytic everywhere in a closed domain D, except at a finite number of isolated singularities z_k ($k = 1, 2, \ldots, n$) lying inside the domain D. Then show that

$$\int_{\Gamma^+} f(\rho)\, d\rho = 2\pi i \sum_{k=1}^{n} Res\,[\, f(z),\, z_k \,]$$

where Γ^+ is the complete boundary of domain D traversed in the positive direction and hence evaluate the integral

$$I = \int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}, \quad |\, a \,| < 1.$$

(b) Construct a function that maps the strip $0 < Re\, z < a$ conformally onto the upper half-plane $Im\,\omega > 0$.

OR

Find the Hamilton's canonical equations of motion of a particle of mass m moving in a force field of potential $V(\rho, \phi, z)$ in cylindrical polar co-ordinates (ρ, ϕ, z).

OR

Show that every Bernoulli sequence of *r.v.s.* obeys the weak law of large numbers.

4. Prove that the family M of Lebesgue measurable sets is an algebra.

OR

Give two examples of non-parametric tests. Discuss the exact and the limiting null distributions of the corresponding test statistics.

OR

Find the rate of convergence of Newton-Raphson method to find the root of an equation $f(x) = 0$.

Download From www.JbigDeaL.com Powered By © JbigDeaL

5. Show that the integral equation

$$y(x) = \int_0^x (x+t)\, y(t)\, dt + 1$$

is equivalent to the differential equation

$$y''(x) - 2xy'(x) - 3y(x) = 0$$

$$y(0) = 1, \quad y'(0) = 0.$$

OR

Suppose $X_1, X_2, \ldots, X_n$ is a random sample from Poisson distribution with parameter θ. The natural conjugate prior for θ is Gamma (α, β). Then obain the posterior density of θ.

OR

Using Ritz method based on the variational principle, show that the approximate solution of the boundary value problem

$$y'' + y = x, \quad y(0) = y(1) = 0$$

is $y = \frac{5}{18}\left(-x + x^2\right)$.

Download From www.JbigDeaL.com Powered By © JbigDeaL

SECTION – III

Note : i) Answer all questions.

ii) Each question carries ten marks. $9 \times 10 = 90$

6. Show that a finite integral domain is a field.

7. In a plane triangle, find the maximum value of $\cos A \cos B \cos C$.

8. Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$, using calculus of variation.

9. Define conformal mapping. What are essential conditions for conformal transformation ? Examine that following transformations are everywhere conformal or not and determine critical points :

   (i) $f(z) = (z-1)^2$

   (ii) $f(z) = \dfrac{z-i}{z+i}$.

10. Use Cayley-Hamilton theorem to find A^{-1}, where

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}.$$

11. Find the eigenvalue and eigenfunctions of the following homogeneous integral equation with degenerate kernels

$$y(x) = \lambda \int_0^1 \left(2xt - 4x^2\right) y(t)\, dt.$$

12. Explain the principle of likelihood ratio test.

13. Define a BIBD and state the situations in which such designs are used.

14. Give the circumstances under which systematic sampling is to be preferred to simple random sampling.

Download From www.JbigDeaL.com Powered By © JbigDeaL

SECTION – IV

Note : i) Answer all questions.

ii) Each question carries five marks. $5 \times 5 = 25$

15. If the vectors (0, 1, a), (1, a, 1), (a, 1, 0) of the vector space $R^3(R)$ be linearly dependent, then find the value of a.

16. If the function $f(z) = \frac{iz}{2}$ is defined on the open disk $|z| < 1$, show that $\lim_{z \to 1} f(z) = \frac{i}{2}$, the point $z = 1$ being on the boundary of definition.

17. Find the general solution of

$$(x^2+1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 6(x^2+1)^2,$$

given that $y = x$ and $y = x^2 - 1$ are linearly independent solutions of corresponding homogeneous equation.

18. Find the curve for which the surface of revolution is minimum.

19. There are two identical urns containing respectively 4 white, 3 red balls and 3 white, 7 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white. If the ball drawn is white, what is the probability that it is from the first urn ?